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On semi-γ-I-open sets and a new mapping

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Abstract

In this paper, we introduce and study the notion of semi-γ-I-open sets in ideal topological spaces and investigate some of their properties. Further, we study continuous functions on the above set and derive some of their properties.

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1 Introduction

The topological notions of γ-semi-open sets and γ-semi-continuity was introduced by Hariwan [5]. Also, the notion of pre-γ-I-open sets introduced by Hariwan [4]. Jankovic and Hamlett [8] introduced the notion of I-open sets in topological spaces via ideals. Hatir and Noiri [2] introduced semi-I-open sets. Kasahara [10] defined an operation α on a topological space to introduce α-closed graphs. Following the same technique, Ogata [13] defined an operation γ on a topological space and introduced γ-open sets.

2 Preliminaries

Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. Let (X, τ) be a topological space and A a subset of X.

An operation γ [10] on a topology τ is a mapping from τ in to power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [13] if for each $x \in A$, there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open set in X. Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-open sets are called γ-closed. The τ_γ-interior [14] of A is denoted by τ_γ-$Int(A)$ and defined to be the union of all γ-open sets of X contained in A. The τ_γ-closure

[13] of A is denoted by τ_γ-$Cl(A)$ and defined to be the intersection of all γ-closed sets containing A.

An ideal is defined as a nonempty collection I of subsets X satisfying the following two conditions:
(1) If $A \in I$ and $B \subseteq A$, then $B \in I$.
(2) $A \in I$ and $B \in I$, then $A \cup B \in I$.

For an ideal I on (X, τ), (X, τ, I) is called an ideal topological space or simply an ideal space. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called a local function [3], [11] of A with respect to τ and I is defined as follows for a subset A of X, $A^*(I, \tau) = \{x \in X : U \cap A \notin I$ for each neighborhood U of $x\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the $*$-topology, finer than τ, is defined by $Cl^*(A) = A \cup A^*(I, \tau)$ [9]. We will simply write A^* for $A^*(I, \tau)$ and τ^* for $\tau^*(I, \tau)$.

Definition 2.1. A subset A of an ideal topological space (X, τ, I) is said to be

1. I-open [8] if $A \subseteq Int(A^*)$.
2. pre-γ-I-open [4] if $A \subseteq \tau_\gamma$-$Int(Cl^*(A))$.
3. semi-I-open [2] if $A \subseteq Cl^*(Int(A))$.
4. semi-open [12] if $A \subseteq Cl(Int(A))$.
5. γ-semi-open [5] if $A \subseteq \tau_\gamma$-$Cl(\tau_\gamma$-$Int(A))$.

Lemma 2.2. [9] Let (X, τ, I) be an ideal topological space and A, B subsets of X. Then

1. If $A \subseteq B$, then $A^* \subseteq B^*$.
2. If $U \in \tau$, then $U \cap A^* \subseteq (U \cap A)^*$.
3. A^* is closed in (X, τ).

Definition 2.3. [1] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be γ-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U containing x such that $f(U) \subseteq V$.

Definition 2.4. [5] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be γ-semi-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-semi-open set U of X containing x such that $f(U) \subset V$.

Definition 2.5. [6] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly γ-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U of X containing x such that $f(U) \subseteq Cl(V)$.

Definition 2.6. [5] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly γ-semi-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-semi-open set U of X containing x such that $f(U) \subseteq Cl(V)$.

Definition 2.7. [7] A function $f : (X, \tau) \to (Y, \sigma)$ is called almost γ-continuous at if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$.

3 Semi-γ-I-Open Sets

Definition 3.1. A subset A of an ideal topological space (X, τ, I) with an operation γ on τ is called semi-γ-I-open set if $A \subseteq Cl^*(\tau_\gamma\text{-}Int(A))$. A subset F of a space X is said to be semi-γ-I-closed if its complement is semi-γ-I-open.

The family of all semi-γ-I-open sets in ideal topological space (X, τ, I) is denoted by $S\gamma IO(X, \tau, I)$ or simply by $S\gamma IO(X, \tau)$ or $S\gamma IO(X)$ when there is no confusion with ideal.

Remark 3.2. The concepts of semi-γ-I-open and pre-γ-I-open sets are independent.

Example 3.3. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, X\}$ and $I = \{\phi\}$. Define an operation γ on τ by $\gamma(A) = A$. Clearly, $\tau_\gamma = \{\phi, X\}$. Then, $\{b\}$ is pre-γ-I-open but not semi-γ-I-open.

Example 3.4. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $I = \{\phi\}$. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \\ X & \text{if } A \neq \{a\}. \end{cases}$$

Clearly, $\tau_\gamma = \{\phi, \{a\}, X\}$. Then, $\{a, b\}$ is semi-γ-I-open but not pre-γ-I-open.

Proposition 3.5. Every γ-open set is semi-γ-I-open.

Proof: Let A be a γ-open subset of an ideal topological space (X, τ, I). Then, $A = \tau_\gamma\text{-}Int(A)$ and $A \subseteq \tau_\gamma\text{-}Int(A) \cup (\tau_\gamma\text{-}Int(A))^* = Cl^*(\tau_\gamma\text{-}Int(A))$. Hence, $A \subseteq Cl^*(\tau_\gamma\text{-}Int(A))$. Thus, A is semi-γ-I-open. ∎

The converse of above proposition need not be true in general as shown in the following example.

Example 3.6. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and $I = \{\phi, \{b\}\}$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Then, $A = \{a, b\}$ is a semi-γ-I-open set which is not γ-open.

Proposition 3.7. Every semi-γ-I-open set is semi-I-open.

Proof: Let A be a semi-γ-I-open subset of an ideal topological space (X, τ, I). Then, $A \subseteq Cl^*(\tau_\gamma\text{-}Int(A)) \subseteq Cl^*(Int(A))$ and so $A \subseteq Cl^*(Int(A))$. Hence, A is semi-I-open. ∎

The converse of above proposition need not be true in general as shown in the following example.

Example 3.8. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, \{c\}, X\}$ and $I = \{\phi, \{c\}\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then, $A = \{c\}$ is a semi-I-open set which is not semi-γ-I-open.

Remark 3.9. Every semi-γ-I-open set is semi-open.

Proposition 3.10. Every semi-γ-I-open set is γ-semi-open.

Proof: Let A be semi-γ-I-open ubset of an ideal topological space (X, τ, I). Then, $A \subseteq Cl^*(\tau_\gamma\text{-}Int(A)) \subseteq Cl(\tau_\gamma\text{-}Int(A)) \subseteq \tau_\gamma\text{-}Cl(\tau_\gamma\text{-}Int(A))$. Therefore, A is γ-semi-open. ∎

The converse of above proposition need not be true in general as shown in the following example.

Example 3.11. Consider $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{c, d\}, X\}$ and $I = P(X)$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Then, $A = \{a, c, d\}$ is a γ-semi-open set which is not semi-γ-I-open.

Proposition 3.12. Let (X, τ, I) be an ideal topological space and $\{A_\alpha : \alpha \in \Delta\}$ be a family of semi-γ-I-open sets in (X, τ, I). Then, $\bigcup_{\alpha \in \Delta} A_\alpha$ is semi-γ-I-open.

Proof: Let $\{A_\alpha : \alpha \in \Delta\}$ be a family of semi-γ-I-open sets in (X, τ, I). Then, $A_\alpha \subseteq Cl^*(\tau_\gamma\text{-}Int(A_\alpha))$ for each $\alpha \in \Delta$ and $\bigcup_{\alpha \in \Delta} A_\alpha \subseteq \bigcup_{\alpha \in \Delta} Cl^*(\tau_\gamma\text{-}Int(A_\alpha)) \subseteq Cl^*(\bigcup_{\alpha \in \Delta} \tau_\gamma\text{-}Int(A_\alpha)) \subseteq Cl^*(\tau_\gamma\text{-}Int(\bigcup_{\alpha \in \Delta} A_\alpha))$. Hence, $\bigcup_{\alpha \in \Delta} A_\alpha$ is semi-γ-I-open. ∎

The intersection of two semi-γ-I-open sets need not be semi-γ-I-open in general as shown in the following example.

Example 3.13. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $I = \{\phi\}$. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a, b\} \text{ or } \{b, c\} \\ X & \text{otherwise.} \end{cases}$$

Set $A = \{a, b\}$ and $B = \{b, c\}$. Since $A^* = B^* = X$, then both A and B are semi-γ-I-open. But on the other hand $A \cap B = \{b\} \notin S\gamma IO(X, \tau)$.

Proposition 3.14. Let (X, τ, I) be an ideal topological space. If A is semi-γ-I-open and U is γ-open, where γ is a regular operation on τ, then $A \cap U$ is semi-γ-I-open.

Proof: Let A be semi-γ-I-open and U be γ-open. Then $A \cap U \subseteq Cl^*(\tau_\gamma\text{-}Int(A)) \cap U = [\tau_\gamma\text{-}Int(A)^* \cup \tau_\gamma\text{-}Int(A)] \cap U = [\tau_\gamma\text{-}Int(A)^* \cap U\ ] \cup [\tau_\gamma\text{-}Int(A)] \cap U\ ]$. By Lemma 2.2, we have $A \cap U \subseteq (\tau_\gamma\text{-}Int(A) \cap U)^* \cup \tau_\gamma\text{-}Int(A \cap U) = Cl^*(\tau_\gamma\text{-}Int(A \cap U))$. Hence, $A \cap U$ is semi-γ-I-open. ∎

Proposition 3.15. Let (X, τ, I) be an ideal topological space and $A \subseteq X$. If $I = P(X)$, then $S\gamma IO(X) = \tau_\gamma$.

Proof: Let $I = P(X)$, then $\tau_\gamma\text{-}Int(A)^* = \emptyset$ and so $Cl^*(\tau_\gamma\text{-}Int(A)) = \tau_\gamma\text{-}Int(A)^* \cup \tau_\gamma\text{-}Int(A)$ $= \tau_\gamma\text{-}Int(A)$. Let A be semi-γ-I-open, then $A \subseteq Cl^*(\tau_\gamma\text{-}Int(A)) \subseteq \tau_\gamma\text{-}Int(A)$. Hence, A is γ-open. By Proposition 3.5, we have $S\gamma IO(X) = \tau_\gamma$. ∎

Definition 3.16. Let (X, τ, I) be an ideal topological space and $A \subseteq X$.

1. The union of all semi-γ-I-open sets contained in A is called the semi-γ-I-*interior* of A and denoted by $s\gamma I\text{-}Int(A)$.
2. The intersection of all semi-γ-I-closed sets containing A is called the semi-γ-I-*closure* of A and denoted by $s\gamma I\text{-}Cl(A)$.

Now, we state the following theorems without proofs.

Theorem 3.17. Let (X, τ, I) be an ideal topological space and γ an operation on τ. For any subsets A, B of X, we have the following:

1. A is semi-γ-I-open if and only if $A = s\gamma I\text{-}Int(A)$.
2. A is semi-γ-I-closed if and only if $A = s\gamma I\text{-}Cl(A)$.
3. If $A \subseteq B$, then $s\gamma I\text{-}Int(A) \subseteq s\gamma I\text{-}Int(B)$ and $s\gamma I\text{-}Cl(A) \subseteq s\gamma I\text{-}Cl(B)$.
4. $s\gamma I\text{-}Int(A) \cup s\gamma I\text{-}Int(B) \subseteq s\gamma I\text{-}Int(A \cup B)$.
5. $s\gamma I\text{-}Int(A \cap B) \subseteq s\gamma I\text{-}Int(A) \cap s\gamma I\text{-}Int(B)$.
6. $s\gamma I\text{-}Cl(A) \cup s\gamma I\text{-}Cl(B) \subseteq s\gamma I\text{-}Cl(A \cup B)$.
7. $s\gamma I\text{-}Cl(A \cap B) \subseteq s\gamma I\text{-}Cl(A) \cap s\gamma I\text{-}Cl(B)$.
8. $s\gamma I\text{-}Int(X \backslash A) = X \backslash\ s\gamma I\text{-}Cl(A)$.
9. $s\gamma I\text{-}Cl(X \backslash A) = X \backslash\ s\gamma I\text{-}Int(A)$.

Theorem 3.18. Let A be a subset of an ideal topological space (X, τ, I) and γ an operation on τ. Then,

1. $x \in s\gamma I\text{-}Cl(A)$ if and only if for every semi-γ-I-open set V of X containing x, $A \cap V \neq \phi$.

2. $x \in s\gamma I\text{-}Int(A)$ if and only if there exists a semi-γ-I-open set U such that $x \in U \subseteq A$.

4 New Continuous Functions Via Semi-γ-I-Open sets

Definition 4.1. A function $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is called semi-γ-I-continuous if for each $x \in X$ and for each open set V of Y containing $f(x)$, there exists a semi-γ-I-open set U of X containing x such that $f(U) \subseteq V$.

Definition 4.2. A function $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is called weakly semi-γ-I-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a semi-γ-I-open set U of X containing x such that $f(U) \subseteq Cl(V)$.

Remark 4.3. It is obvious from the definition that semi-γ-I-continuity implies weakly semi-γ-I-continuity. However, the converse is not true in general as it is shown in the following example:

Example 4.4. Consider $X = \{a, b, c\}$ with the topology
$\tau = \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ and $I = \{\phi\}$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} c & \text{if } x = a \\ b & \text{if } x = b \\ a & \text{if } x = c. \end{cases}$$

Then, f is weakly semi-γ-I-continuous but not semi-γ-I-continuous, because $\{a\}$ is an open set in (X, σ) containing $f(c) = a$, but there exist no semi-γ-I-open set U in (X, τ) containing c such that $f(U) \subseteq \{a\}$.

Corollary 4.5.
1. Every γ-continuous is semi-γ-I-continuous.

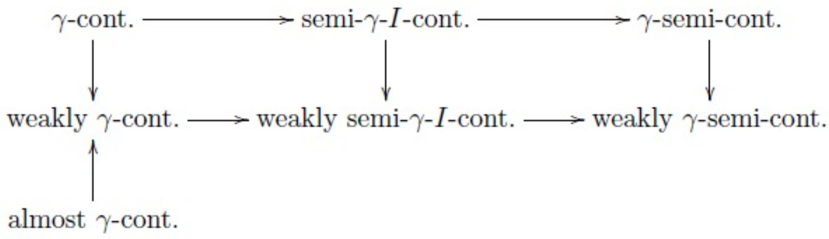
2. Every semi-γ-I-continuous is γ-sem-continuous.

3. Every weakly γ-continuous is weakly semi-γ-I-continuous.

4. Every almost γ-continuous is weakly semi-γ-I-continuous.

5. Every weakly semi-γ-I-continuous is weakly γ-semi-continuous.

Remark 4.6. From Remark 4.3 and Corollary 4.5, we obtain the following diagram of implications:



where cont. means continuous.

Theorem 4.7. For a bijective function $f:(X,\tau,I)\longrightarrow(Y,\sigma)$, the following statements are equivalent:

1. f is semi-γ-I-continuous.
2. $f^{-1}(V)$ is semi-γ-I-open in X, for each open set V of Y.
3. $f^{-1}(V)$ is semi-γ-I-closed in X, for each closed set V of Y.
4. $f(s\gamma I\text{-}Cl(U))\subseteq Cl(f(U))$, for each subset U of X.
5. $s\gamma I\text{-}Cl(f^{-1}(V))\subseteq f^{-1}(Cl(V))$, for each subset V of Y.
6. $f^{-1}(Int(V))\subseteq s\gamma I\text{-}Int(f^{-1}(V))$, for each subset V of Y.
7. $Int(f(U))\subseteq f(s\gamma I\text{-}Int(U))$, for each subset U of X.

Proof: (1) $\Rightarrow$ (2) $\Rightarrow$ (3): Directly from Definition 4.1.

(3) $\Rightarrow$ (4): Let $U\subseteq X$, then $f(U)\subseteq Y$ and $f(U)\subseteq Cl(f(U))$ where $Cl(f(U))$ is closed in Y. Then by Part (3), $f^{-1}(Cl(f(U)))$ is semi-γ-I-closed in X. But $U\subseteq f^{-1}(Cl(f(U)))$. Hence, $s\gamma I\text{-}Cl(U)\subseteq f^{-1}(Cl(f(U)))$. Therefore, $f(s\gamma I\text{-}Cl(U))\subseteq Cl(f(U))$.

(4) $\Rightarrow$ (5): Let V be any subset of Y. Then $f^{-1}(V)$ is a subset of X. By Part (4), $f(s\gamma I\text{-}Cl(f^{-1}(V)))\subseteq Cl(f(f^{-1}(V)))\subseteq Cl(V)$. Hence, $s\gamma I\text{-}Cl(f^{-1}(V))\subseteq f^{-1}(Cl(V))$.

(5) $\Rightarrow$ (6): Let V be any subset of Y. Then apply Part (5) to $Y\backslash V$, we have, $s\gamma I\text{-}Cl(f^{-1}(Y\backslash V))\subseteq f^{-1}(Cl(Y\backslash V))$ which implies, $s\gamma I\text{-}Cl(X\backslash f^{-1}(V))\subseteq f^{-1}(Y\backslash Int(V))$. Hence, $X\backslash s\gamma I\text{-}Int f^{-1}(V))\subseteq X\backslash f^{-1}(Int(V))$ and therefore, $f^{-1}(Int(V))\subseteq s\gamma I\text{-}Int(f^{-1}(V))$.

(6) $\Rightarrow$ (7): Let U be any subset of X. Then $f(U)$ is a subset of Y. By Part (6), we have, $f^{-1}(Int(f(U)))\subseteq s\gamma I\text{-}Int(f^{-1}(f(U)))=s\gamma I\text{-}Int(U)$. Therefore, $Int(f(U))\subseteq f(s\gamma I\text{-}Int(U))$.

(7) $\Rightarrow$ (1): Let $x\in X$ and V be any open subset of Y containing $f(x)$. Then $x\in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X. By (7), $Int(f(f^{-1}(V)))\subseteq f(s\gamma I\text{-}Int(f^{-1}(V)))$. Then $Int(V)\subseteq f(s\gamma I\text{-}Int(f^{-1}(V)))$. Since V is an open set, $V\subseteq f(s\gamma I\text{-}Int(f^{-1}(V)))$ implies, $f^{-1}(V)\subseteq s\gamma I\text{-}Int(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is semi-γ-I-open set in X which contains x and $f(f^{-1}(V))\subseteq V$. Hence, f is semi-γ-I-continuous. ■

Theorem 4.8. For a function $f : (X, \tau, I) \longrightarrow (Y, \sigma)$, the following statements are equivalent:

1. f is weakly semi-γ-I-continuous.
2. $f^{-1}(V) \subseteq s\gamma I\text{-}Int(f^{-1}(Cl(V)))$, for each open set V of Y.
3. $s\gamma I\text{-}Cl(f^{-1}(Int(F))) \subseteq f^{-1}(F)$, for each closed set F of Y.
4. $s\gamma I\text{-}Cl(f^{-1}(Int(Cl(B)))) \subseteq f^{-1}(Cl(B))$, for each subset B of Y.
5. $f^{-1}(Int(B)) \subseteq s\gamma I\text{-}Int(f^{-1}(Cl(Int(B))))$, for each subset B of Y.
6. $s\gamma I\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, for each open subset V of Y.

Proof: (1) $\Rightarrow$ (2): Let V be an open set of Y such that $x \in f^{-1}(V)$. Then $f(x) \in V$ and so there exists a semi-γ-I-open set U in X containing x such that $f(U) \subseteq Cl(V)$. Thus, $x \in U \subseteq f^{-1}(Cl(V))$. Therefore, $x \in s\gamma I\text{-}Int(f^{-1}(Cl(V)))$.

(2) $\Rightarrow$ (3): Let F be a closed subset of Y. Assume that $x \notin f^{-1}(F)$. Then $x \in X\backslash f^{-1}(F)$ where $X\backslash f^{-1}(F) = f^{-1}(Y\backslash F)$ and $Y\backslash F$ is open in Y. By Part (2), $f^{-1}(Y\backslash F) \subseteq s\gamma I\text{-}Int(f^{-1}(Cl(Y\backslash F))) = s\gamma I\text{-}Int(f^{-1}(Y\backslash Int(F))) = s\gamma I\text{-}Int(X\backslash f^{-1}(Int(F))) = X\backslash s\gamma I\text{-}Cl(f^{-1}(Int(F)))$. Therefore, $x \notin s\gamma I\text{-}Cl(f^{-1}(Int(F)))$.

(3) $\Rightarrow$ (4): Let B be a subset of Y. Then $Cl(B)$ is closed in Y. By Part (3), $s\gamma I\text{-}Cl(f^{-1}(Int(Cl(B)))) \subseteq f^{-1}(Cl(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y and $x \in f^{-1}(Int(B))$. But $f^{-1}(Int(B)) = X\backslash f^{-1}(Cl(Y\backslash B))$. Hence, $x \in X\backslash f^{-1}(Cl(Y\backslash B))$ which implies, by Part (4), $x \in X\backslash s\gamma I\text{-}Cl(f^{-1}(Int(Cl(Y\backslash B)))) = s\gamma I\text{-}Int(f^{-1}(Cl(Int(B))))$.

(5) $\Rightarrow$ (6): Let V be open in Y. Suppose that $x \notin f^{-1}(Cl(V))$. Then $f(x) \notin Cl(V)$ and so there exists an open set W containing $f(x)$ such that $W \cap V = \emptyset$ which implies, $Cl(W) \cap V = \emptyset$. By Part (5), $x \in s\gamma I\text{-}Int(f^{-1}(Cl(W)))$ and hence, there exists a semi-γ-I-open set U such that $x \in U \subseteq f^{-1}(Cl(W))$. Since, $Cl(W) \cap V = \emptyset$, $U \cap f^{-1}(V) = \emptyset$ and so, $x \notin s\gamma I\text{-}Cl(f^{-1}(V))$. Therefore, if $x \in s\gamma I\text{-}Cl(f^{-1}(V))$, then $x \in f^{-1}(Cl(V))$.

(6) $\Rightarrow$ (1): Let $x \in X$ and V be an open set of Y containing $f(x)$. Then $x \in f^{-1}(V) \subseteq f^{-1}(Int(Cl(V))) = X\backslash f^{-1}(Cl(Y\backslash Cl(V)))$. By Part (6), $x \notin f^{-1}(Cl(Y\backslash Cl(V)))$ and hence, $x \in s\gamma I\text{-}Int(f^{-1}(Cl(V)))$. Therefore, there exists a semi-γ-I-open set U such that $x \in U \subseteq f^{-1}(Cl(V))$. Hence, $f(U) \subseteq Cl(V)$ and f is weakly semi-γ-I-continuous. ■

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