International Journal of Mathematics and Soft Computing
Vol.5, No.2 (2015), 145 - 153.

ISSN Print : 2249 - 3328
ISSN Online: 2319 - 5215

On m- Neighbourly irregular fuzzy graphs

N.R. Santhi Maheswari[1], C. Sekar[2]

[1] Department of Mathematics
G. Venkataswamy Naidu College, Kovilpatti, India.
nrsmaths@yahoo.com

[2] Department of Mathematics
Aditanar College of Arts and Science
Tiruchendur- 628 216, Tamilnadu, India.
sekar.acas@gmail.com

Abstract

In this paper, d_m-degree and total d_m-degree of a vertex in fuzzy graphs, m- neighbourly irregular fuzzy graphs and m- neighbourly totally irregular fuzzy graphs are introduced. Some properties on m-neighbourly irregular fuzzy graphs are also discussed in this paper. Comparative study between m- neighbourly irregular fuzzy graphs and m- neighbourly totally irregular fuzzy graphs is done and m- neighbourly irregularity on some fuzzy graphs whose underlying crisp graphs are a cycle and a path is also studied.

Keywords: total degree, fuzzy graph, regular fuzzy graph, totally regular fuzzy graph, totally irregular fuzzy graphs, d_2 -degree, total d_2- degree, irregular fuzzy graph.
AMS Subject Classification(2010): 05C12, 03E72, 05C72.

1 Introduction

Azriel Rosenfeld introduced fuzzy graphs in 1975[3]. It has been growing fast and has numerous applications in various fields. A. Nagoor Gani and S.R. Latha [1] introduced irregular fuzzy graphs, total degree and totally irregular fuzzy graphs. N.R. Santhi Maheswari and C. Sekar introduced d_2 - of a vertex in graphs [4] and also discussed some properties on d_2 - of a vertex in graphs and introduced 2-neighbourly irregular graphs (semi neighbourly irregular graphs)and discussed some properties on 2 - neighbourly irregular graphs[4].

N.R. Santhi Maheswari and C. Sekar introduced d_2- of a vertex in fuzzy graphs, $(2, k)$-regular fuzzy graphs, totally $(2, k)$-regular fuzzy graphs [5], $(r, 2, k)$-regular fuzzy graph and totally $(r, 2, k)$-regular fuzzy graph [6], 2- neighbourly irregular fuzzy graphs, 2- neighbourly totally irregular fuzzy graphs[7].

In this paper, we introduce d_m-degree and total d_m-degree of a vertex in fuzzy graphs, m- neighbourly irregular fuzzy graphs and m- neighbourly totally irregular fuzzy graphs and also discussed some properties on m- neighbourly irregular fuzzy graphs. We make comparative study between m- neighbourly irregular fuzzy graphs and m- neighbourly totally irregular fuzzy graphs. Also m- neighbourly irregularity on some fuzzy graphs whose underlying crisp graphs are a cycle and a path is studied.

2 Preliminaries

We present some known definitions and results for a ready reference to go through the work presented in this paper.

Definition 2.1. A Fuzzy graph denoted by $G:(\sigma,\mu)$ on the graph $G^*:(V,E)$ is a pair of functions (σ,μ) where $\sigma : V \rightarrow [0,1]$ is a fuzzy subset V and $\mu : V \times V \rightarrow [0,1]$ is a symmetric fuzzy relation on σ such that for all u, v in V, the relation $\mu(u,v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is complete if $\mu(u,v) = \mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$, where uv denotes the edge joining the vertices u and v. $G^*:(V,E)$ is called the underlying crisp graph of the fuzzy graph $G:(\sigma,\mu)$, where σ and μ are called membership function.

Definition 2.2. Let $G:(\sigma,\mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u) = \sum_{u\neq v} \mu(uv)$, for $uv \in E$ and $\mu(uv) = 0$, for uv not in E; this is equivalent to $d_G(u) = \sum_{uv\in E} \mu(uv)$.

Definition 2.3. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. If $d(v) = k$ for all $v \in V$, then G is said to be a regular fuzzy graph of degree k.

Definition 2.4. The strength of connectedness between two vertices u and v is $\mu^\infty(u,v) = sup\{\mu^k(u,v) : k = 1,2,\dots\}$, where $\mu^k(u,v) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \dots\cdots \wedge \mu(u_{k-1}v) : u, u_1, u_2, \dots, u_{k-1}, v$ is a path connecting u and v of length $k\}$.

Definition 2.5. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The total degree of a vertex u is defined as $td(u) = \sum \mu(u,v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$. If each vertex of G has the same total degree k, then G is said to be totally regular fuzzy graph of degree k or k-totally regular fuzzy graph.

Definition 2.6. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. Then G is an irregular fuzzy graph if no two vertices have the same degree [1].

Definition 2.7. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. Then G is a totally irregular fuzzy graph if no two vertices have the same total degree [1].

Definition 2.8. Let $G:(\sigma,\mu)$ be a connected fuzzy graph on $G^*:(V,E)$. Then G is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct degrees [1].

Definition 2.9. If every two adjacent vertices of a fuzzy graph $G : (\sigma, \mu)$ have distinct total degrees, then G is said to be a neighbourly total irregular fuzzy graph [1].

Definition 2.10. Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_2-degree of a vertex u in G is $d_2(u) = \sum \mu^2(u, v)$, where $\mu^2(u, v) = sup\{\mu(u, u_1) \wedge \mu(u_1, v) : u, u_1, v$ is a shortest path of length of $2\}$. Also $\mu(uv) = 0$, for uv not in E [5].

Definition 2.11. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_2(u) = k$ for all $u \in V$, then G is said to be a $(2, k)$-regular fuzzy graph [5].

Definition 2.12. Let $G : (\sigma, \mu)$ be fuzzy graph on $G^* : (V, E)$. The total d_2-degree of a vertex $u \in V$ is defined as $td_2(u) = \sum \mu^2(u, v) + \sigma(u) = d_2(u) + \sigma(u)$ [5].

Definition 2.13. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d(u) = r$ and $d_2(u) = k$, for all $u \in V$, then G is said to be a $(r, 2, k)$-regular fuzzy graph [6].

Definition 2.14. If each vertex of G has the same total degree r and total d_2-degree k, then G is said to be a totally $(r, 2, k)$ - regular fuzzy graph [6].

Definition 2.15. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a 2-neighbourly irregular fuzzy graph if every two adjacent vertices of G have distinct d_2-degrees [7].

Definition 2.16. If every two adjacent vertices of a fuzzy graph $G : (\sigma, \mu)$ on $G^* : (V, E)$ have distinct total d_2-degrees, then G is said to be a 2-neighbourly totally irregular fuzzy graph [7].

3 d_m-degree and total d_m-degree of a Vertex in Fuzzy Graph

In this section, we define d_m-degree of a vertex in Fuzzy Graph and Total d_m-degree of a vertex in Fuzzy Graph[8].

Definition 3.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The d_m-*degree* of a vertex u in G is $d_m(u) = \sum \mu^m(uv)$, where $\mu^m(uv) = sup\{\mu(uu_1) \wedge \mu(u_1u_2) \wedge \ldots, \mu(u_{m-1}v) : u, u_1, u_2, \ldots, u_{m-1}, v$ is the shortest path connecting u and v of length $m\}$. Also, $\mu(uv) = 0$, for uv not in E.
The minimum d_m-degree of G is $\delta_m(G) = \wedge\{d_m(v) : v \in V\}$.
The maximum d_m-degree of G is $\Delta_m(G) = \vee\{d_m(v) : v \in V\}$[8].

Example 3.2. Consider $G^* : (V, E)$ where $V = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ and $E = \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_1\}$. Define $G : (\sigma, \mu)$ by $\sigma(u_1) = 0.1, \sigma(u_2) = 0.2, \sigma(u_3) = 0.3, \sigma(u_4) = 0.4, \sigma(u_5) = 0.5, \sigma(u_6) = 0.6, \sigma(u_7) = 0.7$ and $\mu(u_1u_2) = 0.1, \mu(u_2u_3) = 0.2, \mu(u_3u_4) =$

$0.3, \mu(u_4u_5) = 0.4, \mu(u_5u_6) = 0.5, \sigma(u_6u_7) = 0.6, \sigma(u_7u_1) = 0.1$

$$\begin{aligned}
d_3(u_1) &= \{0.1 \wedge 0.2 \wedge 0.3\} + \{0.1 \wedge 0.6 \wedge 0.5\} = 0.1 + 0.1 = 0.2.\\
d_3(u_2) &= \{0.2 \wedge 0.3 \wedge 0.4\} + \{0.1 \wedge 0.1 \wedge 0.6\} = 0.2 + 0.1 = 0.3.\\
d_3(u_3) &= \{0.3 \wedge 0.4 \wedge 0.5\} + \{0.2 \wedge 0.1 \wedge 0.1\} = 0.3 + 0.1 = 0.4.\\
d_3(u_4) &= \{0.4 \wedge 0.5 \wedge 0.6\} + \{0.3 \wedge 0.2 \wedge 0.1\} = 0.4 + 0.1 = 0.5.\\
d_3(u_5) &= \{0.5 \wedge 0.6 \wedge 0.1\} + \{0.4 \wedge 0.3 \wedge 0.2\} = 0.1 + 0.2 = 0.3.\\
d_3(u_6) &= \{0.5 \wedge 0.4 \wedge 0.3\} + \{0.6 \wedge 0.1 \wedge 0.1\} = 0.3 + 0.1 = 0.4.\\
d_3(u_7) &= \{0.6 \wedge 0.5 \wedge 0.4\} + \{0.1 \wedge 0.1 \wedge 0.2\} = 0.4 + 0.1 = 0.5.
\end{aligned}$$

Definition 3.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total d_m-degree of a vertex $u \in V$ is defined as $td_m(u) = \sum \mu^m(uv) + \sigma(u) = d_m(u) + \sigma(u)$.
The minimum td_m-degree of G is $t\delta_m(G) = \wedge\{td_m(v) : v \in V\}$.
The maximum td_m-degree of G is $t\Delta_m(G) = \vee\{td_m(v) : v \in V\}$[8].

Example 3.4. Fuzzy graph given in above example 3.2, we have

$$\begin{aligned}
td_3(u_1) &= d_3(u_1) + \sigma(u_1) = 0.2 + 0.1 = 0.3.\\
td_3(u_2) &= d_3(u_2) + \sigma(u_2) = 0.3 + 0.2 = 0.5.\\
td_3(u_3) &= d_3(u_3) + \sigma(u_3) = 0.4 + 0.3 = 0.7.\\
td_3(u_4) &= d_3(u_4) + \sigma(u_4) = 0.5 + 0.4 = 0.9.\\
td_3(u_5) &= d_3(u_5) + \sigma(u_5) = 0.3 + 0.5 = 0.8.\\
td_3(u_6) &= d_3(u_6) + \sigma(u_6) = 0.4 + 0.6 = 1.0\\
td_3(u_7) &= d_3(u_7) + \sigma(u_7) = 0.5 + 0.7 = 1.2.
\end{aligned}$$

4 m- Neighbourly irregular fuzzy graph and m- Neighbourly totally irregular fuzzy graph.

In this section, we define m-neighbourly irregular fuzzy graphs and m-neighbourly totally irregular fuzzy graphs and campared through various examples. The neccessary and sufficient condition under which they are equivalent is provided.

Definition 4.1. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be an m- neighbourlyirregular fuzzy graph if every pair of adjacent vertices of G have distinct d_m-degrees.

Definition 4.2. If every pair of adjacent vertices of a fuzzy graph $G : (\sigma, \mu)$ have distinct total d_m- degrees, then G is said to be an m- neighbourly totally irregular fuzzy graph.

Definition 4.3. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a 3- neighbourly irregular fuzzy graph if every pair of adjacent vertices of G have distinct d_3-degrees.

Definition 4.4. If every pair of adjacent vertices of a fuzzy graph $G : (\sigma, \mu)$ have distinct total d_3- degrees, then G is said to be a 3- neighbourly totally irregular fuzzy graph.

Example 4.5. Consider $G^* : (V, E)$, where $V = \{u, v, w, x, y, z\}$ and $E = \{uv, vw, wx, xy, yz, zu\}$.

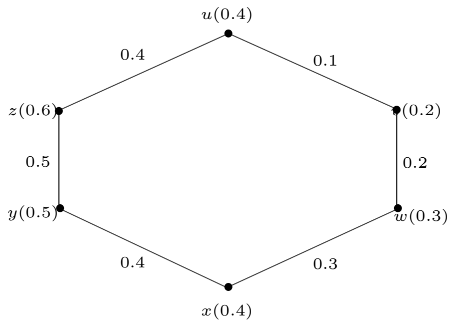

Figure 1: A 3-neighbourly irregular as well as 3-neighbourly totally irregular fuzzy graph.

$$\begin{aligned}
d_3(u) &= Sup\{0.1 \wedge 0.2 \wedge 0.3, 0.4 \wedge 0.5 \wedge 0.4\} = Sup\{0.1, 0.4\} = 0.4.\\
d_3(v) &= Sup\{0.2 \wedge 0.3 \wedge 0.4, 0.1 \wedge 0.4 \wedge 0.5\} = Sup\{0.2, 0.1\} = 0.2.\\
d_3(w) &= Sup\{0.3 \wedge 0.4 \wedge 0.5, 0.2 \wedge 0.1 \wedge 0.4\} = Sup\{0.3, 0.1\} = 0.3.\\
d_3(x) &= Sup\{0.4 \wedge 0.5 \wedge 0.4, 0.3 \wedge 0.2 \wedge 0.1\} = Sup\{0.4, 0.1\} = 0.4.\\
d_3(y) &= Sup\{0.5 \wedge 0.4 \wedge 0.1, 0.4 \wedge 0.3 \wedge 0.2\} = Sup\{0.1, 0.2\} = 0.2.\\
d_3(z) &= Sup\{0.4 \wedge 0.1 \wedge 0.2, 0.5 \wedge 0.4 \wedge 0.3\} = Sup\{0.1, 0.3\} = 0.3.
\end{aligned}$$

Note that in Figure 1, $d_3(u) = 0.4, d_3(v) = 0.2, d_3(w) = 0.3, d_3(x) = 0.4, d_3(y) = 0.2, d_3(z) = 0.3$. Hence G is 3-neighbourly irregular fuzzy graph.
Now, we calculate total d_3-degree of all the vertices of G.

$$\begin{aligned}
td_3(u) &= d_3(u) + \sigma(u) = 0.4 + 0.4 = 0.8.\\
td_3(v) &= d_3(v) + \sigma(v) = 0.2 + 0.2 = 0.4.\\
td_3(w) &= d_3(w) + \sigma(w) = 0.3 + 0.3 = 0.6.\\
td_3(x) &= d_3(x) + \sigma(x) = 0.4 + 0.4 = 0.8.\\
td_3(y) &= d_3(y) + \sigma(y) = 0.2 + 0.5 = 0.7.\\
td_3(z) &= d_3(z) + \sigma(z) = 0.3 + 0.6 = 0.9
\end{aligned}$$

It is noted that $td_3(u) = 0.8, td_3(v) = 0.4, td_3(w) = 0.6, td_3(x) = 0.8$ and $td_3(y) = 0.7, td_3(z) = 0.9$. Hence G is both 3-neighbourly irregular fuzzy graph and 3-neighbourly totally irregular fuzzy graph.

Example 4.6. An m-neighbourly totally irregular fuzzy graph G need not be m-neighbourly irregular fuzzy graph. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, such that $d_m(v) = k$, for all v in G. So, G is not an m-neighbourly irregular graph. If no two vertices of G have same membershiph value, then G is an m-neighbourly totally irregular graph.

Example 4.7. An m-neighbourly irregular fuzzy graph G need not be an m-neighbourly totally irregular fuzzy graph. For example, let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ which is Sub$(B_{2,2})$.

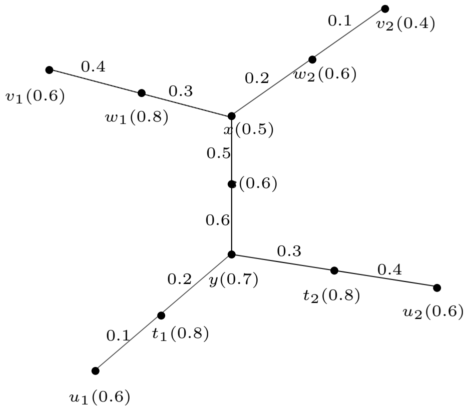

Figure 2: A 3-neighbourly irregular but not 3-neighbourly totally irregular fuzzy graph.

Note that in Figure 2, $d_3(v_1) = 0.5, d_3(w_1) = 0.4, d_3(x) = 0.5, d_3(w_2) = 0.4, d_3(v_2) = 0.2, d_3(s) = 0.8, d_3(y) = 0.5, d_3(t_1) = 0.4, d_3(u_1) = 0.2, d_3(t_2) = 0.4, d_3(u_2) = 0.5$. Hence $d_3(v_i) \neq d_3(w_i)$, $(i = 1, 2)$; $d_3(w_i) \neq d_3(x)$, $(i = 1, 2)$; $d_3(x) \neq d_3(s)$, $d_3(s) \neq d_3(y)$, $d_3(t_i) \neq d_3(y)$, $(i = 1, 2)$ and $d_3(t_i) \neq d_3(u_i)$, $(i = 1, 2)$. Hence G is a 3-neighbourly irregular fuzzy graph. But $td_3(v_1) = 1.1, td_3(w_1) = 1.2, td_3(x) = 1, td_3(w_2) = 1, td_3(v_2) = 0.6, td_3(s) = 1.4, td_3(y) = 1.2, td_3(t_1) = 1.2, td_3(t_2) = 1.2 td_3(u_1) = 0.8 and td_3(u_2) = 1.1$. Hence $td_3(x) = td_3(w_2)$, and $td_3(t_i) = td_3(y)$, $(i = 1, 2)$. Hence the fuzzy graph G is not a 3-neighbourly totally irregular.

Theorem 4.8. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If σ is a constant function, then G is an m-neighbourly totally irregular fuzzy graph if and only if G is an m-neighbourly irregular fuzzy graphs(m, a positive integer).

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Let u and v be any pair of adjacent vertices in the fuzzy graph $G : (\sigma, \mu)$. Let $G : (\sigma, \mu)$ be an m-neighbourly irregular fuzzy graph. Then the d_m-degree of every pair of adjacent vertices are distinct. This implies that $d_m(u) = k_1$ and $d_m(v) = k_2$, where $k_1 \neq k_2$ and $\sigma(u) = \sigma(v) = c$, a constant where $c \in [0, 1]$.

Now, $d_m(u) \neq d_m(v) \Rightarrow k_1 \neq k_2 \Rightarrow k_1 + c \neq k_2 + c \Rightarrow d_m(u) + \sigma(u) \neq d_m(v) + \sigma(v) \Rightarrow td_m(u) \neq td_m(v)$.

Then $td_m(u) \neq td_m(v)$. Hence any two adjacent vertices u and v with distinct d_m- degrees have their total d_m- degrees distinct, provided σ is a constant function. This is true for every pair of adjacent vertices in G.

Conversely, let $G : (\sigma, \mu)$ be an m-neighbourly totally irregular fuzzy graph. Then the total d_m- degrees of every pair of adjacent vertices are distinct. Let u and v be pair of adjacent vertices with d_m- degrees k_1 and k_2. Then $d_m(u) = k_1$ and $d_m(v) = k_2$.

Given that $\sigma(u) = \sigma(v) = c$, a constant where $c \in [0, 1]$ and $td_m(u) \neq td_m(v)$. Hence, $td_m(u) \neq td_m(v) \Rightarrow d_m(u)+\sigma(u) \neq d_m(v)+\sigma(v) \Rightarrow k_1+c \neq k_2+c \Rightarrow k_1 \neq k_2 \Rightarrow d_m(u) \neq d_m(v)$.

Hence any pair of adjacent vertices u and v with distinct total d_m-degrees have their d_m-degrees distinct, provided σ is a constant function. This is true for every pair of adjacent vertices in G. ■

5 m-Neighbourly irregular fuzzy graph on a cycle with some specific membership functions

Theorems 5.1 and 5.3 provide m-neighbourly irregularity on fuzzy graph $G : (\sigma, \mu)$ on a cycle $G^* : (V, E)$.

Theorem 5.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on a cycle $G^* : (V, E)$ of length $2m + 1$. If the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ are respectively $c_1, c_2, c_3, c_4, \ldots, c_{2m+1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m+1}$, then G is an m-neighbourly irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on a cycle $G^* : (V, E)$ of length $2m + 1$.

Let $e_1, e_2, e_3, \ldots, e_{2m+1}$ be the edges of the cycle G^* in that order. Let the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ be $c_1, c_2, c_3, \ldots, c_{2m+1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m+1}$.

$$\begin{aligned}
d_m(v_1) &= c_1 + c_{m+2}. \\
\text{For}, i &= 2, 3, 4, 5, \ldots, m + 1 \\
d_m(v_i) &= c_i + c_1. \\
d_m(v_{m+2}) &= c_2 + c_{m+2}. \\
\text{For}, i &= m + 3, m + 4, m + 5, \ldots, 2m + 1 \\
d_m(v_i) &= c_{i-m} + c_1.
\end{aligned}$$

Hence G is an m- neighbourly irregular fuzzy graph. ■

Remark 5.2. Even if the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m+1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m+1}$, then G need not be an m- neighbourly totally irregular fuzzy graphs.

Theorem 5.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on a cycle $G^* : (V, E)$ of length $2m + 1$. If the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ are respectively $c_1, c_2, c_3, c_4 \ldots, c_{2m+1}$ such that $c_1 > c_2 > c_3 > \cdots > c_{2m+1}$, then G is an m-neighbourly irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ of length $2m+1$. Let $e_1, e_2, e_3, \ldots, e_{2m+1}$ be the edges of the cycle G^* in that order.

Let the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ be respectively $c_1, c_2, c_3, \ldots,$ c_{2m+1} such that $c_1 > c_2 > c_3 > \ldots, > c_{2m+1}$.

$$\begin{aligned}
\text{For}, i &= 1, 2, 3, 4, 5, \ldots, m \\
d_m(v_i) &= c_{m+i-1} + c_{2m+1}. \\
d_m(v_{m+1}) &= c_m + c_{2m}. \\
\text{For}, i &= m + 2, m + 3, m + 4, m + 5, \ldots, 2m + 1 \\
d_m(v_i) &= c_{i-1} + c_{2m+1}.
\end{aligned}$$

Hence G is an m-neighbourly irregular fuzzy graph. ■

Remark 5.4. Even if the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m+1}$ such that $c_1 > c_2 > c_3 > \cdots > c_{2m+1}$, then G need not be an m-neighbourly totally irregular fuzzy graph.

Remark 5.5. Let $G : (\sigma, \mu)$ be a fuzzy graph on a cycle $G^* : (V, E)$ of length $2m + 1$. If the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m+1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m+1}$ and are distinct then G need not be an m-neighbourly irregular fuzzy graph.

6 m-Neighbourly irregular fuzzy graph on a path on 2m vertices with specific membership functions

Theorem 7.1 provides a condition for m-neighbourly irregularity on fuzzy graph $G : (\sigma, \mu)$ on a path $G^* : (V, E)$ on $2m$ vertices.

Theorem 6.1. Let $G : (\sigma, \mu)$ be a fuzzy graph on a path $G^* : (V, E)$ on $2m$ vertices. If the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m-1}$, then G is an m-neighbourly irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on a path $G^* : (V, E)$ on $2m$ vertices. Let $e_1, e_2, e_3, \ldots, e_{2m-1}$ be the edges of the path G^* in that order. Let membership value of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ be respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 < c_2 < c_3, \ldots, < c_{2m-1}$.

$$\begin{aligned}
\text{For}, i &= 1, 2, 3, 4, 5, \ldots, m \\
d_m(v_i) &= \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \ldots \mu(e_{i+m-2}) \wedge \mu(e_{i+m-1})\} \\
d_m(v_i) &= \{c_i \wedge c_{i+1} \wedge \ldots c_{i+m-2} \wedge c_{i+m-1}\} \\
&= c_i.
\end{aligned}$$

$$\begin{aligned}
\text{For}, i &= m + 1, m + 2, m + 3 \ldots, 2m \\
d_m(v_i) &= \{\mu(e_{i-1}) \wedge \mu(e_{i-2}) \ldots \mu(e_{i-(m-1)} \wedge \mu(e_{i-m})\} \\
d_m(v_i) &= \{c_{i-1} \wedge c_{i-2} \ldots c_{i-(m-1)} \wedge c_{i-m}\} \\
&= c_{i-m}
\end{aligned}$$

Hence G is an m-neighbourly irregular fuzzy graph. ■

Remark 6.2. Even if the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, c_4 \ldots, c_{2m-1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m-1}$, then G need not be an m-neighbourly totally irregular fuzzy graph.

Theorem 6.3. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$ vertices. If the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 > c_2 > c_3 > \ldots, > c_{2m-1}$, then G is an m-neighbourly irregular fuzzy graph.

Proof: Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on n vertices. Let $e_1, e_2, e_3, \ldots, e_{2m-1}$ be the edges of the path G^* in that order. Let membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$

are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 > c_2 > c_3 > \cdots > c_{2m-1}$.

$$\begin{aligned}
\text{For}, i &= 1,2,3,4,5,\ldots,m \\
d_m(v_i) &= \{\mu(e_i) \wedge \mu(e_{i+1}) \wedge \ldots \mu(e_{i+m-2}) \wedge \mu(e_{i+m-1})\} \\
d_m(v_i) &= \{c_i \wedge\ c_{i+1} \wedge \ldots\ c_{i+m-2} \wedge c_{i+m-1}\} \\
&= c_{i+m-1}. \\
\text{For}, i &= m+1, m+2, m+3\ldots, 2m \\
d_m(v_i) &= \{\mu(e_{i-1}) \wedge \mu(e_{i-2}) \ldots \mu(e_{i-(m-1)} \wedge \mu(e_{i-m})\} \\
d_m(v_i) &= \{c_{i-1} \wedge\ c_{i-2} \ldots\ c_{i-(m-1)} \wedge\ c_{i-m}\} \\
&= c_{i-1}
\end{aligned}$$

Hence G is an m-neighbourly irregular fuzzy graph. ∎

Remark 6.4. Even if the membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 > c_2 > c_3 > \ldots, > c_{2m-1}$, then G need not be an m-neighbourly totally irregular fuzzy graph.

Acknowledgement: This work is supported by F.No:4-4/2014-15, MRP- 5648/15 of the University Grant Commission, SERO, Hyderabad.

References

[1] A. Nagoor Gani and S. R. Latha, *On Irregular Fuzzy graphs*,Applied Mathematical Sciences, 6(2012), 517–523.

[2] A. Nagoor Gani and K. Radha, On Regular Fuzzy graphs, *Journal of Physical Science*, 12(2008), 33–40.

[3] A. Rosenfeld, *Fuzzy Graphs*, In:L.A.Zadeh,K.S.Fu, M.Shimura,Eds., Fuzzy Sets and Their Applications, Academic press (1975), 77–95.

[4] N. R. Santhi Maheswari and C. Sekar, Semi Neighbourly Irregular graphs, *International Journal of Combinatorial Graph Theory and Applications*, **5**(2) (2012), 135–144.

[5] N. R. Santhi Maheswari and C. Sekar, On $(2, k)$-regular fuzzy graphs and totally $(2, k)$-regular fuzzy graphs, *International Journal of Mathematics and Soft Computing* , 4(2)(2014), 59-69.

[6] N. R. Santhi Maheswari and C. Sekar, $(r, 2, k)$*-regular fuzzy graphs*, accepted in JCMCC.

[7] N. R. Santhi Maheswari and C. Sekar, 2-*Neighbourly Irregular fuzzy graphs*, accepted in Utilitas.

[8] N. R. Santhi Maheswari and C. Sekar, *A Study on Distance d-regular and Neighbourly Irregular fuzzy graphs*, Ph.D Thesis Manonmaniam Sundaranar University, Tirunelveli (2014) (Submitted)