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Some new sets using soft semi $^{\#}g\alpha$-closed sets in soft topological spaces

V. Kokilavani[1], M. Vivek Prabu[2]

[1]Department of Mathematics
Kongunadu Arts and Science College (Autonomous)
Coimbatore-641029, Tamilnadu, India.
vanikasc@yahoo.co.in

[2] Research Scholar
Department of Mathematics
Kongunadu Arts and Science College (Autonomous)
Coimbatore-641029, Tamilnadu, India.
kavithai.vivek@yahoo.in

Abstract

In this paper, we define some new sets namely soft semi $^{\#}g\alpha$-border, soft semi $^{\#}g\alpha$-frontier and soft semi $^{\#}g\alpha$-exterior which are denoted by $\widetilde{s}$semi $^{\#}g\alpha$-bd(F,A), $\widetilde{s}$semi $^{\#}g\alpha$-fr(F,A) and $\widetilde{s}$semi $^{\#}g\alpha$-ext(F,A), where (F,A) is any soft set of (X,E) and also investigate their basic properties.

Keywords: soft semi $^{\#}g\alpha$-closed set, soft semi $^{\#}g\alpha$-open set, soft semi $^{\#}g\alpha$-closure, soft semi $^{\#}g\alpha$-interior, soft semi $^{\#}g\alpha$-border, soft semi $^{\#}g\alpha$-frontier and soft semi $^{\#}g\alpha$-exterior.
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1 Introduction

Kokilavani and Vivek Prabu [6] introduced the concepts of semi $^{\#}g\alpha$-closed sets, semi $^{\#}g\alpha$-continuous functions and semi $^{\#}g\alpha$-irresolute functions in topological spaces. We defined and examined the basic properties of semi $^{\#}g\alpha$-border, semi $^{\#}g\alpha$-frontier and semi $^{\#}g\alpha$-exterior in general topological spaces [7]. In this paper, we define these sets in soft topological spaces and study their properties.

2 Preliminaries

Definition 2.1. A soft set (F,A) of a soft topological space (X,$\widetilde{\tau}$,E) is called

(i) soft α-closed [4] if $\widetilde{s}$cl($\widetilde{s}$int($\widetilde{s}$cl(F,A))) $\widetilde{\subseteq}$ (F,A). The complement of soft α-closed set is called soft α-open.

(ii) soft semi-closed [2] if $\widetilde{s}$int($\widetilde{s}$cl(F,A))$\widetilde{\subseteq}$ (F,A). The complement of soft semi-closed set is called soft semi-open.

(iii) soft g-closed [5] if $\widetilde{s}$cl(F,A) $\widetilde{\subseteq}$ (U,E), whenever (F,A) $\widetilde{\subseteq}$ (U,E) and (U,E) is soft open in (X,τ,E). The complement of soft g-closed set is called soft g-open.

(iv) Soft $g^{\#}\alpha$-closed [10] if $\widetilde{s}\alpha$cl(F,A) $\widetilde{\subseteq}$ (U,E) whenever (F,A) $\widetilde{\subseteq}$ (U,E) and (U,E) is soft g-open in (X,τ,E). The complement of soft $g^{\#}\alpha$-closed set is called soft $g^{\#}\alpha$-open.

(v) soft $^{\#}g\alpha$-closed [9] if $\widetilde{s}\alpha$cl(F,A) $\widetilde{\subseteq}$ (U,E), whenever (F,A) $\widetilde{\subseteq}$ (U,E) and (U,E) is soft $g^{\#}\alpha$-open in (X,τ,E). The complement of soft $^{\#}g\alpha$-closed set is called soft $^{\#}g\alpha$-open.

(vi) soft semi $^{\#}g\alpha$-closed [8] if $\widetilde{s}$scl(F,A) $\widetilde{\subseteq}$ (U,E), whenever (F,A) $\widetilde{\subseteq}$ (U,E) and (U,E) is soft $^{\#}g\alpha$-open in (X,τ,E). The complement of soft semi $^{\#}g\alpha$-closed set is called soft semi $^{\#}g\alpha$-open.

The union (resp. intersection) of all soft semi $^{\#}g\alpha$-open (resp. soft semi $^{\#}g\alpha$-closed) sets, each contained in (resp. containing) a set (F,A) of (X,$\widetilde{\tau}$,E) is called soft semi $^{\#}g\alpha$-interior (resp. soft semi $^{\#}g\alpha$-closure) of (F,A), which is denoted by $\widetilde{s}$semi $^{\#}g\alpha$-int(F,A) (resp.$\widetilde{s}$semi $^{\#}g\alpha$-cl(F,A)).

Theorem 2.2. If (F,A) and (G,B) are soft subsets of (X,E), then

(i) (F,A) is soft semi $^{\#}g\alpha$-open if and only if soft semi $^{\#}g\alpha$-int(F,A)$\widetilde{=}$(F,A).

(ii) soft semi $^{\#}g\alpha$-int(F,A) is soft semi $^{\#}g\alpha$-open.

(iii) (F,A) is soft semi $^{\#}g\alpha$-closed if and only if soft semi $^{\#}g\alpha$-cl(F,A)$\widetilde{=}$(F,A).

(iv) soft semi $^{\#}g\alpha$-cl(F,A) is soft semi $^{\#}g\alpha$-closed.

(v) soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)) $\widetilde{=}$ (X,E) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A).

(vi) soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)) $\widetilde{=}$ (X,E) $\setminus$ soft semi $^{\#}g\alpha$-cl(F,A).

(vii) If (F,A) is soft semi $^{\#}g\alpha$-open in (X,$\widetilde{\tau}$,E) and (G,B) is soft open in (X,$\widetilde{\tau}$,E), then (F,A) $\widetilde{\cap}$ (G,B) is soft semi $^{\#}g\alpha$-open in (X,$\widetilde{\tau}$,E).

(viii) A point x $\widetilde{\in}$ soft semi $^{\#}g\alpha$-cl(F,A) if and only if every soft semi $^{\#}g\alpha$-open set in (X,E) containing x intersects (F,A).

(ix) Arbitrary intersection of soft semi $^{\#}g\alpha$-closed sets in (X,$\widetilde{\tau}$,E) is also soft semi $^{\#}g\alpha$-closed in (X,$\widetilde{\tau}$,E).

Definition 2.3. For any soft subset (F,A) of (X,E),

(i) the soft border of (F,A) is defined by soft bd(F,A) $\widetilde{=}$ (F,A) $\setminus$ soft int(F,A).

(ii) the soft frontier of (F,A) is defined by soft fr(F,A) $\widetilde{=}$ soft cl(F,A) $\setminus$ soft int(F,A).

(iii) the soft exterior of (F,A) is defined by soft ext(F,A)$\widetilde{=}$ soft int((X,E) $\setminus$ (F,A)).

3 Soft Semi $^{\#}g\alpha$-border of a set

Definition 3.1. For any soft subset (F,A) of (X,E), soft semi $^{\#}g\alpha$-border of (F,A) is defined by soft semi $^{\#}g\alpha$-bd(F,A) $\widetilde{=}$ (F,A) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A).

Theorem 3.2. In a soft topological space (X,$\tilde{\tau}$,E), for any soft subset (F,A) of (X,E), the following statements hold.

(i) soft semi $^{\#}g\alpha$-bd(ϕ) $\tilde{=}$ soft semi $^{\#}g\alpha$-bd((X,E)) $\tilde{=}$ ϕ.

(ii) soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{\subseteq}$ (F,A).

(iii) (F,A) $\tilde{=}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-bd(F,A).

(iv) soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{=}\phi$.

(v) soft semi $^{\#}g\alpha$-int(F,A) $\tilde{=}$ (F,A) $\setminus$ soft semi $^{\#}g\alpha$-bd(F,A).

(vi) soft semi $^{\#}g\alpha$-int(soft semi $^{\#}g\alpha$-bd(F,A)) $\tilde{=}\phi$.

(vii) (F,A) is soft semi $^{\#}g\alpha$-open if and only if soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{=}\phi$.

(viii) soft semi $^{\#}g\alpha$-bd(soft semi $^{\#}g\alpha$-int(F,A)) $\tilde{=}$ ϕ.

(ix) soft semi $^{\#}g\alpha$-bd(soft semi $^{\#}g\alpha$-bd(F,A)) $\tilde{=}$ soft semi $^{\#}g\alpha$-bd(F,A).

(x) soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{=}$ (F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)).

Proof: (i), (ii), (iii), (iv) and (v) follow from Definition 3.1.

(vi) If possible let x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(soft semi $^{\#}g\alpha$-bd(F,A)). Then x $\tilde{\in}$ soft semi $^{\#}g\alpha$-bd(F,A), since soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{\subseteq}$ (F,A), x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(soft semi $^{\#}g\alpha$-bd(F,A)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-int(F,A). Therefore x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-bd(F,A) which is a contradiction to (iv). Thus (vi) is proved.

(F,A) is soft semi $^{\#}g\alpha$-open if and only if soft semi $^{\#}g\alpha$-int(F,A) $\tilde{=}$ (F,A) [Theorem 2.2(i)]. But soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{=}$ (F,A) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A) implies soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{=}\phi$. This proves (vii) and (viii).

When (F,A) $\tilde{=}$ soft semi $^{\#}g\alpha$-bd(F,A), Definition 3.1 becomes soft semi $^{\#}g\alpha$-bd(soft semi $^{\#}g\alpha$-bd(F,A)) $\tilde{=}$ soft semi $^{\#}g\alpha$-bd(F,A) $\setminus$ soft semi $^{\#}g\alpha$-int(soft semi $^{\#}g\alpha$-bd(F,A)). Using (viii), we get (ix).

(x) soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{=}$ (F,A) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{=}$ (F,A) $\tilde{\cap}$ ((X,E) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A)) $\tilde{=}$ (F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)) [Theorem 2.2(v)]. Hence (x) is proved. ∎

4 Soft Semi $^{\#}g\alpha$-frontier of a set

Definition 4.1. For any soft subset (F,A) of (X,E), its soft semi $^{\#}g\alpha$-frontier is defined by soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{=}$ soft semi $^{\#}g\alpha$-cl(F,A) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A).

Theorem 4.2. For any soft subset (F,A) of (X,E), in a soft topological space (X,$\tilde{\tau}$,E), the following statements hold.

(i) soft semi $^{\#}g\alpha$-fr(ϕ) $\tilde{=}$ soft semi $^{\#}g\alpha$-fr((X,E)) $\tilde{=}$ $\tilde{\phi}$.

(ii) soft semi $^{\#}g\alpha$-cl(F,A) $\cong$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-fr(F,A).

(iii) soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-fr(F,A) $\cong \tilde{\phi}$.

(iv) soft semi $^{\#}g\alpha$-bd(F,A) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-cl(F,A).

(v) If (F,A) is soft semi $^{\#}g\alpha$-closed, then (F,A) $\cong$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-fr(F,A).

(vi) soft semi $^{\#}g\alpha$-fr(F,A) $\cong$ soft semi $^{\#}g\alpha$-cl(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)).

(vii) A point x $\tilde{\in}$ soft semi $^{\#}g\alpha$-fr(F,A), if and only if every soft semi $^{\#}g\alpha$-open set containing x intersects both (F,A) and its complement (X,E) $\setminus$ (F,A).

(viii) soft semi $^{\#}g\alpha$-cl(soft semi $^{\#}g\alpha$-fr(F,A)) $\cong$ soft semi $^{\#}g\alpha$-fr(F,A), that is, soft semi $^{\#}g\alpha$-fr(F,A) is soft semi $^{\#}g\alpha$-closed.

(ix) soft semi $^{\#}g\alpha$-fr(F,A) $\cong$ semi $^{\#}g\alpha$-fr((X,E) $\setminus$ (F,A)).

(x) (F,A) is soft semi $^{\#}g\alpha$-closed if and only if soft semi $^{\#}g\alpha$-fr(F,A) $\cong$ soft semi $^{\#}g\alpha$-bd(F,A), that is, (F,A) is soft semi $^{\#}g\alpha$-closed if and only if (F,A) contains its soft semi $^{\#}g\alpha$-frontier.

(xi) (F,A) is soft semi $^{\#}g\alpha$-regular if and only if soft semi $^{\#}g\alpha$-fr(F,A) $\cong \tilde{\phi}$.

(xii) soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-int(F,A)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-fr(F,A).

(xiii) soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-cl(F,A)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-fr(F,A).

(xiv) soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(F,A)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-fr(F,A).

(xv) (X,E) $\cong$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-fr(F,A).

(xvi) soft semi $^{\#}g\alpha$-int(F,A) $\cong$ (F,A) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A).

(xvii) If (F,A) is soft semi $^{\#}g\alpha$-open, then (F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-fr(F,A) $\cong \tilde{\phi}$, that is, soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{\subseteq}$ (X,E) $\setminus$ (F,A).

Proof: (i), (ii), (iii) and (iv) follows from Definition 4.1.

(v) follows from (ii) and Theorem 2.2(ii). (vi) follows from Theorem 2.2(v). (vii) can be proved using (vi) and Theorem 2.2(viii).

From (vi), we can prove (viii) by applying the results of Theorem 2.2(iii) and (ix). Proof of (ix) is similar.

(x): If (F,A) is soft semi $^{\#}g\alpha$-closed, then (F,A) $\cong$ soft semi $^{\#}g\alpha$-cl(F,A). Hence by Definition 4.1, soft semi $^{\#}g\alpha$-fr(F,A) $\cong$ (F,A) $\setminus$ soft semi $^{\#}g\alpha$-int(F,A) $\cong$ soft semi $^{\#}g\alpha$-bd(F,A).

Conversely, suppose that soft semi $^{\#}g\alpha$-fr(F,A) $\cong$ soft semi $^{\#}g\alpha$-bd(F,A),using Definitions 4.1 and 3.1, we get soft semi $^{\#}g\alpha$-cl(F,A) $\cong$ (F,A).

From Theorem 2.2(i) and (iii) and Definition 4.1, (xi) can be proved.

Since soft semi $^{\#}g\alpha$-int(F,A) is soft semi $^{\#}g\alpha$-open, (xii) holds. Similarly (xiii) can also be proved.

Since soft semi $^{\#}g\alpha$-fr(F,A) is soft semi $^{\#}g\alpha$-closed, invoking (x), (xiv) can be proved.

To prove (xv), since (X,E) $\widetilde{=}$ soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{\cup}$ ((X,E) $\setminus$ soft semi $^{\#}g\alpha$-cl(F,A)), but from (ii) soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{=}$ soft semi $^{\#}g\alpha$-int(F,A) $\widetilde{\cup}$ soft semi $^{\#}g\alpha$-fr(F,A). Also (X,E) $\setminus$ soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{=}$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)). Hence (X,E) $\widetilde{=}$ soft semi $^{\#}g\alpha$-int(F,A) $\widetilde{\cup}$ soft semi $^{\#}g\alpha$-fr(F,A) $\widetilde{\cup}$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)). Thus (xv) is proved.

Proof of (vi) is obvious. If (F,A) is soft semi $^{\#}g\alpha$-open, (F,A) $\widetilde{=}$ soft semi $^{\#}g\alpha$-int(F,A). Hence (xvii) follows from (iii). ∎

Theorem 4.3. If a soft subset (F,A) of (X,E) is soft semi $^{\#}g\alpha$-open or soft semi $^{\#}g\alpha$-closed in ((X,E),$\widetilde{\tau}$,E), then soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-fr(F,A).

Proof: By Theorem 4.2(vi), we have soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-cl(soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-fr(F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A)).

If (F,A) is soft semi $^{\#}g\alpha$-open in (X,E), by Theorem 4.2(xvii), we have soft semi $^{\#}g\alpha$-fr(F,A) $\widetilde{\cap}$ (F,A) $\widetilde{=}$ $\widetilde{\phi}$. Therefore (F,A) $\widetilde{\subseteq}$ (X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A). Hence soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{\subseteq}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A)). that is, soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-cl(F,A).

If (F,A) is soft semi $^{\#}g\alpha$-closed in (X,E), then (X,E) $\setminus$ (F,A) is soft semi $^{\#}g\alpha$-open and hence from the above case, we have soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ soft semi $^{\#}g\alpha$-fr((X,E) $\setminus$ (F,A))) $\widetilde{=}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)). In both the cases using Theorem 4.2(vi), we get soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-cl(F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl((X,E) $\setminus$ (F,A)) $\widetilde{=}$ soft semi $^{\#}g\alpha$-fr(F,A). ∎

Theorem 4.4. If (F,A) is any soft subset of (X,E), then soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(F,A))) $\widetilde{=}$ soft semi $^{\#}g\alpha$-fr(soft semi $^{\#}g\alpha$-fr(F,A)).

Proof: It follows from Theorem 4.2(viii) and Theorem 4.3. ∎

Theorem 4.5. If (F,A) and (G,B) are soft subsets of (X,E) such that (F,A) $\widetilde{\cap}$ (G,B) $\widetilde{=}$ $\widetilde{\phi}$, where (F,A) is soft semi $^{\#}g\alpha$-open in (X,E), then (F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl(G,B) $\widetilde{=}$ $\widetilde{\phi}$.

Proof: If possible, let x $\widetilde{\in}$ (F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl(G,B). Then (F,A) is a soft semi $^{\#}g\alpha$-open set containing x and also x $\widetilde{\in}$ soft semi $^{\#}g\alpha$-cl(G,B). By Theorem 2.2(viii), (F,A) $\widetilde{\cap}$ (G,B) $\widetilde{=}$ $\widetilde{\phi}$, which is a contradiction. Thus (F,A) $\widetilde{\cap}$ soft semi $^{\#}g\alpha$-cl(G,B) $\widetilde{=}$ $\widetilde{\phi}$. ∎

Theorem 4.6. If (F,A) and (G,B) are soft subsets of (X,E) such that (F,A) $\tilde{\subseteq}$ (G,B) and (G,B) is soft semi $^{\#}g\alpha$-closed in (X,E), then soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{\subseteq}$ (G,B).

Proof: soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{=}$ soft semi $^{\#}g\alpha$-cl(F,A) \ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-cl(G,B) \ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{=}$ (G,B) \ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\subseteq}$ (G,B). ∎

Theorem 4.7. If (F,A) and (G,B) are soft subsets of (X,E) such that (F,A) $\tilde{\cap}$ (G,B) $\tilde{=}$ $\tilde{\phi}$, where (F,A) is soft semi $^{\#}g\alpha$-open in (X,E), then (F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-fr(G,B) $\tilde{=}$ $\tilde{\phi}$.

Proof: Since soft semi $^{\#}g\alpha$-fr(G,B) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-cl(G,B), proof is obvious from Theorem 4.5. ∎

Theorem 4.8. If (F,A) and (G,B) are soft subsets of (X,E) such that soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{\cap}$ fr(G,B) $\tilde{=}$ $\tilde{\phi}$ and fr(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-fr(G,B) $\tilde{=}$ $\tilde{\phi}$, then soft semi $^{\#}g\alpha$-int((F,A) $\tilde{\cup}$ (G,B)) $\tilde{=}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int(G,B).

Proof: We know that soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int(G,B) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-((F,A) $\tilde{\cup}$ (G,B)). Let x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int((F,A) $\tilde{\cup}$ (G,B)). that is, x $\tilde{\in}$ (U,E) $\tilde{\subseteq}$ (F,A) $\tilde{\cup}$ (G,B), (U,E) is a soft semi $^{\#}g\alpha$-open set. Thus either x $\tilde{\in}$ soft semi $^{\#}g\alpha$-fr(F,A), x $\tilde{\notin}$ soft fr(G,B), since soft semi $^{\#}g\alpha$-fr(F,A) $\tilde{\cap}$ soft fr(G,B) $\tilde{=}$ $\tilde{\phi}$. Hence x $\tilde{\in}$ $\tilde{s}$int(G,B). that is, x $\tilde{\notin}$ soft cl(G,B). Since x $\tilde{\in}$ $\tilde{s}$int(G,B) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-int(G,B), x $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-int(G,B). Moreover since x $\tilde{\notin}$ soft cl(G,B), there exists a soft open set (V,E) containing x which is disjoint from (G,B), that is, (V,E) $\tilde{\subseteq}$ (X,E) \ (G,B). So x $\tilde{\in}$ (U,E) $\tilde{\cap}$ (V,E) $\tilde{\subseteq}$ (F,A). Hence (U,E) $\tilde{\cap}$ (V,E) is a soft semi $^{\#}g\alpha$-open subset of (F,A) containing x (By Theorem 2.2(vii)). That is, x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(F,A). Thus x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int(G,B).

If x $\tilde{\notin}$ soft semi $^{\#}g\alpha$-fr(F,A), x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(F,A) or x $\tilde{\notin}$ soft semi $^{\#}g\alpha$-cl(F,A). If x $\tilde{\notin}$ soft semi $^{\#}g\alpha$-cl(F,A), there exists a soft semi $^{\#}g\alpha$-open set (W,E) containing x which is disjoint from (F,A), that is, (W,E) $\tilde{\subseteq}$ (X,E) \ (F,A). That is, x $\tilde{\in}$ (U,E) $\tilde{\cap}$ (W,E) $\tilde{\subseteq}$ (G,B) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-cl(G,B). that is, x $\tilde{\in}$ soft semi $^{\#}g\alpha$-fr(G,B). Hence from the above case, we get x $\tilde{\in}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int(G,B). So soft semi $^{\#}g\alpha$-int((F,A) $\tilde{\cup}$ (G,B)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int(G,B).

Thus soft semi $^{\#}g\alpha$-int((F,A) $\tilde{\cup}$ (G,B)) $\tilde{=}$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-int(G,B). ∎

5 Soft Semi $^{\#}g\alpha$-Exterior of a set

Definition 5.1. For any soft subset (F,A) of (X,E), its soft semi $^{\#}g\alpha$-exterior is defined by soft semi $^{\#}g\alpha$-ext(F,A) $\tilde{=}$ soft semi $^{\#}g\alpha$-int((X,E) \ (F,A)).

Theorem 5.2. For any soft subsets (F,A) and (G,B) of (X,E), in a soft topological space (X,$\tilde{\tau}$,E), the following statements hold.

(i) soft semi $^{\#}g\alpha$-ext($\tilde{\phi}$) $\tilde{=}$ soft semi $^{\#}g\alpha$-ext((X,E)) $\tilde{=}$ $\tilde{\phi}$.

(ii) If (F,A) $\tilde{\subseteq}$ (G,B), then soft semi $^{\#}g\alpha$-ext(G,B) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-ext(F,A).

(iii) soft semi $^{\#}g\alpha$-ext(F,A) is soft semi $^{\#}g\alpha$-open.

(iv) (F,A) is soft semi $^{\#}g\alpha$-closed if and only if soft semi $^{\#}g\alpha$-ext(F,A) $\cong$ (X,E) $\setminus$ (F,A).

(v) soft semi $^{\#}g\alpha$-ext(F,A) $\cong$ (X,E) $\setminus$ soft semi $^{\#}g\alpha$-cl(F,A).

(vi) soft semi $^{\#}g\alpha$-ext(soft semi $^{\#}g\alpha$-ext(F,A)) $\cong$ soft semi $^{\#}g\alpha$-int(soft semi $^{\#}g\alpha$-cl(F,A)).

(vii) If (F,A) is soft semi $^{\#}g\alpha$-regular, then soft semi $^{\#}g\alpha$-ext(soft semi $^{\#}g\alpha$-ext(F,A)) $\cong$ (F,A).

(viii) soft semi $^{\#}g\alpha$-ext(F,A) $\cong$ soft semi $^{\#}g\alpha$-ext((X,E) $\setminus$ soft semi $^{\#}g\alpha$-ext(F,A)).

(ix) soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-ext(soft semi $^{\#}g\alpha$-ext(F,A)).

(x) (X,E) $\cong$ soft semi $^{\#}g\alpha$-int(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-ext(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-fr(F,A).

(xi) soft semi $^{\#}g\alpha$-ext((F,A) $\tilde{\cup}$ (G,B)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-ext(F,A) $\tilde{\cap}$ soft semi $^{\#}g\alpha$-ext((B,E)).

(xii) soft semi $^{\#}g\alpha$-ext((F,A) $\tilde{\cap}$ (G,B)) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-ext(F,A) $\tilde{\cup}$ soft semi $^{\#}g\alpha$-ext((B,E)).

Proof: (i) and (ii) can be proved from Definition 5.1.

Since soft semi $^{\#}g\alpha$-int(F,A) is soft semi $^{\#}g\alpha$-open, proof of (iii) follows from Definition 5.1. Proof of (iv) is obvious.

Since soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)) $\cong$ (X,E) $\setminus$ soft semi $^{\#}g\alpha$-cl(F,A), (v) follows from Definition 5.1. Similarly (vi) can be proved.

If (F,A) is soft semi $^{\#}g\alpha$-regular, from (iv), we have soft semi $^{\#}g\alpha$-ext(F,A) $\cong$ (X,E) $\setminus$ (F,A) which is also soft semi $^{\#}g\alpha$-regular. Thus soft semi $^{\#}g\alpha$-ext(soft semi $^{\#}g\alpha$-ext(F,A)) $\cong$ (F,A), (vii) is proved.

(viii) soft semi $^{\#}g\alpha$-ext((X,E) $\setminus$ soft semi $^{\#}g\alpha$-ext(F,A)) $\cong$ soft semi $^{\#}g\alpha$-ext((X,E) $\setminus$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A))) $\cong$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ ((X,E) $\setminus$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)))) $\cong$ soft semi $^{\#}g\alpha$-int(soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A))) $\cong$ soft semi $^{\#}g\alpha$-int((X,E) $\setminus$ (F,A)) $\cong$ soft semi $^{\#}g\alpha$-ext(F,A). Hence (viii) is proved.

Since (F,A) $\tilde{\subseteq}$ soft semi $^{\#}g\alpha$-cl(F,A), using (vi), (ix) can be proved. (x) follows from Theorem 4.2(xv) and Definition 5.1.

Proof of (xi) and (xii) are obvious. ∎

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