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A displayed inventory model using pentagonal fuzzy number

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Abstract

This paper deals with Fuzzy Multi-item displayed inventory model with alternative power supply cost (power generator). The cost parameters and the constraint are represented by the pentagonal fuzzy number. The model is solved by fuzzy geometric programming method. The optimal order quantity and number of display quantity have been determined. A numerical example is given to illustrate the model.

Keywords: Display inventory, economic order quantity, fuzzy geometric programming, pentagonal fuzzy number, nearest interval approximation.

AMS Subject Classification (2010): 90B05.

1 Introduction

Multi – item classical inventory models under various types of constraints such as capital investment, available storage area, number of orders and available set – up time are presented in well – known books written by Churchman, Ackoff and Arnoff [3], Hardley and Whitin [7] , Silver and Peterson.

While modeling an inventory problem, generally three types of demand are considered. They are (1) constant demand (2) time – dependent demand and (3) stock – dependent demand. In the stock dependent demand, specially displayed inventory level demand has an effect on sales for many retail products.

Whitin [16] stated that for the retail stores the inventory control problem for style goods is further complicated with the fact that the inventory and the sales are not independent to each other. An increase in inventory may bring increased sales of some items. According to Silver and Peterson [13] the sale at the retail level is proportional to the amount of displayed inventory. The most of the retailers displayed some products on shelf following the product variety, choice of the customers towards brand quality, and physical size of the product to influence the customer's attention.

Urban [15] developed a model to identify those products, which should be included in a firm's product line in which the demand rate is a polynomial function of price, advertising and distribution. Corstjens and Doyle [4] developed a shelf – space allocation model in which demand rate is a function of shelf – space allocated to the product.

But all these inventory problems are solved with the assumption that the co-efficient or cost parameters are specified in a precise way. In real life, there are many diverse situations due to uncertainty. Here inventory costs are imprecise, that is fuzzy in nature.

Early works using fuzzy concept in decision making were done by Zadeh [18] and Bellman [2] by introducing fuzzy goals, costs and constraints. Later, the fuzzy linear programming model was formulated and an approach for solving linear programming model with fuzzy numbers has been presented by Zimmermann [19].

Geometric programming method is a relatively new technique to solve a non-linear programming problem. Duffin, Peterson and Zener [5] first developed an idea on GP method. Kotchenberger was the first to use this method on inventory problems. Later on Worrall and Hall [17] analyzed a multi-item inventory model with several constraints using posynomial GP method. Later, the Geometric Programming techniques were discussed by Abou-el-Ata, Fergany, and El-Wakeel [1], Mandal and Roy [8], [9], [11] and [12]. Recently Mandal and Roy [11] presented a displayed inventory model with triangular fuzzy number.

The scarcity of power affects the small scale industries such as Bakery, Restaurants, Packaged food product companies, Retail showrooms. To solve this problem, generators are being installed, it incurs a cost. This paper introduces the cost as 'alternative power supply cost'. Also the pentagonal fuzzy number is defined. So the display inventory model by using pentagonal fuzzy number with Alternative power supply cost has been considered. In this paper, a multi item displayed inventory model under shelf – space constraint in fuzzy environment is formulated. Also power generator has been used in both backroom storage area and display area.

The parameters involved in this paper are assumed to be imprecise in nature and the parameters are represented by pentagonal fuzzy numbers with different types of left and right membership functions. The model is then reduced to multi-objective decision-making inventory problem and is solved by fuzzy geometric programming method. Finally a numerical example is given to illustrate the model.

2 Assumptions and Notations

A multi-item displayed inventory model with generator cost is formulated under the following assumptions.

Assumptions:

1. The unit cost of the item is independent of Q.
2. The display cost does not depend on the length of cycle time T.
3. The outstanding order was never more than one.
4. Lead time is zero.
5. Shortages are not allowed.
6. Demand rate depends on display inventory for i^{th} item $D_i = d_i \, S_i^{d_i'}$ $(d_i > 0, \; 0 < d_i' < 1)$

   Here d_i and d_i' (i=1, 2,….., n) are the scale and shape parameters of the demand function
7. Full-shelf merchandising policy has been adopted, where the display area is always kept fully stocked, so the inventory is replenished as soon as the backroom inventory reaches zero. The displayed inventory will always be at its maximum. The inventory level decreases at a constant rate.
8. Alternative power supply (power generator) cost is allowed.

Notations: Let there be n items. The following are for the i^{th} item,

S_i	-	number of display quantity (decision variable), $(S \equiv (S_1, S_2, ...S_n)^T)$,
Q_i	-	number of order quantity (decision variable), $(Q \equiv (Q_1, Q_2, ...Q_n)^T)$,
θ_i	-	instantaneous inventory level of the entire system including both the backroom storage and the displayed Inventory (net inventory),
p_i	-	selling price per unit,
C_i	-	purchasing price per unit,
C_{1i}	-	holding cost per unit per unit time,
C_{2i}	-	display shelf cost per unit per unit time,
C_{3i}	-	set up cost per cycle,
D_i	-	demand rate,
P_i	-	production rate,
g_i	-	alternative power supply cost (power generator) per unit per unit time,
$\widetilde{PF}$	-	fuzzy profit function,
$T_i = \dfrac{Q_i}{d_i S_i^{d_i'}}$	-	cycle time,
W	-	total display – shelf space.

3 Mathematical Model in Crisp Environment

The inventory model is formulated to maximize the average net profit, which includes the gross revenues, unit purchasing cost, setup cost, holding cost and the display cost under the limited display – space constraint.

Average profit = Gross revenues per unit – purchasing price per unit – setup cost per unit time – holding cost per unit time –generator cost per unit time – display shelf space cost per unit time

Hence, the profit function is

$$PF\ (S,Q)=\sum_{i=1}^{n}\left[D_i\ p_i-D_i\ C_i-\frac{C_{3i}D_i}{Q_i}-\frac{(1-D_i)}{P_i}\ \frac{(C_{1i}+g_i)\ \theta_i}{2}-C_{2i}\ S_i\right] \tag{1}$$

where the average inventory is $\frac{\theta_i}{2}=S_i+\frac{Q_i}{2}$.

Average profit function is reduced to

$$PF(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} d_i\ S_i^{d_i'}(p_i-C_i)-\frac{C_{3i}}{Q_i}d_i\ S_i^{d_i'}-\frac{(g_i+C_{1i})\ Q_i}{2}-(g_i+C_{1i}+C_{2i})\ S_i \\ +\frac{d_i\ S_i^{d_i'}(C_{1i}+g_i)Q_i}{2P_i}+\frac{d_iS_i^{d_i'+1}(C_{1i}+g_i)}{P_i}\end{array}\right] \tag{2}$$

The problem is then stated as

Max $PF(S\ ,Q)$

$$\text{subject to:}\quad \sum_{i=1}^{n}\frac{w_i\ S_i}{W}\le 1\quad ;\quad S_i,Q_i>0 \tag{3}$$

The standard geometric programming problem is

$$\text{Min}\ \ PF(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l}\frac{C_{3i}}{Q_i}d_i\ S_i^{d_i'}-d_i\ S_i^{d_i'}(p_i-C_i)+\frac{(g_i+C_{1i})\ Q_i}{2}-\frac{d_i\ S_i^{d_i'}(C_{1i}+g_i)Q_i}{2P_i} \\ +(g_i+C_{1i}+C_{2i})\ S_i-\frac{d_iS_i^{d_i'+1}(C_{1i}+g_i)}{P_i}\end{array}\right]$$

$$\text{subject to:}\quad \sum_{i=1}^{n}\frac{w_i\ S_i}{W}\le 1\ ;\quad S_i,Q_i>0. \tag{4}$$

This primal problem (4) is a constrained signomial problem with 3n-1 degree of difficulty. The corresponding dual problem is

$$\text{Max}\ D_L=-\left[\begin{array}{l}\prod_{i=1}^{n}\left(\frac{d_iC_{3i}}{w_{1i}}\right)^{w_{1i}}\left(\frac{d_i(p_i-C_i)}{w_{2i}}\right)^{-w_{2i}}\left(\frac{(g_i+C_{1i})}{2w_{3i}}\right)^{w_{3i}}\left(\frac{d_i(g_i+C_{1i})}{P_iw_{4i}}\right)^{-w_{4i}} \\ \left(\frac{(g_i+C_{1i}+C_{2i})}{w_{5i}}\right)^{w_{5i}}\left(\frac{d_i(g_i+C_{1i})}{P_iw_{6i}}\right)^{-w_{6i}}\left(\frac{w_i}{Ww_{7i}}\right)^{w_{7i}}\left(\sum_{i=1}^{n}w_{7i}\right)^{w_{7i}}\end{array}\right]^{-1} \tag{5}$$

subject to:

$$w_{1i}-w_{2i}+w_{3i}-w_{4i}+w_{5i}-w_{6i}=-1$$

$$d_i' w_{1i} - d_i' w_{2i} - d_i' w_{4i} + d_i' - (d_i' + 1) w_{6i} + w_{7i} = 0$$

$$-w_{1i} + w_{3i} - w_{4i} = 0$$

where w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i}, w_{6i} *and* $w_{7i} > 0$.

By using geometric programming theorem [5], the analytical expressions for the decision variables Q_i and S_i are obtained.

$$Q_i^* = \frac{2P_i w_{4i}(P_i - C_i)}{w_{2i}(g_i + C_{1i})} \tag{6}$$

$$S_i^* = \frac{(g_{iR} + C_{1iR})Q_i^* w_{5i}}{2w_{3i}(C_{1i} + C_{2i} + g_i)} \tag{7}$$

4 Pentagonal Fuzzy number and its Nearest Interval Approximation

Definition 4.1. A pentagonal fuzzy number $\tilde{A}$ is a fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad a > x \\ w_A \dfrac{x-a}{b-a} & , \quad a \le x \le b \\ w_A + (1 - w_A)\dfrac{x-b}{c-b} & , \quad b \le x \le c \\ w_A + (1 - w_A)\dfrac{d-x}{d-c} & , \quad c \le x \le d \\ w_A \dfrac{e-x}{e-d} & , \quad d \le x \le e \\ 0 & , \quad e < x \end{cases}$$

where $0.6 \le w_A < 1$ and *a*, *b*, *c*, *d* and *e* are real numbers.

This type of fuzzy number be denoted as $\tilde{A} = (a, b, c, d, e; \ w_A)_{\text{PFN}}$.

$\mu_{\tilde{A}}$ satisfies the following conditions:

1. $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval [0, 1].
2. $\mu_{\tilde{A}}$ is a convex function.
3. $\mu_{\tilde{A}} = 0, -\infty < x \le a$.
4. $\mu_{\tilde{A}}$(x) = L(x) is strictly increasing on (*a*, *c*).
5. $\mu_{\tilde{A}}(x) = 1, \ x = c$.
6. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on (*c*, *e*).

7. $\mu_{\tilde{A}}(x) = 0, e \leq x < \infty$.

Remarks:

1. If $w_A < 0.6$ then $\tilde{A}$ becomes a triangular fuzzy number.

2. If $w_A = 1$ then $\tilde{A}$ becomes a trapezoidal fuzzy number.

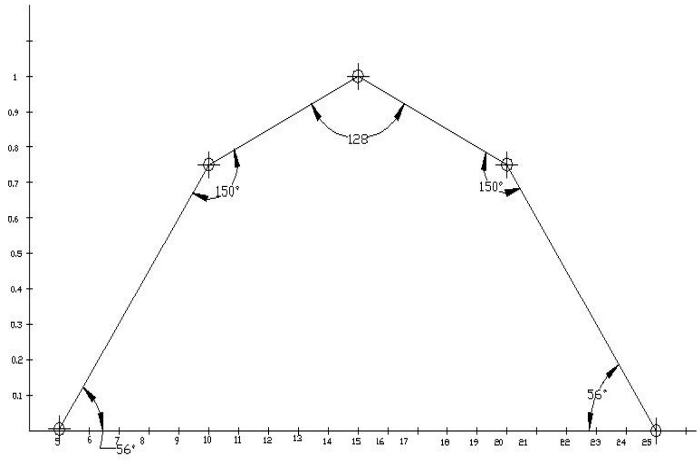

Figure 1: Graphical representation of Pentagonal Fuzzy number for w_A = 0.75.

Nearest Interval Approximation: Here we approximate a fuzzy number by a crisp model. Suppose $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers with α -cuts are [$A_L(\alpha)$, $A_R(\alpha)$] and [$B_L(\alpha)$, $B_R(\alpha)$], respectively. Then the distance between $\tilde{A}$ and $\tilde{B}$ is

$$d(\tilde{A},\tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 d\alpha + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 d\alpha}$$

Given $\tilde{A}$ is a pentagonal fuzzy number. We have to find a closed interval $C_d(\tilde{A})$, which is the nearest to $\tilde{A}$ with respect to metric d. We can do it since each interval is also a fuzzy number with constant α - cut for all α ∈ [0,1]. Hence $(C_d(\tilde{A}))\alpha = [C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_d(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L)^2 d\alpha + \int_0^1 (A_R(\alpha) - C_R)^2 d\alpha}$$ with respect to C_L and C_R.

In order to minimize $d(\tilde{A}, C_d(\tilde{A}))$, it is sufficient to minimize the function

$$D(CL,CR)(= d^2(\tilde{A}, C_d(\tilde{A}))).$$

The first partial derivatives are,

$$\frac{\partial D(C_L, C_R)}{\partial C_L} = -2\int_0^1 A_L(\alpha) d\alpha + 2C_L$$

and $$\frac{\partial D(C_L, C_R)}{\partial C_R} = -2\int_0^1 A_R(\alpha) d\alpha + 2C_R$$

Solving $\frac{\partial D(C_L,C_R)}{\partial C_L}=0$ and $\frac{\partial D(C_L,C_R)}{\partial C_R}=0$ we get, $C_L^*=\int_0^1 A_L(\alpha)\ d\alpha$ and $C_R^*=\int_0^1 A_R(\alpha)\ d\alpha$.

Since $\frac{\partial D^2(C_L,C_R)}{\partial C_L^2}=2>0$ and $\frac{\partial D^2(C_L,C_R)}{\partial C_R^2}=2>0$, we have

$$H(C_L^*,C_R^*)=\frac{\partial D^2(C_L^*,C_R^*)}{\partial C_L^2}.\frac{\partial D^2(C_L^*,C_R^*)}{\partial C_R^2}-\left(\frac{\partial D^2(C_L^*,C_R^*)}{\partial C_L\ \partial C_R}\right)^2=4>0.$$

Hence, D(C$_L$,C$_R$). That is, $d(\tilde{A},C_d(\tilde{A}))$, is global minimum. Therefore, the interval $C_d(\tilde{A})=\left[\int_0^1 A_L(\alpha)d\alpha,\int_0^1 A_R(\alpha)d\alpha\right]$ is the nearest interval approximation of fuzzy number $\tilde{A}$ with respect to the metric *d*.

Let $\tilde{A}$ = (*a, b, c, d, e*) be a pentagonal fuzzy number. The α – level interval of $\tilde{A}$ is defined as $A_\alpha = A_L(\alpha), A_R(\alpha)]$.

When $\tilde{A}$ is a linear fuzzy number (LFN) , the left and right α cuts are

$$A_L(\alpha)=\begin{cases} a+\frac{\alpha(b-a)}{w_A} & if\ a\le x\le b \\ \frac{(\alpha-w_A)\ (c-b)}{(1-w_A)}+b & if\ b\le x\le c\end{cases} \qquad A_R(\alpha)=\begin{cases} d-\frac{(\alpha-w_A)\ (d-c)}{(1-w_A)} & if\ c\le x\le d \\ e-\frac{\alpha(e-d)}{w_A} & if\ d\le x\le e\end{cases}$$

By the nearest interval approximation method, the lower and upper limits of the interval are

$$C_L(\alpha)=\frac{1}{2(1-w_A)}\left[(b+c)+w_A(a-b-2c)-w_A{}^2(a-c)\right]$$

and $$C_R(\alpha)=\frac{1}{2(1-w_A)}\left[(d+c)+w_A(e-d-2c)-w_A{}^2(e-c)\right]$$ respectively.

When $\tilde{A}$ is a parabolic fuzzy number, the limits are given by

$$C_L(\alpha)=\frac{1}{3}\left[w_A\,(a+b-2c)+(b+2c)\right]$$

and $$C_R(\alpha)=\frac{1}{3}\left[w_A\,(e+d-2c)+(d+2c)\right].$$

Similarly, when $\tilde{A}$ is a hyperbolic fuzzy number, the lower and upper limits are

$$C_L(\alpha)=\frac{\sqrt{b^2-a^2}\ w_A}{2}\left[\frac{b}{\sqrt{b^2-a^2}}+\frac{a^2}{b^2-a^2}\log\left(\frac{\sqrt{b^2-a^2}+b}{a}\right)\right]$$
$$+\frac{\sqrt{c^2-b^2}}{2}(1-w_A)\left[\frac{c}{\sqrt{c^2-b^2}}+\frac{b^2}{c^2-b^2}\log\left(\frac{\sqrt{c^2-b^2}+c}{b}\right)\right]$$

and

$$C_R(\alpha) = \frac{\sqrt{d^2 - c^2}}{2}(1 - w_A)\left[\frac{c}{\sqrt{d^2 - c^2}} + \frac{d^2}{d^2 - c^2}\tan^{-1}\left(\frac{\sqrt{d^2 - c^2}}{c}\right)\right]$$
$$+ w_A \frac{\sqrt{e^2 - d^2}}{2}\left[\frac{d}{\sqrt{e^2 - d^2}} + \frac{e^2}{e^2 - d^2}\tan^{-1}\left(\frac{\sqrt{e^2 - d^2}}{d}\right)\right]$$

5 The proposed inventory model in fuzzy environment:

If the cost parameters and total display shelf space parameters are fuzzy numbers, then the problem (3) is transformed to

$$\widetilde{\text{Max}\,PF}\,(S,Q) = \sum_{i=1}^{n}\left[\begin{array}{l} d_i\, S_i^{d_i'}\,(\tilde{p}_i - \tilde{C}_i) - \dfrac{\tilde{C}_{3i}}{Q_i} d_i\, S_i^{d_i'} - \dfrac{(\tilde{g}_i + \tilde{C}_{1i})\, Q_i}{2} - (\tilde{g}_i + \tilde{C}_{1i} + \tilde{C}_{2i})\, S_i \\ + \dfrac{d_i\, S_i^{d_i'}\,(\tilde{C}_{1i} + \tilde{g}_i) Q_i}{2\tilde{P}_i} + \dfrac{d_i S_i^{d_i'+1}(\tilde{C}_{1i} + \tilde{g}_i)}{\tilde{P}_i} \end{array}\right]$$

subject to: $\sum_{i=1}^{n} w_i\, S_i \le \tilde{W}; \quad S_i, Q_i > 0$

where $\sim$ represents the fuzzification of the parameters.

In our proposed model, the cost parameters p_i, C_i, C_{1i}, C_{2i}, C_{3i}, g_i, P_i and W are considered as pentagonal fuzzy numbers.

$\tilde{p}_i = (p_{1i}, p_{2i}, p_{3i}, p_{4i}, p_{5i})$, $\tilde{C}_i = (C_{1i}, C_{2i}, C_{3i}, C_{4i}, C_{5i})$, $\tilde{C}_{2i} = (C_{21i}, C_{22i}, C_{23i}, C_{24i}, C_{25i})$

$\tilde{C}_{1i} = (C_{11i}, C_{12i}, C_{13i}, C_{14i}, C_{15i})$, $\tilde{C}_{3i} = (C_{31i}, C_{32i}, C_{33i}, C_{34i}, C_{35i})$, $\tilde{g}_i = (g_{1i}, g_{2i}, g_{3i}, g_{4i}, g_{5i})$

$\tilde{P}_i = (P_{1i}, P_{2i}, P_{3i}, P_{4i}, P_{5i})$, $\tilde{W}_i = (W_{1i}, W_{2i}, W_{3i}, W_{4i}, W_{5i})$

Our proposed model is reduced to,

$$\text{Max } PF(S,Q) = \sum_{i=1}^{n}\left[\begin{array}{l} d_i\, S_i^{d_i'}\,([p_{iL}, p_{iR}] - [C_{iL}, C_{iR}]) - \dfrac{[C_{3iL}, C_{3iR}]}{Q_i} d_i\, S_i^{d_i'} - \dfrac{([g_{iL}, g_{iR}] + [C_{1iL}, C_{1iR}])\, Q_i}{2} \\ - ([g_{iL}, g_{iR}] + [C_{1iL}, C_{1iR}] + [C_{2iL}, C_{2iR}])\, S_i + \dfrac{d_i\, S_i^{d_i'}\,([g_{iL}, g_{iR}] + [C_{1iL}, C_{1iR}]) Q_i}{2[P_{iL}, P_{iR}]} \\ + \dfrac{d_i S_i^{d_i'+1}([g_{iL}, g_{iR}] + [C_{1iL}, C_{1iR}])}{[P_{iL}, P_{iR}]} \end{array}\right]$$

$$= [PF_L(S,Q), PF_R(S,Q)]$$

subject to: $\sum_{i=1}^{n} w_i\, S_i \le [W_L, W_R]; \quad S_i, Q_i > 0$

where

$$PF_L(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} d_i\,S_i^{d_i'}(p_{iL}-C_{iR})-\dfrac{C_{3iR}}{Q_i}d_i\,S_i^{d_i'}-\dfrac{(g_{iR}+C_{1iR})\,Q_i}{2}-(g_{iR}+C_{1iR}+C_{2iR})\,S_i \\ +\dfrac{d_i\,S_i^{d_i'}(C_{1iL}+g_{iL})Q_i}{2P_{iR}}+\dfrac{d_iS_i^{d_i'+1}(C_{1iL}+g_{iL})}{P_{iR}} \end{array}\right]$$

$$PF_R(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} d_i\,S_i^{d_i'}(p_{iR}-C_{iL})-\dfrac{C_{3iL}}{Q_i}d_i\,S_i^{d_i'}-\dfrac{(g_{iL}+C_{1iL})\,Q_i}{2}-(g_{iL}+C_{1iL}+C_{2iC})\,S_i \\ +\dfrac{d_i\,S_i^{d_i'}(C_{1iR}+g_{iR})Q_i}{2P_{iL}}+\dfrac{d_iS_i^{d_i'+1}(C_{1iR}+g_{iR})}{P_{iL}} \end{array}\right]$$

6 Cases of proposed inventory model with pentagonal fuzzy number

Case 1: All the cost parameters are fuzzified and the total displayed shelf-space parameter is deterministic.

$$\widetilde{\text{Max}\,PF}\,(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} d_i\,S_i^{d_i'}(\tilde{p}_i-\tilde{C}_i)-\dfrac{\tilde{C}_{3i}}{Q_i}d_i\,S_i^{d_i'}-\dfrac{(\tilde{g}_i+\tilde{C}_{1i})\,Q_i}{2}-(\tilde{g}_i+\tilde{C}_{1i}+\tilde{C}_{2i})\,S_i \\ +\dfrac{d_i\,S_i^{d_i'}(\tilde{C}_{1i}+\tilde{g}_i)Q_i}{2\tilde{P}_i}+\dfrac{d_iS_i^{d_i'+1}(\tilde{C}_{1i}+\tilde{g}_i)}{\tilde{P}_i} \end{array}\right] \tag{8}$$

subject to: $\sum_{i=1}^{n} w_i\;S_i\;\le W;\qquad S_i,Q_i>0.$

Using the Nearest Interval Approximation, the above model is defuzzified as follows:

$$\text{Max}\;PF\;(S\;,Q)=[\mathrm{PF_L},\mathrm{PF_R}]$$

subject to: $\sum_{i=1}^{n} w_i\;S_i\;\le W\,;\qquad S_i,Q_i>0.$

The model is converted into a multi-objective non-linear programming problem given below.

$$\text{Max}\;PF_L(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} d_i\,S_i^{d_i'}(p_{iL}-C_{iR})-\dfrac{C_{3iR}}{Q_i}d_i\,S_i^{d_i'}-\dfrac{(g_{iR}+C_{1iR})\,Q_i}{2} \\ -(g_{iR}+C_{1iR}+C_{2iR})\,S_i+\dfrac{d_i\,S_i^{d_i'}(C_{1iL}+g_{iL})Q_i}{2P_{iL}} \\ +\dfrac{d_iS_i^{d_i'+1}(C_{1iL}+g_{iL})}{P_{iR}} \end{array}\right] \tag{9}$$

$$\text{Max } PF_C(S,Q)=\sum_{i=1}^{n}\begin{bmatrix} d_i\, S_i^{d_i'}\,(p_{iC}-C_{iC})-\dfrac{C_{3iC}}{Q_i}\,d_i\,S_i^{d_i'}-\dfrac{(g_{iC}+C_{1iC})\,Q_i}{2} \\ -(g_{iC}+C_{1iC}+C_{2iC})\,S_i+\dfrac{d_i\,S_i^{d_i'}\,(C_{1iC}+g_{iC})Q_i}{2P_{iC}} \\ +\dfrac{S_i^{d_i'+1}(C_{1iC}+g_{iC})\,d_i}{P_{iC}} \end{bmatrix} \tag{10}$$

subject to: $\sum_{i=1}^{n} w_i\ S_i\ \le W; \qquad S_i, Q_i > 0.$

The multi-objective inventory problem (9) is solved by the geometric programming technique and a pay-off matrix of order 2×2 is formed.

The standard geometric programming problem is

$$\text{Min}\quad PF_L(S,Q\ )=\sum_{i=1}^{n}\begin{bmatrix} \dfrac{C_{3iR}}{Q_i}\,d_i\,S_i^{d_i'}-d_i\,S_i^{d_i'}\,(p_{iL}-C_{iR})+\dfrac{(g_{iR}+C_{1iR})\,Q_i}{2} \\ +(g_{iR}+C_{1iR}+C_{2iR})\,S_i-\dfrac{d_i\,S_i^{d_i'}\,(C_{1iL}+g_{iL})Q_i}{2P_{iR}} \\ -\dfrac{d_iS_i^{d_i'+1}(C_{1iL}+g_{iL})}{P_{iR}} \end{bmatrix}$$

subject to: $\sum_{i=1}^{n} w_i\ S_i\ \le W; \qquad S_i, Q_i > 0$ (11)

This primal problem (11) is a constrained signomial problem with 3n-1 degree of difficulty. The corresponding dual problem is

$$\text{Max } D_L=-\begin{bmatrix} \prod_{i=1}^{n}\left(\dfrac{d_iC_{3iR}}{w_{1i}}\right)^{w_{1i}}\left(\dfrac{d_i(p_{iL}-C_{iR})}{w_{2i}}\right)^{-w_{2i}}\left(\dfrac{(g_{iR}+C_{1iR})}{2w_{3i}}\right)^{w_{3i}}\left(\dfrac{d_i(g_{iL}+C_{1iL})}{P_{iR}w_{4i}}\right)^{-w_{4i}} \\ \left(\dfrac{(g_{iR}+C_{1iR}+C_{2iR})}{w_{5i}}\right)^{w_{5i}}\left(\dfrac{d_i(g_{iL}+C_{1iL})}{P_{iR}w_{6i}}\right)^{-w_{6i}}\left(\dfrac{w_i}{Ww_{7i}}\right)^{w_{7i}}\left(\sum_{i=1}^{n}w_{7i}\right)^{w_{7i}} \end{bmatrix}^{-1} \tag{12}$$

subject to:

$w_{1i} - w_{2i} + w_{3i} - w_{4i} + w_{5i} - w_{6i} = -1$

$d_i'\, w_{1i} - d_i'\, w_{2i} - d_i'\, w_{4i} + w_{5i} - (d_i' + 1)w_{6i} + w_{7i} = 0$

$-w_{1i} + w_{3i} - w_{4i} = 0$

where $w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i}, w_{6i}$ & $w_{7i} > 0$.

By using geometric programming theorem [5], the analytical expressions for the decision variables Q_i and S_i are obtained.

$$Q_i^* = \frac{2P_{iR}w_{4i}(P_{iL} - C_{iR})}{w_{2i}(g_{iL} + C_{1iL})} \tag{13}$$

$$S_i^* = \frac{(g_{iR} + C_{1iR})Q_i^* w_{5i}}{2w_{3i}(C_{1iR} + C_{2iR} + g_{iR})} \tag{14}$$

Substituting Q_i^* & S_i^* in PF_L(S ,Q) and PF_C(S ,Q), the optimal values of PF_L^1(S ,Q) and PF_C^1 (S,Q) are obtained.

In a similar way, the optimal values of Q_i and S_i for PF_C[S,Q] subject to the same constraint are obtained.

$$Q_i^* = \frac{2P_{iR}w_{4i}(P_{ic} - C_{ic})}{w_{2i}(g_{ic} + C_{1ic})} \tag{15}$$

$$S_i^* = \frac{(g_{ic} + C_{1ic})Q_i^* w_{5i}}{2w_{3i}(C_{1ic} + C_{2ic} + g_{ic})} \tag{16}$$

Substituting Q_i^* & S_i^* in PF_L(S, Q) and PF_c(S, Q), the optimal values of PF_L^2(S, Q) and PF_C^2 (S, Q) are obtained.

Using the optimal solutions, a payoff matrix of size 2×2 is formed

$$\begin{matrix} 1 \\ 2 \end{matrix}\begin{pmatrix} PF_L^1 & PF_C^1 \\ PF_L^2 & PF_C^2 \end{pmatrix}$$

From the payoff matrix, the lower bounds are $L_L = Min[PF_L^1, PF_L^2]$ and $L_C = Min[PF_C^1, PF_C^2]$ and the upper bounds are $U_L = Max[PF_L^1, PF_L^2]$, $U_C = Max[PF_c^1, PF_c^2]$.

The problem (8) can be formulated as

$$\text{Max } V(S, Q) = [\mu_{PF_L}(S, Q) + \mu_{PF_C}(S, Q)]$$

That is,

$$\text{Max } V(S,Q) = \sum_{i=1}^{n}\left[k_{1i}S_i^{d_i^1} - k_{2i}\frac{S_i^{d_i^1}}{Q_i} - k_{3i}Q_i + k_{4i}S_i^{d_i^1}Q_i - k_{5i}S_i + k_{6i}S_i^{d_i^1+1}\right]$$

subject to: $\sum_{i=1}^{n} w_i\ S_i\ \le W;\qquad S_i, Q_i > 0$

where $k_{1i} = d_i\left(\frac{(p_{iL} - C_{iR})}{U_L - L_L} + \frac{(p_{ic} - C_{ic})}{U_C - L_C}\right)$, $\qquad k_{2i} = \left(\frac{C_{3iR}}{U_L - L_L} + \frac{C_{3ic}}{U_C - L_C}\right)$,

$k_{3i} = \frac{1}{2}\left(\frac{(g_{iR} + C_{1iR})}{(U_L - L_L)} + \frac{(g_{ic} + C_{1ic})}{U_C - L_C}\right)$, $\qquad k_{4i} = \frac{d_i}{2}\left(\frac{((g_{iL} + C_{1iL}))}{P_{iR}(U_L - L_L)} + \frac{((g_{iC} + C_{1iC}))}{P_{iC}(U_C - L_C)}\right)$,

$k_{5i} = \left(\frac{(C_{1iL} + C_{2iL} + g_{iL})}{U_L - L_L} + \frac{(C_{1ic} + C_{2ic} + g_{ic})}{U_C - L_C}\right)$,

$$k_{6i} = d_i\left(\frac{(g_{iL}+C_{1iL})}{P_{iR}(U_L-L_L)}+\frac{(g_{ic}+C_{1ic})}{P_{ic}(U_C-L_C)}\right) \text{ and } k_{7i} = \frac{w_i}{W}.$$

The standard geometric programming problem is,

$$\text{Min } V(S,Q) = \sum_{i=1}^{n}\left[-k_{1i}S_i^{d_i^1} + k_{2i}\frac{S_i^{d_i^1}}{Q_i} + k_{3i}Q_i - k_{4i}S_i^{d_i^1}Q_i + k_{5i}S_i - k_{6i}S_i^{d_i^1+1}\right] \quad (17)$$

$$\text{subject to: } \sum_{i=1}^{n}\frac{w_i\ S_i}{W} \le 1; \qquad S_i, Q_i > 0.$$

This primal problem (17) is a constrained signomial problem with 3n-1 degree of difficulty. The corresponding dual problem is

$$Max\ \ V_L = -\left[\prod_{i=1}^{n}\left(\frac{k_{1i}}{w_{1i}}\right)^{-w_{1i}}\left(\frac{k_{2i}}{w_{2i}}\right)^{w_{2i}}\left(\frac{k_{3i}}{w_{3i}}\right)^{w_{3i}}\left(\frac{k_{4i}}{w_{4i}}\right)^{-w_{4i}}\left(\frac{k_{5i}}{w_{5i}}\right)^{w_{5i}}\left(\frac{k_{6i}}{w_{6i}}\right)^{-w_{6i}}\left(\frac{k_{7i}}{w_{7i}}\right)^{w_{7i}}\left(\sum_{i=1}^{n}w_{7i}\right)^{\sum_{i=1}^{n}w_{7i}}\right]^{-1}$$

subject to:

$$-w_{1i} + w_{2i} + w_{3i} - w_{4i} + w_{5i} - w_{6i} = -1$$

$$-d_i' w_{1i} + d_i' w_{2i} - d_i' w_{4i} + w_{5i} - (d_i' + 1)w_{6i} + w_{7i} = 0$$

$$-w_{2i} + w_{3i} - w_{4i} = 0$$

where $w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i}, w_{6i}$ & $w_{7i} > 0$.

By using geometric programming theorem [5], the analytical expressions for the decision variables Q_i and S_i are obtained.

$$Q_i^* = \frac{w_{4i}k_{1i}}{w_{1i}k_{4i}} \quad (18)$$

$$S_i^* = \frac{k_{3i}Q_i^* w_{5i}}{w_{3i}k_{5i}} \quad (19)$$

Case 2: The cost parameters are deterministic and the display shelf space parameter W is a pentagonal fuzzy number.

Then the problem is

$$\text{Max } PF(S,Q) = \sum_{i=1}^{n}\left[\begin{array}{l} d_i\ S_i^{d_i'}(p_i - C_i) - \dfrac{C_{3i}}{Q_i}d_i\ S_i^{d_i'} - \dfrac{(g_i + C_{1i})\ Q_i}{2} - (g_i + C_{1i} + C_{2i})\ S_i \\ + \dfrac{d_i\ S_i^{d_i'}(C_{1i} + g_i)Q_i}{2P_i} + \dfrac{d_i S_i^{d_i'+1}(C_{1i} + g_i)}{P_i} \end{array}\right] \quad (20)$$

$$\text{subject to: } \sum_{i=1}^{n} w_i\ S_i \le \tilde{W}; \qquad S_i, Q_i > 0.$$

Using the Nearest Interval Approximation, the above model is defuzzified as

$$\text{Max} \quad PF(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} d_i\, S_i^{d_i'}\,(p_i-C_i)-\dfrac{C_{3i}}{Q_i}d_i\, S_i^{d_i'}-\dfrac{(g_i+C_{1i})\,Q_i}{2}-(g_i+C_{1i}+C_{2i})\,S_i \\ +\dfrac{d_i\, S_i^{d_i'}(C_{1i}+g_i)Q_i}{2P_i}+\dfrac{d_i S_i^{d_i'+1}(C_{1i}+g_i)}{P_i} \end{array}\right] \tag{21}$$

subject to: $\sum_{i=1}^{n}\dfrac{w_i\ S_i}{W_R}\le 1,\ \ \sum_{i=1}^{n}\dfrac{w_i\ S_i}{W_L}\ge 1\ ; \quad S_i, Q_i>0.$

The standard geometric programming problem is

$$\text{Min} \quad PF(S,Q)=\sum_{i=1}^{n}\left[\begin{array}{l} \dfrac{C_{3i}}{Q_i}d_i\, S_i^{d_i'}-d_i\, S_i^{d_i'}(p_i-C_i)+\dfrac{(g_i+C_{1i})\,Q_i}{2}-\dfrac{d_i\, S_i^{d_i'}(C_{1i}+g_i)Q_i}{2P_i} \\ +(g_i+C_{1i}+C_{2i})\,S_i-\dfrac{d_i S_i^{d_i'+1}(C_{1i}+g_i)}{P_i} \end{array}\right]$$

subject to: $\sum_{i=1}^{n}\dfrac{w_i\ S_i}{W_R}\le 1,\ \sum_{i=1}^{n}\dfrac{w_i\ S_i}{W_L}\ge 1; \quad S_i, Q_i>0$ (22)

This primal problem (22) is a constrained signomial problem with 4n-1 degree of difficulty. The corresponding dual problem is

$$\text{Max}\ D_L=-\left[\begin{array}{l} \prod_{i=1}^{n}\left(\dfrac{d_i C_{3i}}{w_{1i}}\right)^{w_{1i}}\left(\dfrac{d_i(p_i-C_i)}{w_{2i}}\right)^{-w_{2i}}\left(\dfrac{(g_i+C_{1i})}{2w_{3i}}\right)^{w_{3i}}\left(\dfrac{d_i(g_i+C_{1i})}{P_i w_{4i}}\right)^{-w_{4i}} \\ \left(\dfrac{(g_i+C_{1i}+C_{2i})}{w_{5i}}\right)^{w_{5i}}\left(\dfrac{d_i(g_i+C_{1i})}{P_i w_{6i}}\right)^{-w_{6i}}\left(\dfrac{w_i}{W_R w_{7i}}\right)^{w_{7i}}\left(\dfrac{W_L}{w_i w_{8i}}\right)^{w_{8i}} \\ \left(\sum_{i=1}^{n}w_{7i}\right)^{w_{7i}}\left(\sum_{i=1}^{n}w_{8i}\right)^{w_{8i}} \end{array}\right]^{-1} \tag{23}$$

subject to:

$w_{1i} - w_{2i} + w_{3i} - w_{4i} + w_{5i} - w_{6i} = -1$

$d_i' w_{1i} - d_i' w_{2i} - d_i' w_{4i} + w_{5i} - (d_i' + 1)w_{6i} + w_{7i} - w_{8i} = 0$

$-w_{1i} + w_{3i} - w_{4i} = 0$

where $w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i}, w_{6i}, w_{7i} > 0$.

By using geometric programming theorem [5], the analytical expressions for the decision variables Q_i and S_i are obtained.

$$Q_i^*=\frac{2P_i w_{4i}(P_i-C_i)}{w_{2i}(g_i+C_{1i})} \tag{24}$$

$$S_i^*=\frac{(g_i+C_{1i})Q_i^* w_{5i}}{2w_{3i}(C_{1i}+C_{2i}+g_i)} \tag{25}$$

Case 3: The cost parameters and the total display shelf- space parameter W are considered as pentagonal fuzzy numbers.

$$\widetilde{\text{Max}\,PF}\,(S,\text{Q}) = \sum_{i=1}^{n} \left[\begin{array}{l} d_i\, S_i^{d_i'}\,(\tilde{p}_i - \tilde{C}_i) - \dfrac{\tilde{C}_{3i}}{Q_i} d_i\, S_i^{d_i'} - \dfrac{(\tilde{g}_i + \tilde{C}_{1i})\, Q_i}{2} - (\tilde{g}_i + \tilde{C}_{1i} + \tilde{C}_{2i})\, S_i \\ + \dfrac{d_i\, S_i^{d_i'}\,(\tilde{C}_{1i} + \tilde{g}_i) Q_i}{2\tilde{P}_i} + \dfrac{d_i S_i^{d_i'+1}(\tilde{C}_{1i} + \tilde{g}_i)}{\tilde{P}_i} \end{array} \right]$$

$$\text{subject to: } \sum_{i=1}^{n} w_i\; S_i \;\le \tilde{W}; \qquad S_i, Q_i > 0. \tag{26}$$

Using the nearest interval approximation, the above model is defuzzified as

$$\text{Max }\; PF_L(S\;\;,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} d_i\, S_i^{d_i'}\,(p_{iL} - C_{iR}) - \dfrac{C_{3iR}}{Q_i} d_i\, S_i^{d_i'} - \dfrac{(g_{iR} + C_{1iR})\, Q_i}{2} \\ -(g_{iR} + C_{1iR} + C_{2iR})\; S_i + \dfrac{d_i\, S_i^{d_i'}\,(C_{1iL} + g_{iL}) Q_i}{2P_{iL}} \\ + \dfrac{d_i S_i^{d_i'+1}(C_{1iL} + g_{iL})}{P_{iR}} \end{array} \right] \tag{27}$$

$$\text{Max }\; PF_C(S\;\;,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} d_i\, S_i^{d_i'}\,(p_{iC} - C_{iC}) - \dfrac{C_{3iC}}{Q_i} d_i\, S_i^{d_i'} - \dfrac{(g_{iC} + C_{1iC})\, Q_i}{2} \\ -(g_{iC} + C_{1iC} + C_{2iC})\; S_i + \dfrac{d_i\, S_i^{d_i'}\,(C_{1iC} + g_{iC}) Q_i}{2P_{iC}} \\ + \dfrac{S_i^{d_i'+1}(C_{1iC} + g_{iC})\, d_i}{P_{iC}} \end{array} \right] \tag{28}$$

$$\text{subject to: } \sum_{i=1}^{n} w_i\; S_i \;\le W_R, \qquad \sum_{i=1}^{n} w_i\; S_i \;\ge W_L.$$

By using the same procedure as in cases 1 and 2, multi-objective inventory problem is solved and pay off matrix is formed. Also the membership function for the objective function has been constructed. The problem (26) can be formulated as

$$\text{Max V(S, Q)} = [\mu_{\text{PF}_\text{L}}(\text{S,Q}) + \mu_{\text{PF}_\text{C}}(\text{S,Q})]. \text{ That is,}$$

$$\text{Max V(S,Q)} = \sum_{i=1}^{n} \left[k_{1i} S_i^{d_i^1} - k_{2i} \frac{S_i^{d_i^1}}{Q_i} - k_{3i} Q_i + k_{4i} S_i^{d_i^1} Q_i - k_{5i} S_i + k_{6i} S_i^{d_i^1+1} \right]$$

subject to:

$$\sum_{i=1}^{n} w_i\; S_i \;\le W_R, \qquad \sum_{i=1}^{n} w_i\; S_i \;\ge W_L.$$

The standard signomial geometric programming form can be stated as

$$\text{Min } V(S,Q) = \sum_{i=1}^{n}\left[-k_{1i}S_i^{d_i^1} + k_{2i}\frac{S_i^{d_i^1}}{Q_i} + k_{3i}Q_i - k_{4i}S_i^{d_i^1}Q_i + k_{5i}S_i - k_{6i}S_i^{d_i^1+1}\right] \qquad (29)$$

subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le W_R, \qquad \sum_{i=1}^{n} w_i\, S_i \ge W_L.$$

By using geometric programming theorem [5], the analytical expressions for the decision variables Q_i and S_i are obtained.

$$Q_i^* = \frac{w_{4i}k_{1i}}{w_{1i}k_{4i}} \qquad (30)$$

$$S_i^* = \frac{k_{3i}Q_i^* w_{5i}}{w_{3i}k_{5i}} \qquad (31)$$

7 Numerical Example

Assume that an apparel showroom sells two items. The shop has a total available storage space of 3750 m^2. The relevant data for the two items is given below:

$D_1 = 20S_i^{0.5}$ units, C_1 =₹45, C_{11}=₹1.1, C_{21} =₹ 1.45, C_{31} =₹ 30, p_1 =₹ 100, g_1=₹ 1, P_1=240 units, w_1=0.4m^2, $D_2 = 25S_i^{0.6}$ units, C_2 = ₹100, C_{12}=₹ 0.6, C_{22} =₹1.5, C_{32} =₹ 20, p_2 = ₹ 150, g_2= ₹0.5, P_2=200 units, w_2=0.5 m^2.

$\widetilde{C_1} = [45\ 55\ 65\ 75\ 85\ ]$, $\widetilde{C_{11}} = [1.1\ 1.2\ 1.3\ 1.4\ 1.5\ ]$, $\widetilde{C_{21}} = [1.45\ 1.55\ 1.65\ 1.75\ 2\ ]$,

$\widetilde{C_{31}} = [30\ 31\ 32\ 33\ 34\ ]$, $\widetilde{p_1} = [100\ 110\ 120\ 130\ 140\ ]$, $\widetilde{g_1} = [1\ 2\ 3\ 4\ 5\ ]$

$\widetilde{P_1} = [240\ 250\ 260\ 270\ 280\ ]$, $\widetilde{C_2} = [100\ 105\ 110\ 115\ 120\ ]$, $\widetilde{C_{12}} = [0.6\ 0.65\ 0.7\ 0.75\ 0.8\ ]$

$\widetilde{C_{22}} = [1.5\ 1.6\ 1.7\ 1.8\ 1.9\ ]$, $\widetilde{C_{32}} = [20\ 21\ 22\ 23\ 24]$, $\widetilde{P_2} = [200\ 210\ 220\ 230\ 240\ ]$

$\widetilde{p_2} = [150\ 170\ 190\ 210\ 230]$, $\widetilde{g_2} = [1.5\ 1.6\ 1.7\ 1.8\ 1.9\ ]$

$\widetilde{W} = [3500\ 3600\ 3700\ 3800\ 3900\ ]$, $w_A = 0.75$

Using the analytic expression (6), (7), (9), (10), (18), (19), (24), (25), (30) & (31) for Q_i*, S_i*and PF(S*,Q*) in crisp and fuzzy environment, the following results are obtained.

Table 1: Left and Right Branches of Fuzzy Parameters.

Br	$\widetilde{C_1}$	$\widetilde{C_{11}}$	$\widetilde{C_{21}}$	$\widetilde{C_{31}}$	$\widetilde{p_1}$	$\widetilde{g_1}$	$\widetilde{P_1}$	$\widetilde{C_2}$	$\widetilde{C_{12}}$	$\widetilde{C_{22}}$	$\widetilde{C_{32}}$	$\widetilde{p_2}$	$\widetilde{P_2}$	$\widetilde{g_2}$	$\widetilde{W}$
Left	H	P	P	L	L	H	L	H	L	H	P	P	L	P	L
Right	L	H	L	P	H	h	L	P	H	H	L	P	L	H	L

Here P,L,H stands for Parabolic, Linear and Hyperbolic pentagonal fuzzy membership function respectively.

Table 2: Nearest interval approximation to pentagonal fuzzy numbers for Item 1& 2.

Br	$\widetilde{C_1}$	$\widetilde{C_{11}}$	$\widetilde{C_{21}}$	$\widetilde{C_{31}}$	$\widetilde{p_1}$	$\widetilde{g_1}$	$\widetilde{P_1}$	
Left	50.96	1.19	1.54	30.75	107.5	1.63	247.5	
Right	77.5	1.44	1.83	33.08	134.22	4.43	272.5	
Center	64.23	1.315	1.685	31.91	120.86	3.03	260	
Br	$\widetilde{C_2}$	$\widetilde{C_{12}}$	$\widetilde{C_{22}}$	$\widetilde{C_{32}}$	$\widetilde{p_2}$	$\widetilde{g_2}$	$\widetilde{P_2}$	$\widetilde{W}$
Left	102.93	0.64	1.56	20.92	168.33	1.59	207.5	3575
Right	115.42	0.76	1.81	23.25	211.67	1.84	232.5	3825
Center	109.17	0.7	1.685	22.085	190	1.715	220	3700

Table 3: Optimal Solutions.

Cases	**i**	**S_i***	**Q_i***	**PF(S*,Q*)**
Crisp	1	201.2072	179.5918	35,651
	2	103.7713	67.2403	
Case 1	1	233.5637	156.6133	[30,494 50,50970,436]
	2	109.4986	66.6495	
Case 2	1	108.1143	88.0000	30,398
	2	95.2381	66.6667	
Case 3	1	315.5399	171.2809	[36,165 59,847 83,389]
	2	147.20	66.61	

Observation 7.1. In Table -3, the optimal values are given for the fuzzy model as well as the crisp model, from the same, the following are observed.

(i) In Case 1, the optimal value of the average profit is more than that of crisp model.

(ii) In Case 2, the optimal value of the average profit is less compared to that of Cases 1, 3 and crisp model.

(iii) In Case 3, the optimal value of the average profit is more compared to that of Cases 1, 2 and the crisp model.

(iv) Among the above three cases, Case 3 gives the best optimal solution.

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