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Super root square mean labeling of graphs

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Abstract

Let G be a graph with p vertices and q edges. Let $f: V(G) \to \{1,2,3,\ldots,p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f^*(e=uv)$ is defined by $f^*(e)$ = $\left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil or \left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$. f is called a super root square mean if $f(v(G)) \cup \{f(e)/e \in E(G)\} = \{1,2,3,\ldots p+q\}$. A graph which admits super root square mean labeling is called super root square mean graph. In this paper, we investigate super root square mean labeling of some graphs.

Keywords: Root mean square labeling, path, cycle, comb, triangular snake, ladder, total path.

AMS Subject Classification (2010): 05C78.

1 Introduction

We begin with simple, finite, connected and undirected graph $G(V,E)$ with p vertices and q edges. A labeling of a graph is a map that carries graph elements to numbers (Usually to positive (or) non-negative integers). Some labelings use only the vertex set (or) the edge set. We shall call them vertex labelings (or) edge labelings respectively. Terms are not defined here are used in the sense of Harary [2]. For a detailed survey of graph labeling we refer to Gallian [1].

Mean labeling was introduced by S.Somasundaram and R.Ponraj [3],[4]. Harmonic mean labeling was introduced by S.Somasundaram, R. Ponraj and S.S.Sandhya in [7]. Geometric mean labeling was introduced by S.Somasundaram, P.Vidhyarani and R.Ponraj in [5], [6]. Root square mean labeling was introduced by S.S.Sandhya, S.Somasundaram and S.Anusa in [8]. In this paper, we introduce super root square mean labeling of graphs and investigate super root square mean labeling of paths, cycles, Peterson graph, combs, triangular snake, ladder, total path.We now give the following observation and definitions which are useful for the present investigation.

Observation 1.1. If G is a super root mean square graph, then 1 and 3 must be the labels of the adjacent vertices of G since an edge should get label 2.

Definition 1.2. Let $f : V(G) \rightarrow \{1,2,3,...,p+q\}$ be an injective function. For a vertex labeling f, the induced edge labeling $f*(e)$ is defined by $f*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil or \left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$. f is called a super root square mean labeling if $f(v(G)) \cup \{f(e)/e \in E(G)\} = \{1,2,3,...p+q\}$. A graph which admits super root square mean labeling is called super root square mean graph.

Definition 1.3. The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Definition 1.4. The graph $P_n \odot K_1$ is called a comb.

Definition 1.5. The product graph $P_2 \times P_n$ is called a ladder and it is denoted by L_n.

Definition 1.6. A triangular snake T_n is obtained from a path $u_1, u_2, u_3, ..., u_n$ by joining u_i and $u_i + 1$ to a new vertex v_i for $1 \leq i \leq n-1$.

2 Main Results

Theorem 2.1. Any Path P_n is a super root mean square graph.

Proof : Let the vertices of P_n be $\{v_i : 1 \leq i \leq n\}$ and the edges of P_n be $\{e_i = (v_i v_{i+1}) : 1 \leq i \leq n-1\}$.
Here $|V(P_n)| = n$ and $|E(P_n)| = n-1$.
Define a function $f : V(P_n) \rightarrow \{1,2,3,...,2n-1\}$ by $f(v_i) = 2i-1; 1 \leq i \leq n$.
Then the induced edge labels of P_n is $f^*(e_i) = 2i; 1 \leq i \leq n-1$.

Thus the vertices and edges together get distinct labels. Hence P_n is a super root square mean graph. ■

Example 2.2. Super root square mean labeling of P_7 is given below.

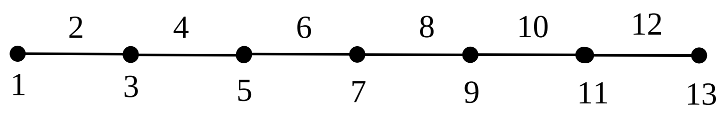

Figure 1: Super root square mean labeling of P_7.

Theorem 2.3. Any cycle C_n is a super root square mean graph.

Proof : Let the vertices of C_n be $\{v_i : 1 \le i \le n\}$ and the edges of C_n be $\{e_i : 1 \le i \le n\}$. Here $|V(P_n)| = |E(C_n)| = n$. We now label the vertices of C_n as follows:

Define a function $f : V(C_n) \to \{1,2,\ldots,2n\}$ by

$$f(v_i) = \begin{cases} 2i-1; 1 \le i \le \frac{k}{2} \\ 2i \quad ; \frac{k+2}{2} \le i \le n \end{cases} \quad \text{if k=} \left\lfloor \sqrt{\tfrac{4n^2+1}{2}} \right\rfloor \textit{is even}$$

$$\text{and } f(v_i) = \begin{cases} 2i-1; 1 \le i \le k-1 \; if \; k = \left\lfloor \sqrt{\frac{4n^2+1}{2}} \right\rfloor is\, odd \\ 2i \quad ; \frac{k+1}{2} \le i \le n \end{cases}$$

Then the induced edge labels of C_n are as follows:

$$f^*(e_i) = \begin{cases} k = \left\lfloor \sqrt{\tfrac{4n^2+1}{2}} \right\rfloor ; i = n \; when\, k\, is\, odd \\ 2i \qquad ; 1 \le i \le \frac{k-1}{2} \\ 2i+1 \qquad ; \frac{k+1}{2} \le i \le n-1 \end{cases}$$

$$f^*(e_i) = \begin{cases} k = \left\lfloor \sqrt{\tfrac{4n^2+1}{2}} \right\rfloor ; i = n \; when\, k\, is\, even \\ 2i \qquad ; 1 \le i \le \tfrac{k}{2} - 1 \\ 2i+1 \qquad ; \tfrac{k}{2} \le i \le n-1 \end{cases}$$

Thus the vertices and edges together get distinct labels. Hence C_n is a super root square mean graph. ■

Example 2.4 Super root square mean labelings of C_4 and C_5 are given below.

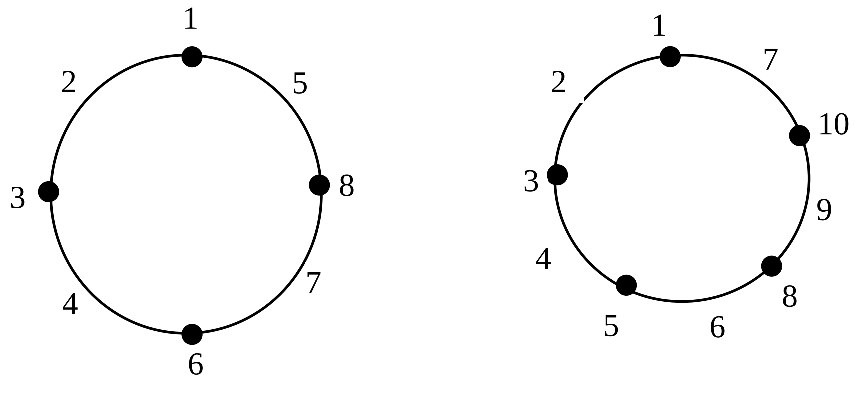

Figure 2: Super root square mean labelings of C_4.

Theorem 2.5. Any comb $P_n \odot K_1$ is super root square mean graph.

Proof: Let $\{v_i, u_i; 1 \le i \le n\}$ be the vertices of comb and let $\{(u_i u_{i+1}); 1 \le i \le n-1\}$ and $\{(u_i v_i); 1 \le i \le n\}$ be the edges of Comb. Here $|E(P_n \odot K_1)| = 2n-1$.

Define a function $f : V(P_n \odot K_1) \to \{1, 2, 3, ..., p+q\}$ by

$$f(u_i) = \begin{cases} 4i-3; 1 \le i \le n, if\ i\ is\ odd \\ 4i-1; 1 \le i \le n, if\ i\ is\ even \end{cases}$$

$$f(v_i) = \begin{cases} 4i-1; 1 \le i \le n, if\ i\ is\ odd \\ 4i-3; 1 \le i \le n, if\ i\ is\ even \end{cases}$$

Then the induced edge labels of Comb as follows:

$f^*(u_i u_{i+1}) = 4i; 1 \le i \le n-1$ and

$f^*(u_i v_i) = 4i-2; 1 \le i \le n-1.$

Thus the vertices and edges together get distinct labels. Hence $P_n \odot K_1$ is a super root square mean graph. ■

Example 2.6. Super root square mean labeling of $P_6 \odot K_1$ is given below.

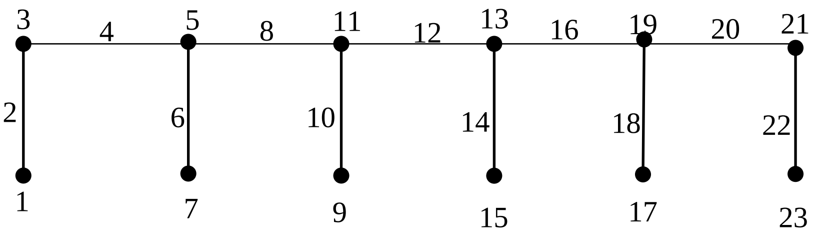

Figure 4: Super root square mean labeling of $P_6 \odot K_1$.

Theorem 2.7. The ladder L_n is a super root square mean graph.

Proof: Let the vertices of L_n be $V(L_n) = \{u_i, v_i : 1 \le i \le n\}$ and let the edges of L_n be $E(L_n) = \{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \le i \le n\}$. Here $|V(L_n)| = 2n$ and $|E(L_n)|$ = 3n-2.

Define a function $f : V(L_n) \to \{1, 2, 3, ... 5n-2\}$ by

$$f(u_i) = \begin{cases} 5i-4\ if\ i\ is\ odd \\ 5i-2\ if\ i\ is\ even \end{cases}$$

and $$f(v_i) = \begin{cases} 5i-2\ if\ i\ is\ odd \\ 5i-4\ if\ i\ is\ even \end{cases}$$

Then the induced edge labels of L_n as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 5i & if\ i\, is\, odd \\ 5i-1 & if\ i\, is\, even \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} 5i-1 & if\ i\, is\, odd \\ 5i & if\ i\, is\, even \end{cases}$$

and $f^*(u_iv_i) = \{5i-3;\ 1 \le i \le n\}$.

Thus the vertices and edges together get distinct labels. Hence ladder L_n is super root square mean graph. ■

Example 2.8. Super root square mean labeling of L_4 is given below.

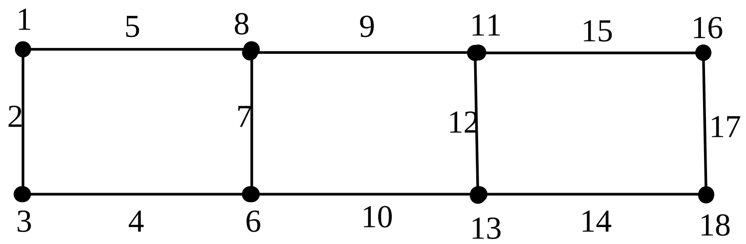

Figure 5: Super root square mean labeling of L_4.

Theorem 2.9. Traingular snake, T_n is a super root square mean graph.

Proof: Let the vertices of T_n be $\{v_i : 1 \le i \le n\}$ and $\{u_i : 1 \le i \le n-1\}$ and let the edges of T_n be $\{e_i : 1 \le i \le n-1\}$ and $\{e_i^{/} : 1 \le i \le 2(n-1)\}$. Here $|V(T_n)| = 2n-1$ and $|E(T_n)| = 3n-3$.

Now we label the vertices of T_n as follows:

Define a function $f : V(T_n) \to \{1,2,3,\ldots,p+q\}$ by

$$f(u_i) = \begin{cases} 3 & ; i = 1 \\ 5i-4; & 2 \le i \le n \end{cases}$$

and $$f(v_i) = \begin{cases} 1 & ; i = 1 \\ 5i-1; & 2 \le i \le n-1. \end{cases}$$

Then the induced edge labels of T_n as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 5 & ; i = 1 \\ 5i-2; & 2 \le i \le n-1 \end{cases}$$

$$f^*(u_iv_i) = 5i-3; 1 \le i \le n-1$$

$$f^*(u_{i+1}v_i) = 6i-2; 1 \le i \le n-1$$

Thus the vertices and edges together get distinct labels. Hence T_n is a super root square mean graph. ■

Example 2.10. Super root square mean labeling of T_5 is given below.

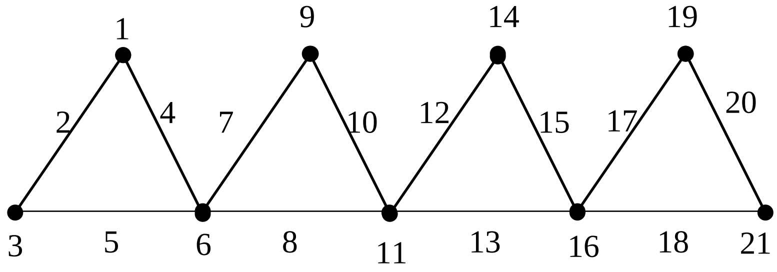

Figure 6: Super root square mean labeling of T_5.

Theorem 2.11. Total path $T(P_n)$ is a super root square mean graph.

Proof: Let the vertices of $T(P_n)$ be $V(T(P_n)) = \{u_i : 1 \le i \le n-1, v_i : 1 \le i \le n\}$ and let the edges of $T(P_n)$ be $E(T(P_n)) = \{(u_i u_{i+1}), (u_i v_i), (v_i v_{i+1}), (u_i v_{i+1}) : 1 \le i \le n-1\}$. Here $|V(T(P_n)| = 2n-1$ and $|E(T(P_n)| = 4n-5$.

Define a function $f : V(T(P_n) \to \{1, 2, \ldots, p+q\}$ by

$$f(u_i) = \begin{cases} 1 & if\ i = 1 \\ 6i-2 & if\ 2 \le i \le n-1 \end{cases}$$

$$f(v_i) = \begin{cases} 3 & if\ i = 1 \\ 6i-2 & if\ 2 \le i \le n-1 \end{cases}$$

Then the induced edge labels of $T(P_n)$ as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} 5i-1 & if\ 1 \le i \le 2 \\ 6i-3 & if\ 3 \le i \le n-1 \end{cases}$$

$$f^*(v_i v_{i+1}) = 6i+1 \ if\ 1 \le i \le n-1$$

$$f^*(u_i v_i) = 6i-4 \ if\ 1 \le i \le n-1$$

$$f^*(u_{i+1} v_i) = 6i-1 \ if\ 1 \le i \le n-1$$

Then the vertices and edges together get distinct labels. Hence $T(P_n)$ is a super root square mean graph. ■

Example 2.12. Super root square mean labeling of P_5 is given below.

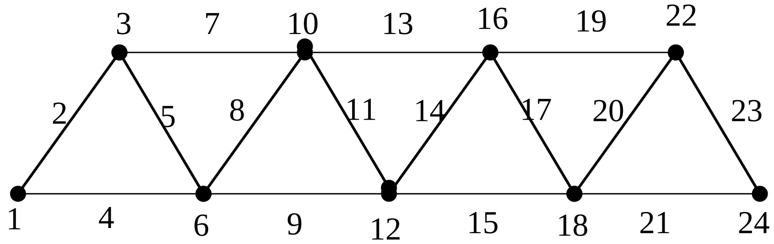

Figure 7: Super root square mean labeling of P_5.

Example 2.13. Peterson graph is a square root square mean graph.

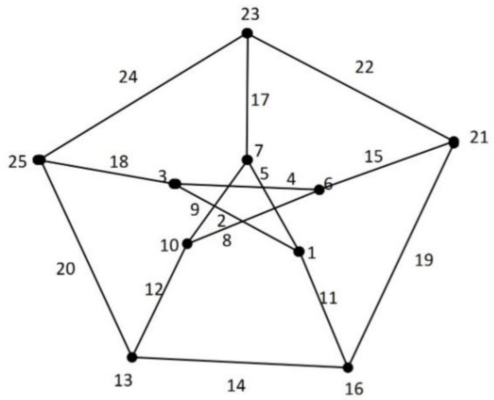

Figure 8: Super root square mean labeling of Peterson graph.

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