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On $(2,(c_1,c_2))$ - regular bipolar fuzzy graph

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Abstract

In this paper d_2- degree and total d_2-degree of a vertex in bipolar fuzzy graphs are defined. Also (2,(c_1,c_2))-regularity and totally (2,(c_1,c_2))-regularity of bipolar fuzzy graphs are defined. A relation between (2,(c_1,c_2))-regularity and totally (2,(c_1,c_2))-regularity on bipolar fuzzy graph is studied. (2,(c_1,c_2))-regularity on path on four vertices, a Barbell graph $B_{m,n}$ (n>1) and a cycle C_n are studied with some specific membership functions.

Keywords: degree of a vertex in fuzzy graph, regular fuzzy graph, bipolar fuzzy graph, total degree, totally regular fuzzy graph, d_2 degree of a vertex in fuzzy graph, semiregular graphs.
AMS Subject Classification(2010): 05C12, 03E72, 05C72.

1 Introduction

In 1965, Lofti A. Zadeh [16] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975 [13]. It has been growing fast and has numerous application in various fields. Nagoor Gani and Radha [11] introduced regular fuzzy graphs, total degree, totally regular fuzzy graphs. Alison Northup introduced semiregular graphs that we call it as $(2,k)$-regular graphs and discussed some properties of (2,k)-regular graphs. In 1994 W.R.Zhang [15] initiated the concept of bipolar fuzzy sets as generalisation of fuzzy sets. Bipolar fuzzy sets are extension of fuzzy sets with membership values in $[-1,1]$. N.R.Santhi Maheswari and C.Sekar [14] introduced d_2- degree of vertex in graphs and discussed some properties of d_2- degree of a vertex in graphs. They also introduced d_2-degree of a vertex in fuzzy graphs, total d_2-degree of a vertex in fuzzy graph and discussed some properties of d_2-degree of the vertex in fuzzy graph [12]. This paper motivated us to introduce d_2 degree in bipolar fuzzy graph. Throughout this paper, the vertices take the membership values (m_1^+,m_1^-) and edges take the membership values (m_2^+,m_2^-) where m_1^+,$m_2^+ \in [0,1]$ and m_1^-,$m_2^- \in [-1,0]$.

2 Preliminaries

We present some known definitions related to fuzzy graphs and bipolar fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1. For a graph G, the d_2 - degree of a vertex v in G is denoted by $d_2(v)$ means the number of vertices at a distance two away from v.

Definition 2.2. A graph G is said to be $(2, k)$- regular (d_2- regular) if $d_2(v) = k$ for all v in G. We observe that $(2, k)$ regular and semiregular and d_2- regular graphs are same.

Definition 2.3. A fuzzy graph G: (σ,μ) is a pair of functions (σ, μ) where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non empty set V and μ: $V \times V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called a complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.4. [12] Let $G : (\sigma, \mu)$ be a fuzzy graph. The d_2*-degree* of a vertex u in G is $d_2(u) = \sum \mu^2(u, v)$, where $\mu^2(uv) = \sup\{\mu(uu_1) \wedge \mu(u_1v) : u, u_1, v$ is the shortest path connecting u and v of length $2\}$. Also, $\mu(uv) = 0$, for uv not in E.
The minimum d_2-degree of G is $\delta_2(G) = \wedge\{d_2(v) : v \in V\}$.
The maximum d_2-degree of G is $\Delta_2(G) = \vee\{d_2(v) : v \in V\}$.

Definition 2.5. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_2(v) = k$ for all $v \in V$, then G is said to be a $(2, k)$*-regular* fuzzy graph.

Definition 2.6. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total d_2-degree of a vertex $u \in V$ is defined as $td_2(u) = \sum \mu^2(u, v) + \sigma(u) = d_2(u) + \sigma(u)$.
The minimum td_2-degree of G is $t\delta_2(G) = \wedge\{td_2(v) : v \in V\}$.
The maximum td_2-degree of G is $t\Delta_2(G) = \vee\{td_2(v) : v \in V\}$.

Definition 2.7. [12] If each vertex of G has the same total d_2 - degree k, then G is said to be a totally $(2, k)$-regular fuzzy graph.

Definition 2.8. A bipolar fuzzy graph with an underlying set V is defined to be the pair $G = (A, B)$ where $A = (m_1^+, m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+, m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+(x, y) \leq \min\{m_1^+(x), m_1^+(y)\}$ and $m_2^-(x, y) \geq \max\{m_1^-(x), m_1^-(y)\}$ for all $(x, y) \in E$. Here A is called a bipolar fuzzy vertex set of V and B is called a bipolar fuzzy edge set of E.

Definition 2.9. The positive degree of a vertex $u \in G$ is $d^+(u) = \sum m_2^+(u, v)$. The negative degree of a vertex $u \in G$ is $d^-(u) = \sum m_2^-(u, v)$. The degree of the vertex u is defined as $d(u) = (d^+(u), d^-(u))$.

Definition 2.10. Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ are two bipolar fuzzy sets on a non-empty finite set V. Then G is said to be a regular bipolar fuzzy graph if all the vertices of G has same degree (c_1, c_2).

Definition 2.11. The strength of connectedness between two vertices u and v is defined as $\mu^\infty(u,v) = \sup\{\mu^k(u,v) : k = 1,2,....\}$ where $\mu^k(u,v) = \sup\{\mu(u,u_1) \wedge \mu(u_1,u_2) \wedge \cdots \wedge \mu(u_{k-1},v) : u, u_1, u_2, \cdots u_{k-1}, v$ is a path connecting u and v of length $k\}$.

Definition 2.12. The total degree of a vertex $u \in V$ is denoted by $td(u)$ and defined as $td(u) = (td^+(u), td^-(u))$ where $td^+(u) = \sum m_2^+(u,v) + m_1^+(u)$ and $td^-(u) = \sum m_2^-(u,v) + m_1^-(u)$.

Definition 2.13. Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ are two bipolar fuzzy sets on a non-empty finite set V. Then G is said to be regular bipolar fuzzy graph if all the vertices have same positive and negative membership values.

Definition 2.14. Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m_1^+, m_1^-)$ and $B = (m_2^+, m_2^-)$ be two bipolar fuzzy sets on a non-empty finite set V. G is said to be a totally regular bipolar fuzzy graph if all the vertices of G has same total degree (k_1, k_2). It is denoted by (k_1, k_2)- totally regular bipolar fuzzy graph.

3 d_2- degree of vertex in Bipolar Fuzzy Graph

Definition 3.1. Let $G = (A, B)$ be a bipolar fuzzy graph. The positive d_2 - degree of a vertex $u \in G$ is defined as $d_2^+(u) = \sum m_2^{(2,+)}(u,v)$ where $m_2^{(2,+)}(u,v) = \sup\{m_2^+(u,u_1) \wedge m_2^+(u_1,v) : u, u_1, v$ is the shortest path connecting u and v of length $2\}$.

The negative d_2- degree of a vertex $u \in G$ is defined as $d_2^-(u) = \sum m_2^{(2,-)}(u,v)$ where $m_2^{(2,-)}(u,v) = \inf\{m_2^-(u,u_1) \vee m_2^-(u_1,v) : u, u_1, v$ is the shortest path connecting u and v of length $2\}$. The d_2 - degree of a vertex u is defined as $d_2(u) = (d_2^+(u), d_2^-(u))$.
The minimum d_2 - degree of G is $\delta_2(G) = \wedge\{d_2(v) : v \in V\}$.
The maximum d_2 - degree of G is $\Delta_2(G) = \vee\{d_2(v) : v \in V\}$.

Example 3.2. The d_2 - degree of the vertices of the bipolar fuzzy graph G= (A,B) on the graph $G^* : (V, E)$ given in Figure 1 are as follows.

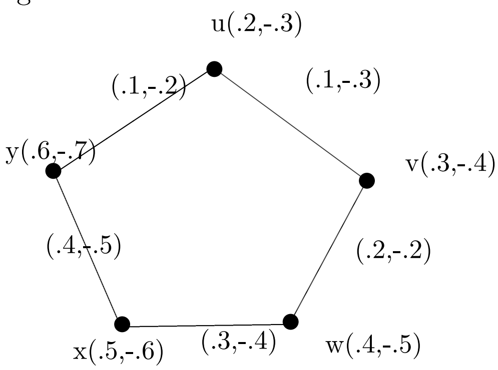

Figure 1: The bipolar fuzzy graph G= (A,B).

$$\begin{aligned}
d_2(u) &= (0.1 \wedge 0.2, -0.3 \vee -0.2) + (0.1 \wedge 0.4, -0.2 \vee -0.5)\\
&= (0.1, -0.2) + (0.1, -0.2) = (0.2, -0.4)\\
d_2(v) &= (0.2 \wedge 0.3, -0.2 \vee -0.4) + (0.1 \wedge 0.1, -0.3 \vee -0.2)\\
&= (0.2, -0.2) + (0.1, -0.2) = (0.3, -0.4)\\
d_2(w) &= (0.3 \wedge 0.4, -0.4 \vee -0.5) + (0.2 \wedge 0.1, -0.2 \vee -0.3)\\
&= (0.3, -0.4) + (0.1, -0.2) = (0.4, -0.6)\\
d_2(x) &= (0.1 \wedge 0.1, -0.2 \vee -0.3) + (0.4 \wedge 0.3, -0.5 \vee -0.4)\\
&= (0.1, -0.2) + (0.2, -0.2) = (0.3, -0.4)\\
d_2(y) &= (0.1 \wedge 0.1, -0.2 \vee -0.3) + (0.4 \wedge 0.3, -0.5 \vee -0.4)\\
&= (0.1, -0.2) + (0.3, -0.4) = (0.4, -0.6)
\end{aligned}$$

Example 3.3. The d_2 - degree of the vertices of the bipolar fuzzy graph G= (A,B) on the graph $G^* : (V, E)$ given in Figure 2 are as follows.

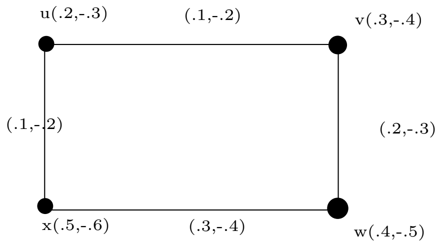

Figure 2: The bipolar fuzzy graph G= (A,B).

$$\begin{aligned}
d_2(u) &= (sup(0.1 \wedge 0.2, 0.1 \wedge 0.3), inf(-0.2 \vee -0.3, -0.2 \vee -0.4))\\
&= (sup(0.1, 0.1), inf(-0.2, -0.2)) = (0.1, -0.2)\\
d_2(v) &= (sup(0.1 \wedge 0.1, 0.2 \wedge 0.3), inf(-0.2 \vee -0.2, -0.3 \vee -0.4))\\
&= (sup(0.1, 0.2), inf(-0.2, -0.3)) = (0.2, -0.3)\\
d_2(w) &= (sup(0.1 \wedge 0.2, 0.1 \wedge 0.3), inf(-0.2 \vee -0.3, -0.2 \vee -0.4))\\
&= (sup(0.1, 0.1), inf(-0.2, -0.2)) = (0.1, -0.2)\\
d_2(x) &= (sup(0.1 \wedge 0.1, 0.2 \wedge 0.3), inf(-0.2 \vee -0.2, -0.3 \vee -0.4))\\
&= (sup(0.1, 0.2), inf(-0.2, -0.3)) = (0.2, -0.3).
\end{aligned}$$

4 (2,(c_1,c_2)) - Regular and Totally (2,(c_1,c_2)) - Regular Bipolar Fuzzy Graph

Definition 4.1. Let $G = (A, B)$ be a bipolar fuzzy graph. If $d_2(u) = (c_1, c_2)$ for all $u \in V$, then G is said to be a (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Example 4.2. A (2,(0.1,-0.2)) - regular bipolar fuzzy graph is given in Figure 3.

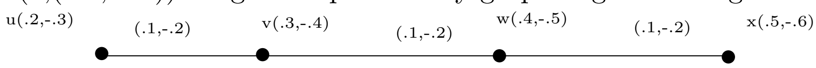

Figure 3: A (2,(0.1,-0.2)) - regular bipolar fuzzy graph.

Note that $d_2(u)$ = (0.1,-0.2), $d_2(v)$ = (0.1,-0.2), $d_2(w)$ = (0.1,-0.2) and $d_2(x)$ = (0.1,-0.2). This graph is (2,(0.1,-0.2)) - regular bipolar fuzzy graph.

Definition 4.3. Let $G = (A, B)$ be a bipolar fuzzy graph. Then the total d_2- degree of a vertex $u \in V$ is defined as $td_2(u) = (td_2^+(u), td_2^-(u))$ where $td_2^+(u) = d_2^+(u) + m_1^+(u)$ and $td_2^-(u) = d_2^-(u)+m_1^-(u)$. Also it can be defined as $td_2(u) = d_2(u)+A(u)$ where $A(u) = (m_1^+(u), m_1^-(u))$.

Definition 4.4. Let $G = (A, B)$ be a bipolar fuzzy graph. If each vertex of G has same total d_2 - degree, then G is said to be totally (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Example 4.5. A totally (2,(0.7,-0.8)) regular bipolar fuzzy graph is given in Figure 4.

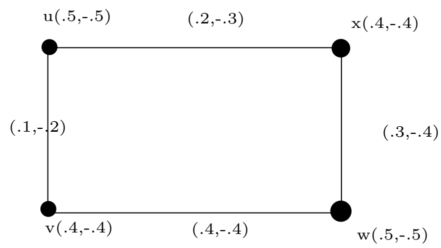

Figure 4: A totally (2,(0.7,-0.8)) regular bipolar fuzzy graph.

$td_2^+(u) = d_2^+(u) + m_1^+(u)$ and $td_2^-(u) = d_2^-(u) + m_1^-(u)$.
Here,

$$\begin{aligned} td_2(u) &= (0.2, -0.3) + (0.5, -0.5) = (0.7, -0.8) \\ td_2(v) &= (0.3, -0.4) + (0.4, -0.4) = (0.7, -0.8) \\ td_2(w) &= (0.2, -0.3) + (0.5, -0.5) = (0.7, -0.8) \\ td_2(x) &= (0.3, -0.4) + (0.4, -0.4) = (0.7, -0.8) \end{aligned}$$

This graph is totally (2,(0.7,-0.8)) regular bipolar fuzzy graph.

Example 4.6. A totally (2,(c_1,c_2)) - regular bipolar fuzzy graph need not be $(2,(c_1,c_2))$ - regular bipolar fuzzy graph. A bipolar fuzzy graph G= (A,B) on $G^* : (V, E)$ which is not $(2,(c_1,c_2))$ - regular bipolar fuzzy graph is given in Figure 5.

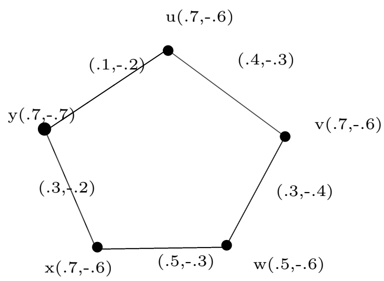

Figure 5: A totally (2,(c_1,c_2)) regular but not (2,(c_1,c_2)) - regular bipolar fuzzy graph.

$td_2(u)$=(1.1,-1.1). So G is a totally(2,(c_1,c_2)) regular bipolar fuzzy graph. But $d_2(u) \neq d_2(w)$. Hence G is not a (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Example 4.7. A (2,(c_1, c_2)) - regular bipolar fuzzy graph need not be a totally (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Consider the bipolar fuzzy graph G= (A,B) on $G^* : (V, E)$ given in Figure 6.

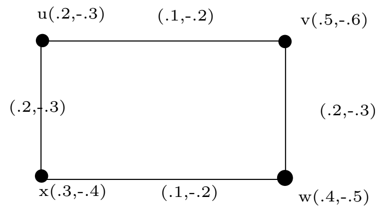

Figure 6

Note that $d_2(u)$ = (0.1,-0.2) for all $u \in V$. Hence G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph.
But $td_2(u) \neq td_2(v)$. Hence G is not a totally (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Example 4.8. A (2,(c_1,c_2)) - regular bipolar fuzzy graph which is also totally (2,(c_1,c_2)) - regular is given below.

Consider the bipolar fuzzy graph G= (A,B) on $G^* : (V, E)$ in Figure 7.

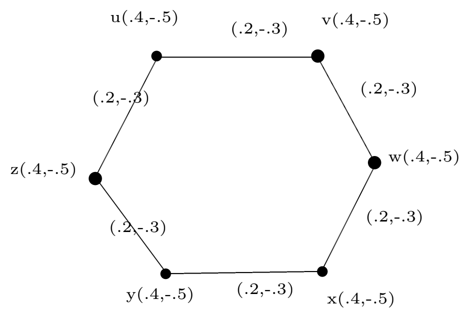

Figure 7

Then $d_2(u)$ = (0.4,-0.6) for all $u \in V$ and $td_2(u)$ = (0.8,-1.1) for all $u \in V$. Hence G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph as well as totally (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Theorem 4.9. Let $G = (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then $A(u) = (k_1, k_2)$ for all $u \in V$ if and only if the following conditions are equivalent.

1. $G = (A, B)$ is a (2,(c_1,c_2)) - regular bipolar fuzzy graph.

2. $G = (A, B)$ is a totally (2,($c_1 + k_1$,$c_2 + k_2$)) - regular bipolar fuzzy graph.

Proof: Suppose $A(u) = (k_1, k_2)$ for all $u \in V$.
Assume that G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph. Then $d_2(u) = (c_1, c_2)$ for all $u \in V$.
So $td_2(u) = d_2(u) + A(u) = (c_1, c_2) + (k_1, k_2) = (c_1 + k_1, c_2 + k_2)$.
Hence G is a totally (2,($c_1 + k_1$,$c_2 + k_2$)) - regular bipolar fuzzy graph.
Thus (i) $\Rightarrow$ (ii) is proved.

Suppose G is a totally (2,($c_1 + k_1$,$c_2 + k_2$)) - regular bipolar fuzzy graph.
$\Rightarrow td_2(u) = (c_1 + k_1, c_2 + k_2)$ for all $u \in V$
$\Rightarrow d_2(u) + A(u) = (c_1 + k_1, c_2 + k_2)$ for all $u \in V$
$\Rightarrow d_2(u) + (k_1, k_2) = (c_1, c_2) + (k_1, k_2)$ for all $u \in V$
$\Rightarrow d_2(u) = (c_1, c_2)$ for all $u \in V$
Hence G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph.
Thus (i) and (ii) are equivalent.

Conversely assume that (i) and (ii) are equivalent. Let G be a (2,(c_1,c_2)) - regular bipolar fuzzy graph and totally (2,($c_1 + k_1$,$c_2 + k_2$)) - regular bipolar fuzzy graph.
$\Rightarrow td_2(u) = (c_1 + k_1, c_2 + k_2)$ and $d_2(u) = (c_1, c_2)$ for all $u \in V$
$\Rightarrow d_2(u) + A(u) = (c_1 + k_1, c_2 + k_2)$ and $d_2(u) = (c_1, c_2)$ for all $u \in V$
$\Rightarrow d_2(u) + A(u) = (c_1, c_2) + (k_1, k_2)$ and $d_2(u) = (c_1, c_2)$ for all $u \in V$
$\Rightarrow A(u) = (k_1, k_2)$ for all $u \in V$.
Hence $A(u) = (k_1, k_2)$. ■

5 (2,(c_1,c_2)) - regularity on path on four vertices with specific membership function

Theorem 5.1. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a path on four vertices. If B is a constant function then G is a (2,(k_1,k_2)) - regular bipolar fuzzy graph.

Proof: Suppose that B is a constant function, say $B(uv) = (k_1, k_2)$, for all $uv \in E$. Then $d_2(u) = (k_1, k_2)$. Hence G is (2,(k_1,k_2)) - regular bipolar fuzzy graph. ■

Remark 5.2. The converse of Theorem 5.1 need not be true. For example consider $G = (A, B)$ be bipolar fuzzy graph such that $G^*(V, E)$ is path on four vertices.

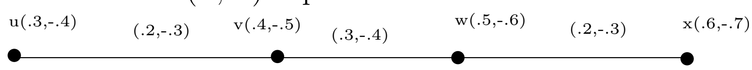

Figure 8

Note that, $d_2(u)$ = (0.2,-0.3) for all $u \in V$. So, G is a (2,(0.2,-0.3)) regular bipolar fuzzy graph. But B is not a constant function.

Theorem 5.3. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a path on four vertices. If alternate edges have the same membership values then G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph where $c_1 = min\{m_2^{(2,+)}\}$ and $c_2 = max\{m_2^{(2,-)}\}$.

Theorem 5.4. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a path on four vertices. If the middle edge has positive membership value less than positive membership values of the remaining edges and negative membership value greater than the negative membership value of remaining edges, then G is a $(2, (c_1, c_2))$ - regular bipolar fuzzy graph where c_1 and c_2 are membership values of the middle edge.

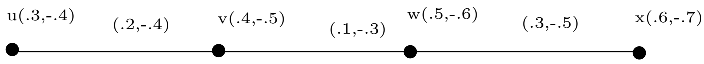

Figure 9

Note that $d_2(u) = (0.1, -0.3)$, $d_2(v) = (0.1, -0.3)$, $d_2(w) = (0.1, -0.3)$ and $d_2(x) = (0.1, -0.3)$. Hence G is a (2,(0.1,-0.3)) regular bipolar fuzzy graph.

Remark 5.5. If A is a constant function, then Theorems 5.1, 5.3 and 5.4 hold good for totally (2,(c_1,c_2)) - regular bipolar fuzzy graphs.

6 (2,(c_1,c_2)) - regularity on Barbell graph $B_{n,n}(n > 1)$ with some specific membership function

Theorem 6.1. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a barbell graph $B_{n,n}$ of order $2n$. If B is a constant function, then G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph where $(c_1, c_2) = nB(uv)$ where $uv \in E$.

Remark 6.2. The converse of Theorem 6.1 need not be true. For example, consider a bipolar fuzzy graph $G = (A, B)$ such that $G^*(V, E)$ is a barbell graph $B_{2,2}$ of order 6.

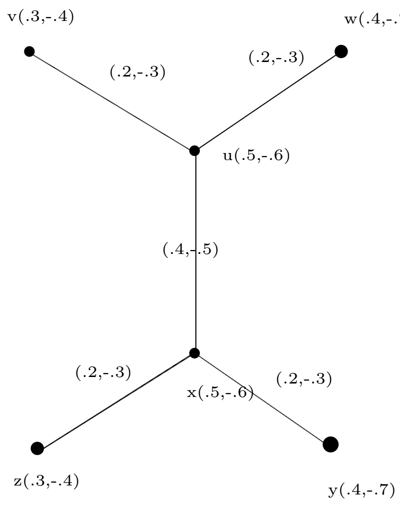

Figure 10

Note that $d_2(u) = (0.4, -0.6)$ for all $u \in V$. So, G is a (2,(0.4,-0.6)) regular bipolar fuzzy graph. But B is not a constant function.

Theorem 6.3. Let $G = (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$, a barbell graph $B_{n,n}(n > 1)$. If the pendant edges have positive membership values less than the positive membership value of the middle edge and negative membership values greater than the negative membership value of the middle edge, then G is a (2,n(c_1,c_2)) - regular bipolar fuzzy graph where (c_1, c_2) is the membership value of pendant edge.

Remark 6.4. If A is a constant function, then Theorem 6.1 and 6.3 hold good for totally (2,(c_1,c_2)) - regular bipolar fuzzy graphs.

7 $(2,(c_1,c_2))$ - regularity on cycle with some specific membership functions

Theorem 7.1. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a cycle of length ≥ 5. If m_2^+ and m_2^- are constant functions, then G is a $(2,(c_1,c_2))$ - regular bipolar fuzzy graph where $(c_1, c_2) = 2(m_2^+, m_2^-)$.

Remark 7.2. The converse of Theorem 7.1 need not be true. For example, consider a bipolar fuzzy graph $G = (A, B)$ such that $G^*(V, E)$ is an odd cycle of length five.

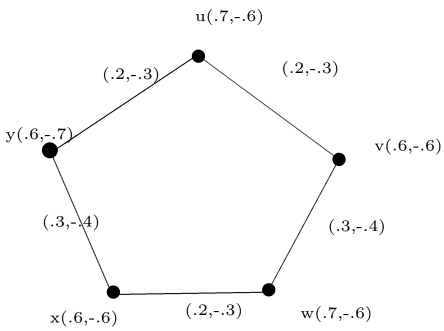

Figure 11

Note that $d_2(u) = (0.4, -0.6)$ for all $u \in V$. So G is a (2,(0.4,-0.6)) regular bipolar fuzzy graph. But m_2^+ and m_2^- are not constant functions.

Theorem 7.3. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is an even cycle of length ≥ 6. If alternate edges have same positive and negative membership values then G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Proof: If alternate edges have the same positive and negative membership values then,
$m_2^+(e_i) = c_1$ when i is odd and c_2 when i is even.
$m_2^-(e_i) = c_3$ when i is odd and c_4 when i is even.
Here we have the following 4 possible cases.

(i) $c_1 > c_2$ and $c_3 > c_4$

(ii) $c_1 > c_2$ and $c_3 < c_4$

(iii) $c_1 < c_2$ and $c_3 > c_4$

(iv) $c_1 < c_2$ and $c_3 < c_4$

In all cases, $d_2(u)$ is a constant for all $u \in V$. Hence G is a $(2,(c_1,c_2))$ - regular bipolar fuzzy graph where $d_2(u) = (c_1, c_2)$. ∎

Remark 7.4. If all the vertices take same positive and negative membership values then Theorem 7.3 holds good for totally $(2,(c_1,c_2))$ - regular bipolar fuzzy graphs.

Remark 7.5. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is an odd cycle of length greater than 5. Even if the alternate edges have same positive and same negative membership values, then G need not be a $(2,(c_1,c_2))$ - regular bipolar fuzzy graph.

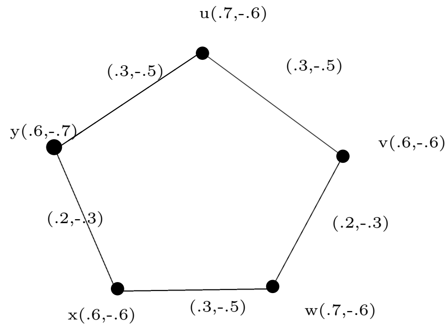

Figure 12

Note that $d_2(u) \neq d_2(v)$. Hence G is not a $(2,(c_1,c_2))$ - regular bipolar fuzzy graph.

Theorem 7.6. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a cycle of length greater than 4. Let $k_2 \geq k_1$ and $k_4 \geq k_3$. Let $m_2^+(e_i) = k_1$ when i is odd and k_2 when i is even. $m_2^-(e_i) = k_3$ when i is odd and k_4 when i is even. Then G is a $(2,(c_1,c_2))$ - regular bipolar fuzzy graph.

Proof: We prove the theorem in two cases.

Case (i): G^* is an even cycle.

$$\begin{aligned} d_2(v_i) &= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4) \\ &= (k_1, k_3) + (k_1, k_3) = (2k_1, 2k_3) \\ d_2(v_i) &= (c_1, c_2) where c_1 = 2k_1, c_2 = 2k_3 \end{aligned}$$

Hence G is a $(2,(c_1,c_2))$ - regular bipolar fuzzy graph.

Case(ii): G^* is an odd cycle.

Let $e_1, e_2 ....e_{2n+1}$ be the edges of G^*

$d_2(v_1) = (m_2^+(e_1) \wedge m_2^+(e_2), m_2^-(e_1) \vee m_2^-(e_2)) + (m_2^+(e_{2n}) \wedge m_2^+(e_{2n+1}), m_2^-(e_{2n}) \vee m_2^-(e_{2n+1}))$

$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$
$= (k_1, k_3) + (k_1, k_3) = (2k_1, 2k_3)$
$d_2(v_1) = (c_1, c_2)$ where $c_1 = 2k_1,\ c_2 = 2k_3$
$d_2(v_2) = (m_2^+(e_2) \wedge m_2^+(e_3), m_2^-(e_2) \vee m_2^-(e_3)) + (m_2^+(e_1) \wedge m_2^+(e_{2n+1}), m_2^-(e_1) \vee m_2^-(e_{2n+1}))$
$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$
$= (k_1, k_3) + (k_1, k_3) = (2k_1, 2k_3)$
$d_2(v_2) = (c_1, c_2)$ where $c_1 = 2k_1,\ c_2 = 2k_3$.
Proceeding like this we get $d_2(v_n) = (c_1, c_2)$ where $c_1 = 2k_2,\ c_2 = 2k_4$.
Hence $d_2(v_i) = (c_1, c_2)$. So G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph. ■

Remark 7.7. Theorem 7.6 holds good for totally (2,(c_1,c_2)) - regular bipolar fuzzy graphs if all the vertices take same positive and same negative membership values.

Theorem 7.8. Let $G = (A, B)$ be a bipolar fuzzy graph such that $G^*(V, E)$ is a cycle of length length greater than 4 with

$$m_2^+(e_i) = \begin{cases} k_1 & \text{if i is odd,} \\ k_2 & \text{if i is even for some } k_2 < k_1. \end{cases} \quad \text{and}$$

$$m_2^-(e_i) = \begin{cases} k_3 & \text{if i is odd,} \\ k_4 & \text{if i is even for some } k_4 < k_3. \end{cases}$$

where k_2 and k_4 are not constants. Then G is (2,(c_1,c_2)) - regular bipolar fuzzy graph.

Proof: We prove the theorem in two cases.
Case (i): G^* is an even cycle.
$d_2(v_i) = (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$
$= (k_2, k_4) + (k_2, k_4) = (2k_2, 2k_4)$
$d_2(v_i) = (c_1, c_2)$ where $c_1 = 2k_2,\ c_2 = 2k_4$
Hence G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph.
Case(ii): G^* is an odd cycle.
Let $e_1, e_2, ..., e_{2n+1}$ be the edges of G^*.
$d_2(v_1) = (m_2^+(e_1) \wedge m_2^+(e_2), m_2^-(e_1) \vee m_2^-(e_2)) + (m_2^+(e_{2n}) \wedge m_2^+(e_{2n+1}), m_2^-(e_{2n}) \vee m_2^-(e_{2n+1}))$
$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$
$= (k_2, k_4) + (k_2, k_4) = (2k_2, 2k_4)$.
$d_2(v_1) = (c_1, c_2)$ where $c_1 = 2k_2,\ c_2 = 2k_4$.
$d_2(v_2) = (m_2^+(e_2) \wedge m_2^+(e_3), m_2^-(e_2) \vee m_2^-(e_3)) + (m_2^+(e_1) \wedge m_2^+(e_{2n+1}), m_2^-(e_1) \vee m_2^-(e_{2n+1}))$
$= (k_1 \wedge k_2, k_3 \vee k_4) + (k_1 \wedge k_2, k_3 \vee k_4)$
$= (k_2, k_4) + (k_2, k_4) = (2k_2, 2k_4)$.
$d_2(v_2) = (c_1, c_2)$ where $c_1 = 2k_2,\ c_2 = 2k_4$.

Proceeding like this we get $d_2(v_n) = (c_1, c_2)$ where $c_1 = 2k_2$, $c_2 = 2k_4$. Hence $d_2(v_i) = (c_1, c_2)$. So, G is a (2,(c_1,c_2)) - regular bipolar fuzzy graph. ∎

Remark 7.9. Theorem 7.8 holds good for totally (2,(c_1,c_2)) - regular bipolar fuzzy graphs if all the vertices take the same positive and the same negative membership values.

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