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Multi-item inventory model using hexagonal fuzzy number

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Abstract

This paper explores a fuzzy multi-item inventory model with alternative power supply costs. The cost parameters and the constraint are represented by a hexagonal fuzzy numbers. This model is worked out through fuzzy non − linear programming technique. The optimal lot size and backlog quantity are determined. A numerical example with sensitivity analysis are given to illustrate the model.

Keywords: Inventory, hexagonal fuzzy number, Karush–Kuhn –Tucker necessary condition.

AMS Subject Classification(2010): 90B05.

1 Introduction

Inventory planning and control is concerned with the acquisition and storage of materials required for supporting various business operations. In classical model such as EOQ and EPQ developed by Harris [5] and Taft [15], it is assumed that the rate of replenishment / production and unit price of an item are constant. But later, theorists worked out more comprehensive, realistic market − sensitive models. Multi −item classical inventory models under various types of constraints such as capital investment, available storage area, number of orders and available setup time are presented in well known books by Hadley and Whitin[4], Silver and Peterson [13], Taha[16]. Cheng[3] related EOQ model with demand dependent unit cost using geometric programming technique. Pureto et. al[9] considered inventory models in both constrained and unconstrained situation.

While modeling an inventory problem, it is assumed that demand and various relevant costs are defined with certainty. But, in real life, demand and various relevant costs are not exactly known. In this situation, uncertainties are treated as randomness and are handled through probability theory. In certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory, originally introduced by Zadeh[19] may be applied. After Bellman and Zadeh[2] initiated fuzzy optimization through aggregation operations that combine fuzzy goals and fuzzy − decision space. There have been many attempts to apply fuzzy set theory in inventory management. Later, the fuzzy linear programming model was formulated and an approach for solving problem in linear programming model with fuzzy numbers has been presented by Zimmermann[20]. Park[8] examined the EOQ formula to inventory management from a fuzzy set theoretical perspective associating the fuzziness with cost data. The Economic Production Quantity(EPQ) model seeks to tackle the storage space constraint in a fuzzy environment by using geometric programming technique. Such a model was developed by Sahidul Islam et. al[12].

A multi − item inventory model was formulated in the fuzzy environment involving budgetary constraints, using fuzzy non-linear techniques. An EPQ model with demand dependent unit production cost in a fuzzy environment has been developed by Mahapatra et.al[7]. Roy and Maiti[11] have rewritten the classical economic order quantity problem using a fuzzy nonlinear programming technique. A fuzzy inventory model with quantity dependent unit production cost seen as inversely related to available storage space is tackled by the penalty function method considered by Sudipta sinha[14]. Detailed reviews on multi − item inventory models with constraints are given by Arun Prasnth et.al[1], Kasthuri.et.al[6], Ranganayaki.et.al[10]. More recently, using the Karush-Kuhn Tucker method Vasanthi et.al[17],[18] worked out a model that could tackle factors such as limited storage space, low investment and the like along similar lines.

In recent time, power scarcity affects the large scale industries. To solve this problem, solar plants are installed by industries and it incurs an additional cost. This paper introduces the cost as alternative power supply cost. In this study a multi-item inventory model using a hexagonal fuzzy number is considered by including the additional operating and maintenance cost due to the solar plants which is directly proportional to the demand.

The parameters involved in this paper are assumed to be imprecise in nature and the parameters are represented by hexagonal fuzzy number with left and right membership functions. The model is solved by fuzzy non-linear programming technique. Finally a numerical example and sensitivity analysis are given to illustrate the model.

2 Assumptions and Notations

A multi-item inventory model with solar plants and their operating and maintenance costs is formulated under the following assumptions and notations.

Assumptions:

1. Production rate is finite.
2. Shortages are treated as fully backlogged.

3. Lead time is Zero.
4. Time horizon is infinite.
5. Alternative power supply (solar plants, their operating and maintenance) costs are allowed and such costs depend upon demand.(ie,$S_{e_i} = D_i^{1-a}, 0 < a < 1$).

Notations:

Let there be n items in the inventory. The following notations are used for the i^{th} item ($i = 1, 2, 3 \cdots, n$).

D_i	–	demand.
Q_i	–	optimal lot size per cycle.
M_i	–	backlog quantity per cycle.
S_E	–	fixed solar plants cost for plan period.
S_{e_i}	–	operating and maintenance cost of solar plants per cycle.
k_i	–	production rate.
d_i	–	demand rate.
B	–	maximum investment cost.
α	–	aspiration Level.
$\tilde{s}_i$	–	fuzzy setup cost per cycle.
$\tilde{H}_i$	–	fuzzy holding cost per unit per unit time.
$\tilde{m}_i$	–	fuzzy backlogged cost per unit per unit time.
$\tilde{p}_i$	–	fuzzy production cost per unit per unit time.
p_{i0}	–	tolerance limit for production cost, similarly to other parameters.
$TC(Q_i, M_i)$	–	Average total cost.
$\lambda, \lambda_1, \lambda_2$	–	Lagrange multipliers.

3 Mathematical Model in Crisp Environment

The inventory model is formulated to minimize the average total cost under the limited investment constraint.

Total cost = Production cost + Setup cost + Holding cost + Backlogged cost+Solar plants operating and maintenance cost+Solar plants cost.

Minimum average total cost is given by,

$$TC(Q_i, M_i) = \sum_{i=1}^{n} \left[p_i D_i + \frac{s_i D_i}{Q_i} + \frac{H_i k_i (\frac{k_i - d_i}{k_i} Q_i - M_i)^2}{2Q_i(k_i - d_i)} + \frac{m_i k_i M_i^2}{2Q_i(k_i - d_i)} + \frac{s_{e_i} D_i}{Q_i} \right] + S_E$$

Investment amount on total production cost cannot be infinite,it may have an upper limit on the maximum investment.That is, $\sum_{i=1}^{n} p_i Q_i \leqslant B$.

The problem is then stated as

Minimize $TC(Q_i, M_i)$

$$\text{subject to : } \sum_{i=1}^{n} p_i Q_i \leqslant B \qquad \cdots (1)$$

$$Q_i, M_i \geqslant 0$$

The Lagrange function is

$$L(Q_i, M_i, \lambda) = \begin{Bmatrix} \left[\sum_{i=1}^{n}\left(p_i D_i + \frac{s_i D_i}{Q_i} + \frac{H_i k_i (\frac{k_i - d_i}{k_i} Q_i - M_i)^2}{2Q_i(k_i - d_i)} + \frac{m_i k_i M_i^2}{2Q_i(k_i - d_i)} + \frac{s_{e_i} D_i}{Q_i}\right) + S_E\right] \\ +\lambda\left[\sum_{i=1}^{n} p_i Q_i - B\right] \end{Bmatrix} \qquad \cdots (2)$$

By using necessary Kuhn-Tucker conditions in equation(2), the analytical expressions for the decision variables Q_i and M_i are obtained.

$$Q_i = \frac{(H_i + m_i)k_i}{H_i(k_i - d_i)} M_i \qquad \cdots (3)$$

and

$$M_i = \sqrt{\frac{2(\frac{H_i}{H_i + m_i})^2(\frac{k_i - d_i}{k_i})^2(s_i D_i + D_i^{2-a})}{(\frac{H_i m_i}{H_i + m_i})(\frac{k_i - d_i}{k_i}) + 2\lambda p_i}} \qquad \cdots (4)$$

4 Hexagonal fuzzy number and its properties

A Hexagonal fuzzy number $\tilde{A}$ is described as a fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for } x \leq (a_i - a_{i0}) \\ W_A\left(\frac{x - (a_i - a_{i0})}{a_{i0}}\right) & \text{for } (a_i - a_{i0}) \leq x \leq a_i \\ W_A + (1 - W_A)\left(\frac{x - a_i}{a_{i0}}\right) & \text{for } a_i \leq x \leq (a_i + a_{i0}) \\ 1 & \text{for } (a_i + a_{i0}) \leq x \leq (a_{i1} - a_{i0}) \\ W_A + (1 - W_A)\left(\frac{a_{i1} - x}{a_{i0}}\right) & \text{for } (a_{i1} - a_{i0}) \leq x \leq a_{i1} \\ W_A\left(\frac{(a_{i1} + a_{i0}) - x}{a_{i0}}\right) & \text{for } a_{i1} \leq x \leq (a_{i1} + a_{i0}) \\ 0 & \text{for } x \geq (a_{i1} + a_{i0}) \end{cases}$$

where $0.6 \leq W_A < 1$, a_{i0}, a_i and a_{i1} are real numbers.

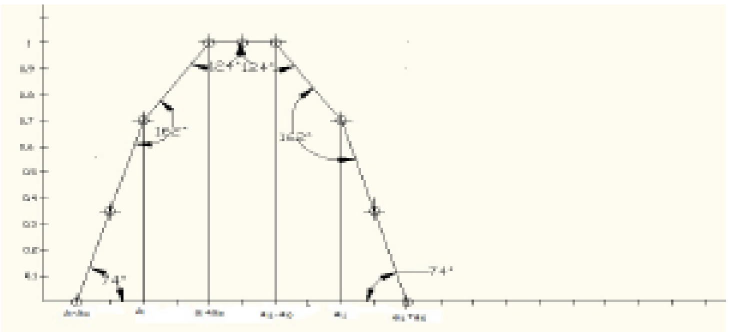

Figure 1: Graphical representation of hexagonal fuzzy number.

This type of fuzzy number is denoted by
$\tilde{A} = (a_i - a_{i0}, a_i, a_i + a_{i0}, a_{i1} - a_{i0}, a_{i1}, a_{i1} + a_{i0}; W_A)_{HFN}$
$L(x) = \min(\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^R}(x))$
$R(x) = \max(\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^R}(x))$

$\mu_{\tilde{A}}$ **satisfies the following conditions:**

1. $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval [0,1].
2. $\mu_{\tilde{A}}$ is a convex function.
3. $\mu_{\tilde{A}} = 0, -\infty < x \leq (a_i - a_{i0})$.
4. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $(a_i - a_{i0}, a_i + a_{i0})$.
5. $\mu_{\tilde{A}} = 1, x = a_i + a_{i0}, a_{i1} - a_{i0}$.
6. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $(a_{i1} - a_{i0}, a_{i1} + a_{i0})$.
7. $\mu_{\tilde{A}}(x) = 0, (a_{i1} + a_{i0}) \leq x < \infty$.

Remark 4.1. If $0 < W_A < 0.6$, then $\tilde{A}$ becomes a trapezoidal fuzzy number.

5 The proposed inventory model in Fuzzy Environment

If the cost parameters are fuzzy number, then the problem (1) is transformed to

$$\mathrm{Min}\tilde{TC}(Q_i, M_i) = \left\{ \sum_{i=1}^{n} \left(\tilde{p}_i D_i + \frac{\tilde{s}_i D_i}{Q_i} + \frac{\tilde{H}_i k_i (\frac{k_i - d_i}{k_i} Q_i - M_i)^2}{2Q_i(k_i - d_i)} + \frac{\tilde{m}_i k_i {M_i}^2}{2Q_i(k_i - d_i)} + \frac{S_{ei} D_i}{Q_i} \right) + \tilde{S_E} \right\}$$

$$\text{subject to :} \qquad \sum_{i=1}^{n} \tilde{p}_i Q_i \leq \tilde{B}; \qquad Q_i, M_i \geq 0 \qquad \cdots (5)$$

where $\sim$ represents the fuzzification of the parameters.
In the proposed model, the cost parameters p_i, s_i, H_i, m_i and S_E are considered as hexagonal fuzzy numbers.
$\tilde{p_i} = (p_i - p_{i0}, p_i, p_i + p_{i0}, p_{i1} - p_{i0}, p_{i1}, p_{i1} + p_{i0})$
$\tilde{s_i} = (s_i - s_{i0}, s_i, s_i + s_{i0}, s_{i1} - s_{i0}, s_{i1}, s_{i1} + s_{i0})$

$\tilde{H_i} = (H_i - H_{i0}, H_i, H_i + H_{i0}, H_{i1} - H_{i0}, H_{i1}, H_{i1} + H_{i0})$
$\tilde{m_i} = (m_i - m_{i0}, m_i, m_i + m_{i0}, m_{i1} - m_{i0}, m_{i1}, m_{i1} + m_{i0})$
$\tilde{S_{Ei}} = (S_{Ei} - S_{Ei0}, S_{Ei}, S_{Ei} + S_{Ei0}, S_{Ei1} - S_{Ei0}, S_{Ei1}, S_{Ei1} + S_{Ei0})$

The corresponding fuzzy non-linear programming problem is
Max α

subject to :

$$\left\{\begin{array}{l}\sum_{i=1}^{n}\left[\mu_{p_i}^{-1}(\alpha)D_i + \frac{\mu_{s_i}^{-1}(\alpha)D_i}{Q_i} + \frac{\mu_{H_i}^{-1}(\alpha)k_i(\frac{k_i-d_i}{k_i}Q_i - M_i)^2}{2Q_i(k_i-d_i)} + \frac{\mu_{m_i}^{-1}(\alpha)k_iM_i^2}{2Q_i(k_i-d_i)} + \frac{D_i^{2-a}}{Q_i}\right] \\ +\mu_{SE}^{-1}(\alpha) \leq \mu_{TC}^{-1}(\alpha)(Q_i, M_i)\end{array}\right\}$$

$$\sum_{i=1}^{n}\mu_{p_i}^{-1}(\alpha)Q_i \leq \mu_B^{-1}(\alpha)$$

Consider left membership function of hexagonal fuzzy number $\tilde{p}_i$

$$\mu_{\tilde{p}_i{}^L}(x) = \begin{cases} 0 & \text{for } x \leq (p_i - p_{i0}) \\ W_A\left(\dfrac{x-(p_i-p_{i0})}{p_{i0}}\right) & \text{for } (p_i - p_{i0}) \leq x \leq p_i \\ W_A + (1-W_A)\left(\dfrac{x-p_i}{p_{i0}}\right) & \text{for } p_i \leq x \leq (p_i + p_{i0}) \\ 1 & \text{for } (p_i + p_{i0}) \leq x \end{cases}$$

Similarly, left membership functions are defined for $\tilde{s}_i$, $\tilde{H}_i$, $\tilde{m}_i$ and $\tilde{S_E}$.

Consider right membership function of hexagonal fuzzy number $\tilde{B}$

$$\mu_{\tilde{B}^R}(x) = \begin{cases} 1 & \text{for } x \leq (B - B_0) \\ W_A + (1-W_A)\left(\dfrac{B-x}{B_0}\right) & \text{for } (B - B_0) \leq x \leq B \\ W_A\left(\dfrac{(B+B_0)-x}{B_0}\right) & \text{for } B \leq x \leq (B + B_0) \\ 0 & \text{for } (B + B_0) \leq x \end{cases}$$

Similarly right membership function can be defined for $\tilde{TC}$.

From the above equations,

$\mu^{-1}_{\tilde{p_i}^L}(\alpha) = \frac{1}{2}\left(2p_i - p_{i0} + \frac{p_{i0}}{W_A}\alpha + \frac{p_{i0}(\alpha - W_A)}{1 - W_A}\right)$ and

$\mu_{\tilde{B^R}^{-1}}(\alpha) = \frac{1}{2}\left(2B + B_0 - \frac{B_0}{W_A}\alpha - \frac{B_0(\alpha - W_A)}{1 - W_A}\right)$ similarly for $\mu^{-1}_{\tilde{TC}^R}(\alpha)$.

The Lagrangian function becomes

$L(\alpha, Q_i, M_i, \lambda_1, \lambda_2)$=

$$\left\{\begin{array}{l} \alpha - \lambda_1 \left[\begin{array}{l} \sum_{i=1}^{n} \left[\begin{array}{l} \frac{1}{2} D_i \left(2p_i - p_{i0} + \frac{p_{i0}}{W_A}\alpha + \frac{p_{i0}(\alpha - W_A)}{1 - W_A}\right) \\ + \frac{\frac{1}{2}\left(2s_i - s_{i0} + \frac{s_{i0}}{W_A}\alpha + \frac{s_{i0}(\alpha - W_A)}{1 - W_A}\right) D_i}{Q_i} \\ + \frac{k_i\left(\frac{k_i - d_i}{k_i}Q_i - M_i\right)^2 \frac{1}{2}\left(2H_i - H_{i0} + \frac{H_{i0}}{W_A}\alpha + \frac{H_{i0}(\alpha - W_A)}{1 - W_A}\right)}{2Q_i(k_i - d_i)} \\ + \frac{\frac{1}{2}\left(2m_i - m_{i0} + \frac{m_{i0}}{W_A}\alpha + \frac{m_{i0}(\alpha - W_A)}{1 - W_A}\right) k_i M_i^2}{2Q_i(k_i - d_i)} + \frac{D_i^{2-a}}{Q_i} \end{array} \right] \\ + \frac{1}{2}\left(2S_E - S_{E0} + \frac{S_{E0}}{W_A}\alpha + \frac{S_{E0}(\alpha - W_A)}{1 - W_A}\right) \\ - \frac{1}{2}\left(2TC + TC_0 - \frac{TC_0}{W_A}\alpha - \frac{TC_0(\alpha - W_A)}{1 - W_A}\right) \end{array} \right] \\ - \lambda_2 \left[\begin{array}{l} \sum_{i=1}^{n} \frac{1}{2}\left(2p_i - p_{i0} + \frac{p_{i0}}{W_A}\alpha + \frac{p_{i0}(\alpha - W_A)}{1 - W_A}\right) Q_i \\ - \frac{1}{2}\left(2B + B_0 - \frac{B_0}{W_A}\alpha - \frac{B_0(\alpha - W_A)}{1 - W_A}\right) \end{array} \right] \end{array}\right\} \cdots (6)$$

By using necessary Kuhn-Tucker conditions in equation(6), the analytical expressions for the decision variables Q_i and M_i are obtained.

$$M_i = (\frac{k_i - d_i}{k_i}) \left[\frac{\left(2H_i - H_{i0} + \dfrac{H_{i0}}{W_A}\alpha + \dfrac{H_{i0}(\alpha - W_A)}{1 - W_A}\right)}{\left(2H_i - H_{i0} + \dfrac{H_{i0}}{W_A}\alpha + \dfrac{H_{i0}(\alpha - W_A)}{1 - W_A} + 2m_i - m_{i0} + \dfrac{m_{i0}}{W_A}\alpha + \dfrac{m_{i0}(\alpha - W_A)}{1 - W_A}\right)} \right] Q_i \quad \cdots (7)$$

and

$$Q_i = \sqrt{\frac{\lambda_1 \left[(2H_i - H_{i0} + \dfrac{H_{i0}}{W_A}\alpha + \dfrac{H_{i0}(\alpha - W_A)}{1 - W_A}) + (2m_i - m_{i0} + \dfrac{m_{i0}}{W_A}\alpha + \dfrac{m_{i0}(\alpha - W_A)}{1 - W_A}) \right] \left[D_i^{2-a} + D_i \left(\dfrac{1}{2} (2s_i - s_{i0} + \dfrac{s_{i0}}{W_A}\alpha + \dfrac{s_{i0}(\alpha - W_A)}{1 - W_A}) \right) \right]}{\lambda_1 \left[\dfrac{1}{4} (2H_i - H_{i0} + \dfrac{H_{i0}}{W_A}\alpha + \dfrac{H_{i0}(\alpha - W_A)}{1 - W_A})(2m_i - m_{i0} + \dfrac{m_{i0}}{W_A}\alpha + \dfrac{m_{i0}(\alpha - W_A)}{1 - W_A}) \left(\dfrac{k_i - d_i}{k_i} \right) \right] + \lambda_2 \left[\dfrac{1}{2} \left(2p_i - p_{i0} + \dfrac{p_{i0}}{W_A}\alpha + \dfrac{p_{i0}(\alpha - W_A)}{1 - W_A} \right) \right] \left[(2H_i - H_{i0} + \dfrac{H_{i0}}{W_A}\alpha + \dfrac{H_{i0}(\alpha - W_A)}{1 - W_A}) + (2m_i - m_{i0} + \dfrac{m_{i0}}{W_A}\alpha + \dfrac{m_{i0}(\alpha - W_A)}{1 - W_A}) \right]}} \quad \cdots (8)$$

where α is the root of the equation

$(a_1a_4 + a_8a_{12})\alpha^5 + (a_1a_5 + a_2a_4 + a_8a_{13} + a_9a_{12})\alpha^4 + (a_1a_6 + a_2a_5 + a_3a_4 + a_8a_{14} + a_9a_{13} + a_{10}a_{12})\alpha^3 + (a_1a_7 + a_2a_6 + a_3a_5 + a_9a_{14} + a_{10}a_{13} + a_{11}a_{12})\alpha^2 + (a_2a_7 + a_3a_6 + a_{10}a_{14} + a_{11}a_{13})\alpha + (a_3a_7 + a_{11}a_{14}) = 0$

where

$a_1 = p_{i0}(H_{i0} + m_{i0})$

$a_2 = \left[(2p_i - p_{i0})W_A - 2p_iW_A{}^2)(H_{i0} + m_{i0})\right]p_{i0}\left[(2H_i - H_{i0} + 2m_i - m_{i0})W_A - (2H_i + 2m_i)W_A{}^2\right]$

$a_3 = \left[(2p_i - p_{i0})W_A - 2p_iW_A{}^2\right]\left[(2H_i - H_{i0})W_A - 2H_iW_A{}^2 + (2m_i - m_{i0}W_A - 2m_iW_A{}^2)\right]$

$$a_4 = 2(H_{i0} + m_{i0})[D_i p_{i0}(-B_0 + s_{i0}) - B_0(TC_0 + S_{E0})]$$

$$a_5 = \Big[(2p_i - p_{i0})W_A - 2p_i {W_A}^2][2D_i(H_{i0} + m_{i0})(-B_0 + s_{i0})\Big] + \Big[(2TC + TC_0 - 2S_E + S_{E0})W_A - (2TC - 2S_E){W_A}^2][2B_0(H_{i0} + m_{i0})\Big] + \Big[(2B + B_0)W_A - 2B{W_A}^2][2(H_{i0} + m_{i0})(D_i p_{i0} + (TC_0 + S_{E0}))\Big] + \Big[2D_i p_{i0}(-B_0 + s_{i0}) - 2B_0(TC_0 + S_{E0})\Big]\Big[(2H_i - H_{i0})W_A - 2H_i{W_A}^2 + (2m_i - m_{i0})W_A - 2m_i{W_A}^2\Big] + \Big[((2s_i - s_{i0})W_A - 2s_i{W_A}^2)(2p_{i0}D_i(H_{i0} + m_{i0}))\Big] + 4D_i^{2-a}W_A(1 - W_A)p_{i0}(H_{i0} + m_{i0})$$

$$a_6 = \Big[(2p_i - p_{i0})W_A - 2p_i{W_A}^2\Big]\Big[\Big((2B + B_0)W_A - 2B{W_A}^2\Big)2D_i(H_{i0} + m_{i0})\Big((-2D_iB_0)((2H_i - H_{i0})W_A - 2H_i{W_A}^2 + (2m_i - m_{i0})W_A - 2m_i{W_A}^2)\Big)\Big]$$

$$a_7 = \Big[(2H_i - H_{i0})W_A - 2H_i{W_A}^2 + (2m_i - m_{i0})W_A - 2m_i{W_A}^2\Big]\Big[((2p_i - p_{i0})W_A - 2p_i{W_A}^2)\big(2D_i((2B + B_0)W_A - 2B{W_A}^2) + 2D_i((2s_i - s_{i0})W_A - 2s_i{W_A}^2) + 4D_i^{2-a}W_A(1 - W_A)\big) - 2((2B + B_0)W_A - 2B{W_A}^2)((2TC + TC_0 - 2S_E + S_{E0})W_A - (2TC - 2S_E){W_A}^2)\Big]$$

$$a_8 = -m_{i0}H_{i0}B_0\left(\frac{k_i - d_i}{k_i}\right)$$

$$a_9 = \left(\frac{k_i - d_i}{k_i}\right)\Big[m_{i0}H_{i0}\Big((2B + B_0)W_A - 2B{W_A}^2\Big) - B_0H_{i0}((2m_i - m_{i0})W_A - 2m_i{W_A}^2) - B_0m_{i0}((2H_i - H_{i0})W_A - 2H_i{W_A}^2)\Big]$$

$$a_{10} = \left(\frac{k_i - d_i}{k_i}\right)\Big[((2B + B_0)W_A - 2B{W_A}^2)\Big(H_{i0}((2m_i - m_{i0})W_A - 2m_i{W_A}^2) + m_{i0}((2H_i - H_{i0})W_A - 2H_i{W_A}^2)\Big) - B_0((2H_i - H_{i0})W_A - 2H_i{W_A}^2)((2m_i - m_{i0})W_A - 2m_i{W_A}^2)\Big]$$

$$a_{11} = \left(\frac{k_i - d_i}{k_i}\right)\Big((2B + B_0)W_A - 2B{W_A}^2\Big)\Big((2H_i - H_{i0})W_A - 2H_i{W_A}^2\Big)\Big((2m_i - m_{i0})W_A - 2m_i{W_A}^2\Big)$$

$$a_{12} = -B_0(m_{i0} + H_{i0})$$

$$a_{13} = \Big[((2B + B_0)W_A - 2B{W_A}^2)(m_{i0} + H_{i0}) - B_0\Big((2m_i - m_{i0})W_A - 2m_i{W_A}^2 + (2H_i - H_{i0})W_A - 2H_i{W_A}^2\Big)$$

$$a_{14} = \Big[\Big((2B + B_0)W_A - 2B{W_A}^2\Big)\Big((2m_i - m_{i0})W_A - 2m_i{W_A}^2 + (2H_i - H_{i0})W_A - 2H_i{W_A}^2\Big)\Big]$$

6 Numerical Example

A computer parts manufacturing company produces two items. The relevant data for the two items are given below:

$k_1 = 3400$ units,	$d_1 = 2900$ units,	$D_1 = 30000$ units	$p_1 = \$\ 11$,	$H_1 = \$\ 8$,
$s_1 = \$\ 700$,	$m_1 = \$\ 8$	$p_{10} = \$\ 2$	$H_{10} = \$\ 1$,	$s_{10} = \$\ 50$,
$m_{10} = \$\ 1$,	$k_2 = 2700$ units,	$d_2 = 2200$ units,	$D_2 = 24000$ units,	$p_2 = \$\ 14$,

$H_2 = \$ 6$, $s_2 = \$ 700$, $m_2 = \$ 6$, $p_{20} = \$ 2$, $H_{20} = \$1$,
$s_{20} = \$ 50$, $m_{20} = \$ 1$, B= \$ 1000000, $B_0 = \$ 5000$, $S_{E0} = \$ 300$,
$S_E = \$10000$, TC = \$ 800000 $TC_0 = \$ 1000$, $W_A = 0.7$.

Using the analytical expressions (3), (4), (7) and (8) for Q_i, M_i and $TC(Q_i, M_i)$ in a crisp and fuzzy environment, the following results are obtained.

Table 1: Solution for Crisp Model.

a	Item	Q_i^*(units)	M_i^*(units)	Total cost for the item($)	Total quantity (units)	Minimum average total cost($)
0.3	1	1853	136	368910	3519	742250
	2	1400	130	373340		
0.5	1	1206	89	357070	2331	719970
	2	948	88	362900		
0.7	1	1097	81	355070	2130	716180
	2	871	81	361110		

Table 2: Solution for Fuzzy Model.

a	Item	α	Q_i^*(units)	M_i^*(units)	Total cost for the item($)	Total quantity (units)	Minimum average total cost($)
0.3	1	0.9283	2016	148	399000	4160	798000
	2	0.9831	1827	169	399000		
0.5	1	0.9314	1323	97	389000	2785	780000
	2	0.9863	1249	116	391000		
0.7	1	0.9317	1207	89	387000	2554	777000
	2	0.9867	1151	107	390000		

Table 3: Comparison table between Crisp and Fuzzy Model.

Model	Item	Q_i^*(units)	M_i^*(units)	Total quantity for the item(units)	Total cost for the item($)	Expenditure of one unit ($)
Crisp	1	1853	136	1989	368910	185
(a=0.3)	2	1400	130	1530	373340	244
Fuzzy	1	2016	148	2164	399000	184
(a=0.3)	2	1827	169	1996	399000	200

Sensitivity Analysis

By giving variations in the tolerance limit for total cost the sensitivity analysis on the optimum lot size, back logged quantity and other costs are worked out. The results of this analysis are tabulated below.

Table 4: Effect of variation in TC_0 : $(a = 0.3)$.

TC_0	$\tilde{p}_i^*$	$\tilde{H}_i^*$	$\tilde{m}_i^*$	$\tilde{s}_i^*$	α	$Q_i{}^*$	$M_i{}^*$	$TC^*(Q_i, M_i)$
1000	12.0871	8.5436	8.5436	727.1786	0.9283	2016	148.20	399000
2000	12.0695	8.5348	8.5348	726.7381	0.9246	2017	148.30	399000
3000	12.0524	8.5262	8.5262	726.3095	0.9210	2018	148.39	398000
4000	12.0357	8.5179	8.5179	725.8929	0.9175	2019	148.47	398000
5000	12.0195	8.5098	8.5098	725.4881	0.9141	2020	148.56	397000
6000	12.0038	8.5019	8.5019	725.0952	0.9108	2022	148.64	397000
7000	11.9886	8.4943	8.4943	724.7143	0.9076	2023	148.72	396000
8000	11.9738	8.4869	8.4869	724.3452	0.9045	2024	148.80	396000
9000	11.9595	8.4798	8.4798	723.9881	0.9015	2025	148.88	395000
10000	11.9457	8.4729	8.4729	723.6429	0.8986	2026	148.95	395000

Similarly sensitivity analysis can be done for variations in the tolerance limits of production cost, holding cost, backlogged cost and setupcost respectively.

Observations:

From Table (1) and (2), it is noted that $'a'$ increases where as Q_i^*, M_i^* and minimum average total cost decrease in crisp and fuzzy environment.

From Table (3) ,it should be noted that the fuzzy model is better than the crisp model in the sense that

- the optimum lot size and backlogged level obtained in the fuzzy model are greater than that of the crisp model.
- cost of one unit in the fuzzy model is less than that in the crisp model.

From Table (4), it should be noted that as TC_0 increases,

- α^* decreases but it never becomes 0 as is expected.
- Q_i^* and M_i^* remain increasing.
- production cost, holding cost, backlogged cost and setupcost are decreasing.

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