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b-g-compactness in ditopological texture spaces

Hariwan Z. Ibrahim

Department of Mathematics, Faculty of Science
University of Zakho, Kurdistan-Region, Iraq.
hariwan_math@yahoo.com

Abstract

The main goal of this paper is to introduce and study new notions of continuity, compactness and stability in ditopological texture spaces based on the notions of b-g-open and b-g-closed sets and some of their characterizations are obtained.

Keywords: Texture, difunction, b-g-bi-irresolute, b-g-stability.
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1 Introduction

Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study fuzzy topology. The study of compactness and stability in ditopological texture spaces was begun in [5]. In this paper, we introduce and study the concepts of b-g-bicontinuity, b-g-bi-irresolute, b-g-compactness and b-g-stability in ditopological textures spaces.

2 Preliminaries

The following are some basic definitions of textures we will need later on.
Texture space: [5] Let S be a set. Then $\varphi \subseteq P(S)$ is called a texturing of S, and S is said to be textured by φ if

1. $(\varphi, \subseteq)$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \varphi$, $i \in I$, the meet $\bigwedge_{i\in I} A_i$ and the join $\bigvee_{i\in I} A_i$ in φ are related with the intersection and union in $P(S)$ by the equalities

   $\bigwedge_{i\in I} A_i = \bigcap_{i\in I} A_i$

   for all I, while

   $\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i$

   for all finite I.

2. φ is completely distributive.

3. φ separates the points of S. That is, given $s_1 \neq s_2$ in S we have $L \in \varphi$ with $s_1 \in L$, $s_2 \notin L$, or $L \in \varphi$ with $s_2 \in L$, $s_1 \notin L$.

If S is textured by φ then (S, φ) is called a texture space, or simply a texture.

Complementation: [5] A mapping $\sigma : \varphi \to \varphi$ satisfying $\sigma(\sigma(A)) = A$, $\forall A \in \varphi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$, $\forall A, B \in \varphi$ is called a complementation on (S, φ) and (S, φ, σ) is then said to be a complemented texture.

For a texture (S, φ), most properties are conveniently denoted in terms of the p-sets

$$P_s = \bigcap\{A \in \varphi : s \in A\}$$

and the q-sets,

$$Q_s = \bigvee\{A \in \varphi : s \notin A\}.$$

Ditopology: [5] A dichotomous topology on a texture (S, φ), or ditopology for short, is a pair (τ, k) of subsets of φ, where the set of open sets τ satisfies

1. $S, \phi \in \tau$,
2. $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$, and
3. $G_i \in \tau, i \in I \Rightarrow \bigvee_i G_i \in \tau$,

and the set of closed sets k satisfies

1. $S, \phi \in k$,
2. $K_1, K_2 \in k \Rightarrow K_1 \cup K_2 \in k$, and
3. $K_i \in k, i \in I \Rightarrow \bigcap K_i \in k$.

Hence a ditopology is essentially a "topology" for which there is no a priori relation between the open and closed sets.

For $A \in \varphi$ we define the closure $[A]$ and the interior $]A[$ of A under (τ, k) by the equalities

$[A] = \bigcap\{K \in k : A \subseteq K\}$ and $]A[= \bigvee\{G \in \tau : G \subseteq A\}$

We refer to τ as the topology and k as the cotopology of (τ, k).

If (τ, k) is a ditopology on a complemented texture (S, φ, σ), then we say that (τ, k) is complemented if the equality $k = \sigma[\tau]$ is satisfied. In this study, a complemented ditopological texture space is denoted by $(S, \varphi, \tau, k, \sigma)$.

In this case we have $\sigma([A]) =]\sigma(A)[$ and $\sigma(]A[) = [\sigma(A)]$.

We denote by $O(S, \varphi, \tau, k)$, or when there can be no confusion by $O(S)$, the set of open sets in φ. Likewise, $C(S, \varphi, \tau, k)$, $C(S)$ will denote the set of closed sets.

Let (S_1, φ_1) and (S_2, φ_2) be textures. In the following definition we consider the product texture [2] $P(S_1) \otimes \varphi_2$, and denote by $\overline{P}_{s,t}$, $\overline{Q}_{s,t}$, respectively the p-sets and q-sets for the product texture $(S_1 \times S_2, P(S_1) \otimes \varphi_2)$.

Direlation: [4] Let (S_1, φ_1) and (S_2, φ_2) be textures. Then

1. $r \in P(S_1) \otimes \varphi_2$ is called a relation from (S_1, φ_1) to (S_2, φ_2) if it satisfies

   R1 $r \not\subseteq \overline{Q}_{s,t}$, $P_{s'} \not\subseteq Q_s \Rightarrow r \not\subseteq \overline{Q}_{s',t}$.

   R2 $r \not\subseteq \overline{Q}_{s,t} \Rightarrow \exists s' \in S_1$ such that $P_s \not\subseteq Q_{s'}$ and $r \not\subseteq \overline{Q}_{s',t}$.

2. $R \in P(S_1) \otimes \varphi_2$ is called a corelation from (S_1, φ_1) to (S_2, φ_2) if it satisfies

   CR1 $\overline{P}_{s,t} \not\subseteq R$, $P_s \not\subseteq Q_{s'} \Rightarrow \overline{P}_{s',t} \not\subseteq R$.

   CR2 $\overline{P}_{s,t} \not\subseteq R \Rightarrow \exists s' \in S_1$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{s',t} \not\subseteq R$.

3. A pair (r, R), where r is a relation and R a corelation from (S_1, φ_1) to (S_2, φ_2) is called a direlation from (S_1, φ_1) to (S_2, φ_2).

One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.

Difunctions: [4] Let (f, F) be a direlation from (S_1, φ_1) to (S_2, φ_2). Then (f, F) is called a difunction from (S_1, φ_1) to (S_2, φ_2) if it satisfies the following two conditions.

DF1 For $s, s' \in S_1$, $P_s \not\subseteq Q_{s'} \Rightarrow \exists t \in S_2$ such that $f \not\subseteq \overline{Q}_{s,t}$ and $\overline{P}_{s',t} \not\subseteq F$.

DF2 For $t, t' \in S_2$ and $s \in S_1$, $f \not\subseteq \overline{Q}_{s,t}$ and $\overline{P}_{s,t'} \not\subseteq F \Rightarrow P_{t'} \not\subseteq Q_t$.

Image and Inverse Image: [4] Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction.

1. For $A \in \varphi_1$, the image $f^{\to}A$ and the co-image $F^{\to}A$ are defined by

$$f^{\to}A = \bigcap\{Q_t : \forall s, f \not\subseteq \overline{Q}_{s,t} \Rightarrow A \subseteq Q_s\},$$
$$F^{\to}A = \bigvee\{P_t : \forall s, \overline{P}_{s,t} \not\subseteq F \Rightarrow P_s \subseteq A\}.$$

2. For $B \in \varphi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by

$$f^{\leftarrow}B = \bigvee\{P_s : \forall t, f \not\subseteq \overline{Q}_{s,t} \Rightarrow P_t \subseteq B\},$$
$$F^{\leftarrow}B = \bigcap\{Q_s : \forall t, \overline{P}_{s,t} \not\subseteq F \Rightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

Bicontinuity: [3] The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1) \to (S_2, \varphi_2, \tau_2, k_2)$ is called continuous if $B \in \tau_2 \Rightarrow F^{\leftarrow}B \in \tau_1$, cocontinuous if $B \in k_2 \Rightarrow f^{\leftarrow}B \in k_1$, and bicontinuous if it is both continuous and cocontinuous.

Surjective difunction: [4] Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condition

SUR. For $t, t' \in S_2$, $P_t \not\subseteq Q_{t'} \Rightarrow \exists s \in S_1$ with $f \not\subseteq \overline{Q}_{s,t'}$ and $\overline{P}_{s,t} \not\subseteq F$.

If (f, F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$ for all $B \in \varphi_2$ [[4], Corollary 2.33]

Definition 2.1. [4] Let (f, F) be a difunction between the complemented textures $(S_1, \varphi_1, \sigma_1)$ and $(S_2, \varphi_2, \text{Îÿ}\sigma_2)$. The complement $(f, F)' = (F', f')$ of the difunction (f, F) is a difunction, where $f' = \bigcap\{\overline{Q}_{s,t} | \exists u, v$ with $f \not\subseteq \overline{Q}_{u,v}, \sigma_1(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \sigma_2(P_t)\}$ and $F' = \bigvee\{\overline{P}_{s,t} | \exists u, v$ with $\overline{P}_{u,v} \not\subseteq F, P_u \not\subseteq \sigma_1(P_s)$ and $\sigma_2(Q_t) \not\subseteq Q_v\}$.

If $(f, F) = (f, F)'$ then the difunction (f, F) is called complemented.

Definition 2.2. [1] Let (S, φ, τ, k) be a ditopological texture space. A set $A \in \varphi$ is called b-open (b-closed) if $A \subseteq][A][\cup[]A[]$ $(][A][\cap[]A[] \subseteq A)$.

We denote by $bO(S, \varphi, \tau, k)$, or when there can be no confusion by $bO(S)$, the set of b-open sets in φ. Likewise, $bC(S, \varphi, \tau, k)$, or $bC(S)$ will denote the set of pre-closed sets.

Definition 2.3. [7] Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be generalized closed (g-closed for short) if $A \subseteq G \in \tau$ then $[A] \subseteq G$.

Definition 2.4. [7] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is said to be generalized open (g-open for short) if $\sigma(A)$ is g-closed.

We denote by $gc(S, \varphi, \tau, k)$, or when there can be no confusion by $gc(S)$, the set of g-closed sets in φ. Likewise, $go(S, \varphi, \tau, k, \sigma)$, or $go(S)$ will denote the set of g-open sets.

Definition 2.5. [6] Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be b-g-closed if $A \subseteq G \in bO(S)$ then $[A] \subseteq G$.

We denote by $bgc(S, \varphi, \tau, k)$, or when there can be no confusion by $bgc(S)$, the set of b-g-closed sets in φ.

Definition 2.6. [6] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is called b-g-open if $\sigma(A)$ is b-g-closed.

We denote by $bgo(S, \varphi, \tau, k, \sigma)$, or when there can be no confusion by $bgo(S)$, the set of b-g-open sets in φ.

Definition 2.7. [6] Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. For $A \in \varphi$, we define the b-g-closure $[A]_{b-g}$ and the b-g-interior $]A[_{b-g}$ of A under (τ, k) by the equalities

$$[A]_{b-g} = \bigcap\{K \in bgc(S) : A \subseteq K\} \text{ and }]A[_{b-g} = \bigcup\{G \in bgo(S) : G \subseteq A\}.$$

3 b-g-bicontinuous, b-g-bi-irresolute, b-g-compact and b-g-stable

Definition 3.1. The difunction $(f,F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is called:

1. b-g-continuous (b-g-irresolute), if $F^{\leftarrow}(G) \in bgo(S_1)$, for every $G \in O(S_2)$ ($G \in bgo(S_2)$).
2. b-g-cocontinuous (b-g-co-irresolute), if $f^{\leftarrow}(G) \in bgc(S_1)$, for every $G \in k_2$ ($G \in bgc(S_2)$).
3. b-g-bicontinuous, if it is b-g-continuous and b-g-cocontinuous.
4. b-g-bi-irresolute, if it is b-g-irresolute and b-g-co-irresolute.

Corollary 3.2. Let $(f,F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then:

1. Every continuous is b-g-continuous.
2. Every cocontinuous is b-g-cocontinuous.
3. Every b-g-irresolute is b-g-continuous.
4. Every b-g-co-irresolute is b-g-cocontinuous.

Proof: Clear. ∎

Theorem 3.3. Let $(f,F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then:

1. The following are equivalent:
   (a) (f,F) is b-g-continuous.
   (b) $]F^{\rightarrow}A[^{S_2} \subseteq F^{\rightarrow}]A[^{S_1}_{b-g}$, $\forall A \in \varphi_1$.
   (c) $f^{\leftarrow}]B[^{S_2} \subseteq]f^{\leftarrow}B[^{S_1}_{b-g}$, $\forall B \in \varphi_2$.
2. The following are equivalent:
   (a) (f,F) is b-g-cocontinuous.
   (b) $f^{\rightarrow}[A]^{S_1}_{b-g} \subseteq [f^{\rightarrow}A]^{S_2}$, $\forall A \in \varphi_1$.
   (c) $[F^{\leftarrow}B]^{S_1}_{b-g} \subseteq F^{\leftarrow}[B]^{S_2}$, $\forall B \in \varphi_2$.

Proof: We prove (1), leaving the dual proof of (2) to the interested reader.
$(a) \Rightarrow (b)$. Let $A \in \varphi_1$. From [[4], Theorem 2.24 (2 a)] and the definition of interior,

$$f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2} \subseteq f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A.$$

Since inverse image and co-image under a difunction is equal, $f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}= F^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}$. Thus, $f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}\in bgo(S_1)$, by b-g-continuity. Hence

$$f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}\subseteq]A[^{S_1}_{b-g}$$

and applying [[4], Theorem 2.4 (2 b)] gives

$$]F^{\rightarrow}(A)[^{S_2}\subseteq F^{\rightarrow}(f^{\leftarrow}(]F^{\rightarrow}(A)[^{S_2}) \subseteq F^{\rightarrow}]A[^{S_1}_{b-g},$$

which is the required inclusion.
$(b) \Rightarrow (c)$. Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^{\leftarrow}(B)$ and using [[4], Theorem 2.4 (2 b)] gives

$$]B[^{S_2}\subseteq]F^{\rightarrow}f^{\leftarrow}(B)[^{S_2}\subseteq F^{\rightarrow}]f^{\leftarrow}(B)[^{S_1}_{b-g}.$$

Hence, we have $f^{\leftarrow}]B[^{S_2}\subseteq f^{\leftarrow}F^{\rightarrow}]f^{\leftarrow}(B)[^{S_1}_{b-g}\subseteq]f^{\leftarrow}(B)[^{S_1}_{b-g}$ by [[4], Theorem 2.24 (2 a)].
$(c) \Rightarrow (a)$. Applying (c) for $B \in O(S_2)$ gives

$$f^{\leftarrow}(B) = f^{\leftarrow}]B[^{S_2}\subseteq]f^{\leftarrow}(B)[^{S_1}_{b-g},$$

so $F^{\leftarrow}(B) = f^{\leftarrow}(B) =]f^{\leftarrow}(B)[^{S_1}_{b-g}\in bgo(S_1)$. Hence, (f, F) is b-g-continuous. ∎

Theorem 3.4. Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a difunction. Then:

1. The following are equivalent:
   (a) (f, F) is b-g-irresolute.
   (b) $]F^{\rightarrow}A[^{S_2}_{b-g}\subseteq F^{\rightarrow}]A[^{S_1}_{b-g}$, $\forall A \in \varphi_1$.
   (c) $f^{\leftarrow}]B[^{S_2}_{b-g}\subseteq]f^{\leftarrow}B[^{S_1}_{b-g}$, $\forall B \in \varphi_2$.
2. The following are equivalent:
   (a) (f, F) is b-g-co-irresolute.
   (b) $f^{\rightarrow}[A]^{S_1}_{b-g} \subseteq [f^{\rightarrow}A]^{S_2}_{b-g}$, $\forall A \in \varphi_1$.
   (c) $[F^{\leftarrow}B]^{S_1}_{b-g} \subseteq F^{\leftarrow}[B]^{S_2}_{b-g}$, $\forall B \in \varphi_2$.

Proof: We prove (1), leaving the dual proof of (2) to the interested reader.
$(a) \Rightarrow (b)$. Take $A \in \varphi_1$. Then

$$f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{b-g}\subseteq f^{\leftarrow}(F^{\rightarrow}A) \subseteq A$$

by [[4], Theorem 2.24 (2 a)]. Now $f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{b-g}= F^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{b-g}\in bgo(S_1)$ by b-g-irresolute, so $f^{\leftarrow}]F^{\rightarrow}A[^{S_2}_{b-g}\subseteq]A[^{S_1}_{b-g}$ and applying [[4], Theorem 2.4 (2 b)] gives

$$]F^{\to}A[^{S_2}_{b-g} \subseteq F^{\to}(f^{\leftarrow}]F^{\to}A[^{S_2}_{b-g} \subseteq F^{\to}]A[^{S_1}_{b-g},$$

which is the required inclusion.
$(b) \Rightarrow (c)$. Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^{\leftarrow}B$ and using [[4], Theorem 2.4 (2 b)] gives

$$]B[^{S_2}_{b-g} \subseteq]F^{\to}(f^{\leftarrow}B)[^{S_2}_{b-g} \subseteq F^{\to}]f^{\leftarrow}B[^{S_1}_{b-g}.$$

Hence, $f^{\leftarrow}]B[^{S_2}_{b-g} \subseteq f^{\leftarrow}F^{\to}]f^{\leftarrow}B[^{S_1}_{b-g} \subseteq]f^{\leftarrow}B[^{S_2}_{b-g}$ by [[4], Theorem 2.24 (2 a)].
$(c) \Rightarrow (a)$. Applying (c) for $B \in bgo(S_2)$ gives

$$f^{\leftarrow}B = f^{\leftarrow}]B[^{S_2}_{b-g} \subseteq]f^{\leftarrow}B[^{S_1}_{b-g},$$

so $F^{\leftarrow}B = f^{\leftarrow}B =]f^{\leftarrow}B[^{S_1}_{b-g} \in bgo(S_1)$. Hence, (f, F) is b-g-irresolute. ■

Theorem 3.5. Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be complemented difunction. If (f, F) is b-g-continuous then (f, F) is b-g-cocontinuous.

Proof: Since (f, F) is complemented, $(F', f') = (f, F)$. From [[4], Lemma 2.20], $\sigma_1((f')^{\leftarrow}(B)) = f^{\leftarrow}(\sigma_2(B))$ and $\sigma_1((F')^{\leftarrow}(B)) = F^{\leftarrow}(\sigma_2(B))$ for all $B \in \varphi_2$. The proof is clear from these equalities. ■

Corollary 3.6. Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be complemented difunction. If (f, F) is b-g-irresolute then (f, F) is b-g-co-irresolute.

Proof: Clear. ■

Definition 3.7. A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called b-g-compact if every cover of S by b-g-open sets has a finite subcover. Here we recall that $C = \{A_j : j \in J\}$, $A_j \in \varphi$ is a cover of S if $\bigvee C = S$.

Corollary 3.8. Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:

1. Every b-g-compact is compact.

2. Every g-compact is b-g-compact.

Proof: Clear. ■

Theorem 3.9. If $(S, \varphi, \tau, k, \sigma)$ is b-g-compact and $L = \{F_j : j \in J\}$ is a family of b-g-closed sets with $\cap L = \phi$, then $\cap\{F_j : j \in J'\} = \phi$ for $J' \subseteq J$ finite.

Proof: Suppose that $(S, \varphi, \tau, k, \sigma)$ is b-g-compact and let $L = \{F_j : j \in J\}$ be a family of b-g-closed sets with $\cap L = \phi$. Clearly $C = \{\sigma(F_j) : j \in J\}$ is a family of b-g-open sets. Moreover,

$$\bigvee C = \bigvee\{\sigma(F_j) : j \in J\} = \sigma(\cap\{F_j : j \in J\}) = \sigma(\phi) = S,$$

and so we have $J^{'} \subseteq J$ finite with $\bigvee\{\sigma(F_j) : j \in J^{'}\} = S$. Hence $\cap\{F_j : j \in J^{'}\} = \phi$. ■

Theorem 3.10. Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be an b-g-irresolute difunction. If $A \in \varphi_1$ is b-g-compact then $f^{\to}A \in \varphi_2$ is b-g-compact.

Proof: Take $f^{\to}A \subseteq \bigvee_{j\in J} G_j$, where $G_j \in bgo(S_2)$, $j \in J$. Now by [[4], Theorem 2.24 (2 a) and Corollary 2.12 (2)] we have

$$A \subseteq F^{\leftarrow}(f^{\to}A) \subseteq F^{\leftarrow}(\bigvee_{j\in J} G_j) = \bigvee_{j\in J} F^{\leftarrow}G_j.$$

Also, $F^{\leftarrow}G_j \in bgo(S_1)$ because (f, F) is b-g-irresolute. So by the b-g-compactness of A there exists $J^{'} \subseteq J$ finite such that $A \subseteq \cup_{j\in J^{'}} F^{\leftarrow}G_j$. Hence

$$f^{\to}A \subseteq f^{\to}(\cup_{j\in J^{'}} F^{\leftarrow}G_j) = \cup_{j\in J^{'}} f^{\to}(F^{\leftarrow}G_j) \subseteq \cup_{j\in J^{'}} G_j$$

by [[4], Corollary 2.12 (2) and Theorem 2.24 (2 b)]. This establishes that $f^{\to}A$ is b-g-compact. ■

Corollary 3.11. Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a surjective b-g-irresolute difunction. Then, if $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ is b-g-compact so is $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$.

Proof: This follows by taking $A = S_1$ in Theorem 3.10 and noting that $f^{\to}S_1 = f^{\to}(F^{\leftarrow}S_2) = S_2$ by [[4], Proposition 2.28 (1 c) and Corollary 2.33 (1)]. ■

Definition 3.12. A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called b-g-stable if very b-g-closed set $F \in \varphi|\{S\}$ is b-g-compact in S.

Corollary 3.13. Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:

1. Every b-g-stable is stable.
2. Every g-stable is b-g-stable.

Proof: Clear. ■

Theorem 3.14. Let $(S, \varphi, \tau, k, \sigma)$ be b-g-stable. If G is an b-g-open set with $G \neq \phi$ and $D = \{F_j : j \in J\}$ is a family of b-g-closed sets with $\cap_{j\in J} F_j \subseteq G$ then $\cap_{j\in J^{'}} F_j \subseteq G$ for a finite subsets $J^{'}$ of J.

Proof: Let $(S, \varphi, \tau, k, \sigma)$ be b-g-stable, let G be an b-g-open set with $G \neq \phi$ and $D = \{F_j : j \in J\}$ be a family of b-g-closed sets with $\cap_{j\in J} F_j \subseteq G$. Set $K = \sigma(G)$. Then K is b-g-closed and satisfies $K \neq S$. Hence K is b-g-compact. Let $C = \{\sigma(F) | F \in D\}$. Since $\cap D \subseteq G$ we have $K \subseteq \bigvee C$, that is C is an b-g-open cover of K. Hence there exists $F_1, F_2, ..., F_n \in D$ so that

$$K \subseteq \sigma(F_1) \cup \sigma(F_2) \cup ... \cup \sigma(F_n) = \sigma(F_1 \cap F_2 \cap ... \cap F_n).$$

This gives $F_1 \cap F_2 \cap ... \cap F_n \subseteq \sigma(K) = G$, so $\cap_{j \in J'} F_j \subseteq G$ for a finite subsets $J' = \{1, 2, ..., n\}$ of J. ■

Theorem 3.15. Let $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$, $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be two complemented ditopological texture spaces with $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ is b-g-stable, and $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be an b-g-bi-irresolute surjective difunction. Then $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is b-g-stable.

Proof: Take $K \in bgc(S_2)$ with $K \neq S_2$. Since (f, F) is b-g-co-irresolute, so $f^{\leftarrow}K \in bgc(S_1)$. Let us prove that $f^{\leftarrow}K \neq S_1$. Assume the contrary. Since $f^{\leftarrow}S_2 = S_1$, by [[4], Lemma 2.28 (1 c)] we have $f^{\leftarrow}S_2 \subseteq f^{\leftarrow}K$, whence $S_2 \subseteq K$ by [[4], Corollary 2.33 (1 ii)] as (f, F) is surjective. This is a contradiction, so $f^{\leftarrow}K \neq S_1$. Hence $f^{\leftarrow}(K)$ is b-g-compact in $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ by b-g-stability. As (f, F) is b-g-irresolute, $f^{\rightarrow}(f^{\leftarrow}K)$ is b-g-compact for the ditopology (τ_2, k_2) by Theorem 3.10, and by [[4], Corollary 2.33 (1)] this set is equal to K. This establishes that $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is b-g-stable. ■

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