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sb* - Separation axioms

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Abstract

The aim of this paper is to introduce some new type of seperation axioms and study some of their basic properties. Some implications between T_0, T_1 and T_2 axioms are also obtained.

Keywords: sb*-open sets, sb*- closed sets, sb*-T_0, sb*-T_1, sb*- T_2.
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1 Introduction

Andrijevic[1] introduced a new class of generalized open sets called b-open sets in topological spaces. This type of sets was discussed by [5] under the name of γ - open sets. Several research papers [2,3,4,13,15] with advance results in different aspects came into existence. Further, Caldas and Jafari [4], introduced and studied b-T_0, b-T_1, b- T_2, b-D_0, b-D_1 and b-D_2 via b-open sets. After to that Keskin and Noiri [7], introduced the notion of b-$T_{1/2}$. Recently, the authors[16,17,18] introduced and studied about the sb* - closed sets, sb*-open map, sb*-continuous map, sb*- irresolute and Homeomorphisms in topological spaces. In the present paper, sb*-seperation axioms are introduced via sb*-open sets and some of its basic properties are discussed.

2 Preliminaries

Throughout this paper, X and Y denote the topological spaces (X, τ) and (Y,σ) respectively and on which no seperation axioms are assumed unless otherwise explicitly stated. Let A be a subset of the space X. The interior and closure of a set A in X are denoted by int(A) and cl(A) respectively.The complement of A is denoted by (X-A) or A^c. In this section, let us recall some definitions and results which are useful in the sequel.

Definition 2.1. [1] A subset A of a topological space (X, τ) is called *b*-open set if A $\subseteq$ (cl(int(A))$\cup$ int(cl(A))). The complement of a b-open set is said to be b-closed. The family of all b-open subsets of a space X is denoted by BO(X).

Definition 2.2. A subset A of a space X is called
(1) semi-open if A $\subseteq$ (cl(int(A))[8];
(2) α-open if A $\subseteq$ int(cl(int(A)))[14].
The complement of a semi-open (resp. α-open) set is called semiclosed [12](resp. α-closed[19]).

Definition 2.3. [16] A subset A of a topological space (X, τ) is called a sb*-closed set (briefly sb*-closed) if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is b-open in X.The complement of sb*-closed set is called sb*-open. The family of all sb*-open sets of a space X is denoted by sb*O(X).

Definition 2.4. [4] A space X is said to be :
(1) b-T_0 if for each pair of distict points x and y in X, there exists a b-open set A containing x but not y or a b-open set B containing y but not x.
(2) b-T_1 if for each pair x; y in X, x$\neq$y, there exists a b-open set G containing x but not y and a b-open set B containing y but not x.

Definition 2.5. [15] A space X is said to b-T_2 if for any pair of distinct points x and y in X, there exist U$\in$BO(X,x) and V$\in$BO(X,y) such that U$\cap$V=ϕ.

Definition 2.6. A space X is said to be :
(1) α-T_0 if for each pair of distinct points in X, there is an α - open set containing one of the points but not the other[9].
(2) α-T_1 if for each pair of distinct points x and y of X, there exists α-open sets U and V containing x and y respectively such that y$\notin$U and x $\notin$V[9].
(3) α-T_2 if for each pair of distinct points x and y of X, there exist disjoint α-open sets U and V containing x and y respectively[11].

Definition 2.7. [10] (i) Let X be a topological space. For each x$\neq$y$\in$X, there exists a set U, such that x$\in$U, y$\notin$U, and there exists a set V such that y$\in$ V, x$\notin$ V, then X is called w-T_1 space, if U is open and V is w-open sets in X.
(ii) Let X be a topological space. And for each x$\neq$y$\in$ X, there exist two disjoint sets U and V with x$\in$U and y$\in$ V, then X is called w- T_2 space if U is open and V is w-open sets in X.

Definition 2.8. [10] A topological space X is (1) semi T_0 if to each pair of distinct points x,y of X, there exists a semi open set A containing x but not y or a semi open set B containing y but not x.
(2) semi T_1 if to each pair of distinct points x, y of X, there exists a semi open set A containing x but not y and a semi open set B containing y but not x.
(3) semi T_2 if to each pair of distinct points x, y of X, there exist disjoint semi open sets A and B in X s.t. x$\in$ A, y $\in$ B.

Definition 2.9. [20] A topological space X is called a T_0 space if and only if it satisfies the following axiom of Kolmogorov. (T_0) If x and y are distinct points of X, then there exists an open set which contains one of them but not the other.

Definition 2.10. [20] A topological space X is a T_1 -space if and only if it satisfies the following seperation axiom of Frechet. (T_1) If x and y are two distinct points of X, then there exists two open sets, one containing x but not y and the other containing y but not x.

Definition 2.11. [20] A topological space X is said to be a T_2 - space or hausdorff space if and only if for every pair of distinct points x,y of X, there exists two disjoint open sets one containing x and the other containing y.

Theorem 2.12. [16] (i)Every open set is sb*-open.
(ii)Every α open set is sb*-open.
(iii)Every w-open set is sb*-open.
(iv)Every sb*-open set is b - open.

Definition 2.13. Let A be a subset of a space X. Then the sb*-closure of A is defined as the intersection of all sb*-closed sets containing A. ie., sb*-cl(A) = $\cap$ {F: F is sb*-closed, A$\subseteq$F}.

Definition 2.14. [17] Let X and Y be topological spaces. A map f: X $\rightarrow$ Y is called strongly b* - continuous (sb*- continuous) if the inverse image of every open set in Y is sb* - open in X.

Definition 2.15. [17] Let X and Y be a topological spaces. A map f : X $\rightarrow$ Y is called strongly b* -closed (sb* - closed) map if the image of every closed set in X is sb* - closed in Y.

Definition 2.16. [18] Let X and Y be topological spaces. A map f: (X,τ) $\rightarrow (Y, \sigma)$ is said to be sb* - Irresolute if the inverse image of every sb* - closed set in Y is sb* - closed set in X.

Definition 2.17. Let X be a topological space. A subset A$\subseteq$ X is called a sb* - neighbourhood (Briefly sb* - nbd) of a point x $\in$ X if there exists a sb* - open set G such that x $\in$ G $\subseteq$ A.

3 sb* - T_0 Spaces

In this section, we define sb* - T_0 space and study some of their properties.

Definition 3.1. A topological space X is said to be sb*-T_0 if for every pair of distinct points x and y of X, there exists a sb*-open set G such that x$\in$G and y$\notin$ G or y$\in$G and x $\notin$G.

Theorem 3.2. Every α-T_0 space is sb*-T_0.

Proof: Let X be a α-T_0 space. Let x and y be any two distinct points in X. Since X is α-T_0, there exists a α open set U such that x$\in$ U and y$\notin$U or y$\in$U and x$\notin$U. By Theorem 2.11(ii), U is a sb*-open set such that x$\in$ U and y$\notin$U or x$\notin$U and y $\in$ U. Thus X is sb*-T_0. ∎

Theorem 3.3. Every topological space X is sb*-T_0.

Proof: Since every topological space is α-T_0 and by the above Theorem every topological space X is sb*-T_0. ■

Theorem 3.4. A space X is sb*-T_0 space if and only if sb*-closures of distinct points are distinct.

Proof: Necessity: Let x,y $\in$ X with x$\neq$y and X be a sb*-T_0 space. Since X is sb*-T_0, by Definition 3.1, there exists an sb*-open set G such that x$\in$G but y$\notin$G. Also x$\notin$ X-G and y$\in$ X-G, where X-G is a sb*-closed set in X. Since sb*cl({y}) is the smallest sb*-closed set containing y, sb*cl({y})$\subseteq$X-G. Hence y$\in$sb*cl({y}) but x $\notin$sb*cl({y}) as x$\notin$X-G. Consequently sb*cl({x})$\neq$sb*cl({y}).
Sufficicency: Suppose that for any pair of distinct points x,y$\in$ X, sb*cl({x})$\neq$sb*cl({y}). Then there exists atleast one point z$\in$ X such that z$\in$sb*cl({x}) but z$\notin$sb*cl({y}). Suppose we claim that x$\notin$sb*cl({y}). For, if x$\in$ sb*cl({y}), then sb*cl({x})$\subseteq$sb*cl({y}). So z$\in$ sb*cl({y}),which is a contradiction. Hence x$\notin$sb*cl({y}). Which implies that x$\in$ X-sb*cl({y}) is a sb*-open set in X containing x but not y. Hence X is a sb*-T_0 space. ■

Theorem 3.5. Every subspace of a sb*-T_0 space is sb*-T_0.

Proof: Let (Y, τ^*) be a subspace of a space X where τ^* is the relative topology of τ on Y. Let y_1, y_2 be two distinct points of Y. As Y$\subseteq$ X, y_1 and y_2 are distinct points of X and there exists a sb*-open set G such that $y_1 \in$ G but $y_2 \notin$G since X is sb*-T_0. Then G$\cap$Y is a sb*-open set in (Y, τ^*) which contains y_1 but does not contain y_2. Hence (Y, τ^*) is a sb*-T_0 space. ■

4 sb*- T_1 Spaces

Definition 4.1. A space X is said to be sb*-T_1 if for every pair of distinct points x and y in X, there exist sb* - open sets U and V such that x $\in$ U but y$\notin$U and y $\in$ V but x$\notin$ V.

Proposition 4.2. (i) Every w-T_1 space is sb*- T_1.
(ii) Every sb*-T_1 space is b-T_1.

Proof: (i) Suppose X is a w- T_1 space. Let x and y be two distinct points in X. Since X is w-T_1, there exist w- open sets U and V such that x$\in$U but y $\notin$ U and y $\in$V but x $\notin$V. By Theorem 2.11(iii), U and V are sb*- open sets such that x$\in$ U but y $\notin$ U and y $\in$V but x$\notin$ V. Hence X is sb*-T_1.
(ii) Suppose X is a sb*-T_1 space. Let x and y be two distinct points in X. Since X is sb*-T_1, there exist sb*-open sets U and V such that x$\in$U but y $\notin$ U and y $\in$ V and x $\notin$V. By Theorem 2.11(iv) , U and V are b-open sets such that x$\in$ U but y $\notin$ U and y$\in$ V but x $\notin$ V. Thus X is b-T_1. ■

Remark 4.3. The converse of the above proposition is not true as shown in the following examples.

Example 4.4. Consider the space (X, τ), where X = {a,b,c} and τ = {ϕ, {a,b}, X}. Clearly (X, τ) is sb* -T_1 but not w-T_1. This shows that sb*-T_1 does not imply w- T_1.

Example 4.5. Consider the space (X, τ) where X = {a,b,c, d} and τ = {ϕ,{a},{b},{a,b},{a,b,c},X }. Then (X, τ) is b-T_1 but not sb*-T_1. This shows that b-T_1 does not imply sb*-T_1.

Remark 4.6. The concepts of sb*-T_1 and semi -T_1 are independent as shown in the following examples.

Example 4.7. Consider the space (X, τ), where X = {a,b,c,d} and τ = {ϕ,{a},{b},{c}, {a,b},{a,c},{b,c},{a,b,c},X}. Clearly (X, τ) is semi-T_1 but not sb*-T_1. This shows that semi-T_1 does not imply sb*-T_1.

Example 4.8. Consider the space (X,τ) , where X= {a,b,c,d} and τ = { ϕ, {a,b}, {a,b,c},{a,b,d}, X }. Then (X,τ) is sb*-T_1 but not semi - T_1. This shows that sb*-T_1 does not imply semi-T_1.

Theorem 4.9. Let f: X$\rightarrow$ Y be a sb* - irresolute, injective map. If Y is sb*-T_1 , then X is sb*-T_1.

Proof: Assume that Y is sb*-T_1. Let x, y $\in$Y be such that x$\neq$ y. Then there exists a pair of sb*-open sets U , V in Y such that f(x)$\in$U, f(y)$\in$ V and f(x) $\notin$ V, f(y) $\notin$ U. Then x$\in f^{-1}$(U), y $\notin f^{-1}$(U)and y$\in f^{-1}$(V), x$\notin f^{-1}$(V). Since f is sb*-irresolute, X is sb*-T_1. ■

Theorem 4.10. A space (X, τ) is sb*- T_1 if and only if for every x$\in$X, sb*cl{x}={x}.

Proof: Let (X, τ) be sb*-T_1 and x$\in$X. Then for each y $\neq$ x, there exists a sb*-open set G such that x$\in$G but y$\notin$G. This implies that y$\notin$sb*cl{x}, for every y$\in$ X and y $\neq$ x. Thus {x} = sb*cl{x}.

Conversely, suppose sb*cl{x}={x} for every x$\in$ X. Let x, y be two distinct points in X. Then x$\notin${y} = sb*cl{y}implies there exists a sb*-closed set B_1 such that y$\in B_1$, x$\notin B_1$ implies B_1^c is a sb*-open set such that x$\in B_1^c$ but y $\notin B_1^c$.
Also y$\notin${x} = sb*cl{x} $\Rightarrow$ there exists a sb*-closed set B_2 such that x $\in B_2$, y$\notin B_2$. Which implies that B_2^c is a sb*-open set such that y $\in B_2^c$ but x $\notin B_2^c$. By Definition 4.1, (X, τ) is sb*-T_1. ■

Theorem 4.11. Let f: X $\rightarrow$ Y be bijective.
(i) If f is sb* continuous and (Y,τ_2) is T_1 , then (X, τ_1) is sb*-T_1.
(ii) If f is sb*-open and (X, τ) is sb*-T_1 then (Y, τ_2) is sb*-T_1.

Proof: Let f: (X, τ_1) $\rightarrow$ (X, τ_2) be bijective.

(i) Suppose f: (X, τ_1)$\rightarrow$(Y, τ_2) is sb*-continuous and (Y, τ_2) is T_1. Let x_1, $x_2 \in$ X with $x_1 \neq x_2$. Since f is bijective, $y_1 = f(x_1) \neq f(x_2) = y_2$ for some y_1 , $y_2 \in$ Y. Since (Y, τ_2) is T_1, there exist open sets G and H such that $y_1 \in$G but $y_2 \notin$G and $y_2 \in$H but $y_1 \notin$H. Since f is bijective, $x_1 = f^{-1}(y_1) \in f^{-1}$(G) but $x_2 = f^{-1}(y_2) \notin f^{-1}$(G) and $x_2 = f^{-1}(y_2) \in f^{-1}$(H) but $x_1 = f^{-1}(y_1) \notin f^{-1}$(H). Since f is sb*-continuous, f^{-1}(G) and f^{-1}(H)are sb*- open sets in (X, τ_1). It follows that (X, τ_1) is sb* - T_1. This proves (i).

(ii) Suppose f is sb*-open and (X, τ_1) is sb*-T_1. Let $y_1 \neq y_2 \in$ Y. Since f is bijective, there exist x_1, x_2 in X, such that $f(x_1) = y_1$ and $f(x_2) = y_2$ with $x_1 \neq x_2$. Since (X, τ_1) is sb* - T_1, there exist sb*-open sets G and H in X such that $x_1 \in$ G but $x_2 \notin$ G and $x_2 \in$H but $x_1 \notin$ H. Since f is sb* -open, f(G) and f(H) are sb*-open in Y such that $y_1 = f(x_1) \in$ f(G) and $y_2 = f(x_2) \in$ f(H). Again since f is bijective, $y_2 = f(x_2) \notin$ f(G) and $y_1 = f(x_1) \notin$ f(H). Thus (Y, τ_2) is sb* - T_1. This proves (iii). ∎

5 sb*- T_2 Spaces

In this section we introduce sb*-T_2 space and investigate some of their basic properties.

Definition 5.1. A space X is said to be sb*-T_2 if for every pair of distinct points x and y in X, there are disjoint sb*- open sets U and V in X containing x and y respectively.

Theorem 5.2. (i) Every w-T_2 space is sb*-T_2.
(ii) Every α - T_2 space is sb*-T_2.

Proof: (i)Let X be a w-T_2 space. Let x and y be two distinct points in X. Since X is w-T_2, there exist disjoint w-open sets U and V such that x$\in$ U and y $\in$V. By Theorem 2.11(ii), U and V are disjoint sb*-open sets such that x$\in$ U and y $\in$V. Hence X is sb*-T_2.

ii) Suppose X is α-T_2 space. Let x and y be two disjoint α open sets U and V such that x $\in$U and y $\in$V. By Theorem 2.11(i), U and V are disjoint sb*-open sets such that x$\in$U and y$\in$V. Hence X is sb*-T_2. ∎

Remark 5.3. The converse of the statements (i) and(ii) of the above Theorem is not true as shown in the following examples.

Example 5.4. Consider the space (X, τ), where X= {a,b,c} and τ= {ϕ, {a,b}, X}. Then (X, τ) is sb*-T_2 but not w-T_2. This shows that sb*-T_2 does not imply w-T_2.

Example 5.5. Consider the space (X, τ), where X= {a,b,c} and τ={ϕ, {b},{a,c},X}. It can be verified that (X, τ) is sb*-T_2 but not α-T_2. This shows that sb*-T_2 does not imply α-T_2.

Remark 5.6. The concepts of semi-T_2 and sb*-T_2 are independent as shown in the following examples.

Example 5.7. Consider the space (X, τ), wher X={a,b,c,d} and τ = {ϕ,{a}, {b},{a,b},X}. It can be verified that (X, τ) is semi - T_2 but not sb*-T_2. This shows that semi-T_2 does not imply sb*-T_2.

Example 5.8. Consider the space (X, τ), where X= {a,b,c,d} and τ={ϕ, {a,b},X}. Then (X, τ) is sb*-T_2 but not semi - T_2. This shows that sb*-T_2 does not imply semi-T_2.

Remark 5.9. Every sb*-T_2 space is b-T_2. But the converse is not true as shown in the following example.

Example 5.10. Consider the space (X, τ), where X = {a,b,c} and τ= {ϕ, {a}, {c},{a,c},X}. Clearly (X, τ) is b- T_2 but not sb*-T_2. This shows that b-T_2 does not imply sb*-T_2.

Theorem 5.11. Every sb*-T_2 space is sb*-T_1.

Proof: Let X be a sb*-T_2 space. Let x and y be two distinct points in X. Since X is sb*-T_2, there exist disjoint sb*-open sets U and V such that x$\in$ U and y$\in$ V. Since U and V are disjoint, x$\in$ U but y$\notin$ U and y$\in$V but x $\notin$ V. Hence X is sb*-T_1. ■

However the converse is not true as shown in the following example.

Example 5.12. Consider the space (X, τ), where X = {a,b,c,d} and τ = {ϕ,{a,b},X}. Then (X, τ) is sb*-T_1 but not sb*-T_2. This shows that sb*-T_1 does not imply sb*-T_2.

Theorem 5.13. For a topological space X, the following are equivalent:
(i) X is a sb*-T_2 space.
(ii) Let x$\in$ X. Then for each y$\neq$ x there exists a sb*-open set U such that x$\in$U and y $\notin$ sb*cl(U).
(iii) For each x$\in$X, $\cap$ { sb*-cl(U): U$\in$ sb*O(X) and x$\in$ U} = {x}.

Proof: (i)$\Rightarrow$ (ii): Suppose X is a sb*-T_2 space. Then for each y$\neq$ x there exist disjoint sb*-open sets U and V such that x$\in$ U and y$\in$ V. Since V is sb*-open, V^c is sb* - closed and U$\subseteq$ V^c. This implies that sb*cl(U) $\subseteq$ V^c. Since y $\notin$ V^c, y $\notin$ sb*cl(U).

(ii)$\Rightarrow$ (iii) : If y$\neq$ x, then there exists a sb*-open set U such that x $\in$ U and y$\notin$ sb*cl(U). Therefore y$\notin$ $\cap${sb*cl(U):U$\in$sb*O(X) and x$\in$ U}. Therefore $\cap${sb*cl(U):U$\in$sb*O(X) and x$\in$ U}={x}. This proves (iii).

(iii)$\Rightarrow$ (i): Let y$\neq$ x in X. Then y $\notin$ {x} = $\cap${sb*cl(U):U$\in$sb*O(X) and x$\in$ U}. This implies that there exists a sb*-open set U such that x$\in$ U and y $\notin$ sb*cl(U). Let V= $(sb^*cl(U))^c$. Then V is sb*-open and y $\in$V. Now U$\cap$V = U$\cap$$(sb^*cl(U))^c$ $\subseteq$U$\cap$$(U)^c$=$\phi$. Therefore X is sb*-$T_2$ space. ■

Theorem 5.14. Let f:X$\rightarrow$Y be a bijection.
(i) If f is sb*-open and X is T_2, then Y is sb*-T_2.
(ii) If f is sb*-contiuous and Y is T_2, then X is sb*-T_2.

Proof: Let f:X $\rightarrow$ Y be a bijection.

(i) Suppose f is sb*-open and X is T_2. Let $y_1 \neq y_2 \in$ Y. Since f is a bijection, there exist x_1, x_2 in X such that $f(x_1) = y_1$ and $f(x_2) = y_2$ with $x_1 \neq x_2$. Since X is T_2, there exist disjoint open sets U and V in X such that $x_1 \in$U and $x_2 \in$V. Since f is sb*-open, f(U) and f(V) are sb*-open in Y such that y_1=f(x_1) $\in$f(U) and y_2=f(x_2)$\in$f(V). Again since f is a bijection, f(U) and f(V) are disjoint in Y. Thus Y is sb*-T_2.

(ii) Suppose f: X $\rightarrow$Y is sb*-continuous and Y is T_2. Let x_1, x_2 $\in$X with $x_1 \neq x_2$. Let y_1 = f(x_1) and y_2 = f(x_2). Since f is one-one, $y_1 \neq y_2$. Since Y is T_2, there exist disjoint open sets U and V containing y_1 and y_2 respectively. Since f is sb*-continuous bijective, f^{-1}(U) and f^{-1}(V) are disjoint sb*-open sets containing x_1 and x_2 respectively. Thus X is sb*-T_2. ■

Theorem 5.15. A topological space (X, τ) is sb*-T_2 if and only if the intersection of all sb*-closed, sb*-neighbourhoods of each point of the space is reduced to that point.

Proof: Let (X, τ) be sb*-T_2 and x$\in$ X. Then for each y $\neq$ x in X, there exist disjoint sb*-open sets U and V such that x$\in$U , y$\in$V. Now U$\cap$ V = ϕ implies x$\in$U$\subseteq V^c$. Therefore V^c is a sb*-neighbourhood of x. Since V is sb*-open, V^c is sb* closed and sb* -neighbourhood of x to which y does not belong. That is there is a sb*-closed, sb*-neighbourhoods of x which does not contain y. so we get the intersection of all sb* - closed, sb*-neighbourhood of x is {x}.

Conversely, let x, y$\in$X such that x $\neq$y in X. Then by assumption, there exist a sb*-closed , sb*-neighbourhood V of x such that y $\notin$V. Now there exists a sb*-open set U such that x $\in$ U $\subseteq$ V. Thus U and V^c are disjoint sb*-open sets containing x and y respectively. Thus (X, τ) is sb*-T_2. ■

Theorem 5.16. If f: X $\rightarrow$ Y be bijective, sb*-irresolute map and X is sb*-T_2, then (X, τ_2) is sb*-T_2.

Proof: Suppose f: (X, τ_1 $\rightarrow$ (Y,τ_2) is bijective. And f is sb*-irresolute, and (Y, τ_2) is sb*-T_2. Let x_1, x_2 $\in$X with $x_1 \neq x_2$. Since f is bijective, y_1= f(x_1) $\neq$f(x_2) = y_2 for some y_1, y_2 $\in$ Y. Since (Y, τ_2) is sb*-T_2, there exist disjoint sb*-open sets G and H such that $y_1\in$G and $y_2\in$H. Again since f is bijective, x_1 = $f^{-1}(y_1)\in f^{-1}$(G) and x_2 = $f^{-1}(y_2)\in f^{-1}$(H). Since f is sb*-irresolute, f^{-1}(G) and f^{-1}(H) are sb*-open sets in (X, τ_1). Also f is bijective, G$\cap$H= ϕ implies that f^{-1}(G)$\cap$ f^{-1}(H) = f^{-1}(G$\cap$ H)==$f^{-1}(\phi)$=ϕ. It follows that (X, τ_2) is sb*-T_2. ■

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