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Fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy closure space

U.D.Tapi, Bhagyashri A. Deole

Department of Applied Mathematics and Computational Science
Shri G. S. Institute of Technology and Science
Indore (M.P.), India.
utapi@sgsits.ac.in, deolebhagyashri@gmail.com

Abstract

In this paper we introduce the concepts of fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy Čech closure space and study some of their fundamental properties.

Keywords: Fuzzy Čech closure space, fuzzy connectedness in fuzzy Čech closure space, Fs-continuous mapping, Fp-continuous mapping, fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy Čech closure space.

AMS Subject Classification (2010): 54A40.

1 Introduction

In 1965, Zadeh generalized characteristic functions to fuzzy sets in [11]. In 1968, Chang [4] introduced the topological structure of fuzzy sets. Pu and Liu [8] defined the concept of fuzzy connectedness using fuzzy closed set. Lowen [6] defined an extension of a connectedness in a restricted family of fuzzy topologies. Fuzzy Čech closure operator and fuzzy Čech closure space were first studied by Mashhour and Ghanim [7].

The concept of fuzzy semi-open set was introduced by Azad [1] and fuzzy pre-open set was introduced by Bin Shahna [2] in fuzzy topological space. In this paper, we [10] introduce the semi-connectedness and pre-connectedness in Čech closure space. We are introducing the concepts of fuzzy semi-connectedness and fuzzy pre-connectedness in fuzzy Čech closure space and study some of their fundamental properties.

2 Preliminaries

Definition 2.1.[5] Let X be a non-empty fuzzy set. A function $k: I^X \to I^X$ is called fuzzy Čech closure operator on X if it satisfies the following conditions

1. $k(\emptyset) = \emptyset$.
2. $A \leq k(A)$, for all $A \in I^X$.
3. $k(A_1 \vee A_2) = k(A_1) \vee k(A_2)$ for all $A_1, A_2 \in I^X$.

The pair (X, k) is called fuzzy Čech closure space.

Definition 2.2.[9] Let X be a nonempty fuzzy set. A function $k: I^X \rightarrow I^X$ is called fuzzy Čech closure operator on X. A fuzzy Čech closure space (X, k) is said to be fuzzy connected if and only if there exist any F-continuous map f from X to the fuzzy discrete space $\{0, 1\}$ is constant.

Definition 2.3.[3] A subset A in a Čech closure space (X, k) is called Čech semi-open in X if $A \subseteq k(int(A))$. The class of all semi-open sets of Čech closure space (X, k) is denoted by $SO(X, k)$.

Definition 2.4.[3] A subset A in Čech closure space X is called Čech pre-open if $A \subseteq int(k(A))$.The family of all pre-open sets of Čech closure space (X, k) is denoted by $PO(X, k)$.

3 Fuzzy Semi-connectedness In Fuzzy Closure Space

Definition 3.1. A fuzzy set A in a fuzzy Čech closure space (X, k) is said to be fuzzy semi-open set if $A \leq k(int(A))$. The complement of fuzzy semi-open set is called a fuzzy semi-closed set. The class of all fuzzy semi-open sets of fuzzy Čech closure space (X, k) is denoted by $FSO(X, k)$.

Example 3.2. Let $X = \{a, b, c\}$ be a non-empty fuzzy set. Define fuzzy Čech closure operator

$k: I^X \rightarrow I^X$ such that

$$k(A) = \begin{cases} 0_X; & \text{if } A = 0_X \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b,c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{c\}} \\ 1_X; & \text{otherwise.} \end{cases}$$

Then (X, k) is called a fuzzy Čech closure space.

Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.

$FSO(X, k) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.

Definition 3.3. Let (X, k_1) and (Y, k_2) be two fuzzy Čech closure spaces. A mapping $f: X \rightarrow Y$ is Fs-continuous if the inverse image of every fuzzy open set in Y is fuzzy semi-open in X.

Example 3.4. Let $X = \{a, b, c\}$ be a non-empty fuzzy set. Define fuzzy Čech closure operator

$k_1: I^X \rightarrow I^X$ such that

$$k_1(A) = \begin{cases} 0_X; & \text{if } A = 0_X. \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b,c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{c\}} \\ 1_X; & \text{otherwise.} \end{cases}$$

Then (X, k_1) is called a fuzzy Čech closure space.

Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.

$FSO(X, k_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.

Let $Y = \{a, b, c\}$ be a non-empty fuzzy set. Define fuzzy Čech closure operator $k_2: I^Y \rightarrow I^Y$ such that

$$k_2(A) = \begin{cases} 0_Y; & A = 0_Y. \\ 1_{\{a,b\}}; & \text{if } 0 \prec A \le 1_{\{a\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \le 1_{\{b\}} \\ 1_{\{c,a\}}; & \text{if } 0 \prec A \le 1_{\{c\}} \\ 1_Y; & \text{otherwise.} \end{cases}$$

Then (Y, k_2) is called a fuzzy Čech closure space.
Fuzzy open sets= $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, 1_Y, 0_Y\}$,
$FSO(Y, k_2) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, 1_Y, 0_Y\}$,

Define F-mapping $f: X \to Y$ such that
$f^{1}\{a\} = \{a, b\}, f^{1}\{b\} = \{b\}, f^{1}\{c\} = \{a, c\}, f^{1}\{a, b\} = \{a\}$,
$f^{1}\{b, c\} = \{c\}, f^{1}\{a, c\} = X, f^{1}\{1_Y\} = 1_X$,
$f^{1}\{0_Y\} = 0_X$.
Hence, f is an Fs-continuous mapping.

Definition 3.5. A fuzzy Čech closure space (X, k) is said to be a fuzzy semi-connected fuzzy Čech closure space if and only if there exists a constant Fs-continuous map f from X to the fuzzy discrete space $\{0, 1\}$. A fuzzy subset A in a fuzzy Čech closure space (X, k) is said to be a fuzzy semi-connected fuzzy Čech closure space if A with the subspace topology is a fuzzy semi-connected fuzzy Čech closure space.

Example 3.6. Let $X= \{a, b, c\}$ be a non-empty fuzzy set. Define fuzzy Čech closure operator

$$k_1: I^X \to I^X \text{ such that } k_1(A)= \begin{cases} 0_X; & A = 0_X. \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \le 1_{\{b,c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \le 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \le 1_{\{c\}} \\ 1_X; & \text{otherwise.} \end{cases}$$

Then (X, k_1) is called a fuzzy Čech closure space.
Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.
$FSO(X, k_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.

Consider an Fs-continuous map $f: X \to \{0, 1\}$ such that
$f^{1}\{1\}=\{a\}=\{b\}=\{c\}=\{a, b\}=\{b, c\}=\{a, c\}=1_X$.
$f^{1}\{0\} =0_X$ is constant.
Hence, (X, k_1) is a fuzzy semi-connected fuzzy Čech closure space.

Definition 3.7. A fuzzy Čech closure space (X, k) is called a fuzzy semi-disconnected fuzzy Čech closure space if and only if there exists a surjective Fs-continuous map f from X to the fuzzy discrete space $\{0, 1\}$.

Theorem 3.8. A fuzzy Čech closure space (X, k) is fuzzy semi-connected if and only if every *Fs*-continuous mapping f from X into a fuzzy discrete space $Y= \{0, 1\}$ with at least two points is constant.

Proof: Let a fuzzy Čech closure space (X, k) be fuzzy semi-connected. Then there exists an *Fs*-continuous mapping f from the fuzzy Čech closure space X into the fuzzy discrete space $Y= \{0, 1\}$. For each $y \in 1_Y$, $f^{-1}\{y\} = 0_X$ or 1_X. If $f^{-1}\{y\} = 0_X$ for all $y \in 1_Y$, then f ceases to be a mapping. Therefore, $f^{-1}\{y_0\} = 1_X$ for a unique $y_0 \in 1_Y$. This implies that $f\{1_X\} = \{y_0\}$ and hence f is a constant mapping.

Conversely, let *Fs*-continuous mapping f from X into a fuzzy discrete space $Y= \{0, 1\}$ be constant. Suppose U is a fuzzy semi-open set in a fuzzy Čech closure space (X, k). If $U \neq 0_X$, we show that $U=1_X$. Otherwise, choose two fixed points y_1 and y_2 in 1_Y. Define $f: X \to Y$ by

$$f(x)= \begin{cases} y_1; & \text{if } x \in U \\ y_2; & \text{otherwise.} \end{cases}$$

Then for any open set V in 1_Y,

$$f^{-1}(V) = \begin{cases} U; & \text{if V contains } y_1 \text{ only,} \\ 1_X/U; & \text{if V containsy}_2 \text{ only,} \\ 1_X; & \text{if V containboth } y_1 \text{ and } y_2, \\ 0_X; & \text{otherwise.} \end{cases}$$

In all the cases $f^{-1}(V)$ is fuzzy semi-open in 1_X. Hence *Fs*-continuous mapping f is not constant, which is a contradiction. This proves that the only fuzzy semi–open subsets of X are 0_X and 1_X. Hence, (X, k) is a fuzzy semi-connected fuzzy Čech closure space.

Theorem 3.9. The following assertions are equivalent:

1. (Y, k) is a fuzzy semi-connected fuzzy Čech closure space.
2. The only fuzzy subsets of Y both *Fs*-open and *Fs*-closed are 0_Y and 1_Y.
3. No *Fs*-continuous mapping $f: Y \to \{0, 1\}$ is surjective.

Proof: [1] ⇒ [2]: Let (Y, k) be a fuzzy semi-connected fuzzy Čech closure space. Suppose $G \leq Y$ is both fuzzy semi-open and fuzzy semi-closed such that $G \neq 0_Y$ and $G \neq 1_Y$, then $1_Y = G \vee G^c$, where G^c is the complement of G in Y. Hence, *Fs*-continuous mapping $f: Y \to \{0, 1\}$ is not constant. That is, (Y, k) is not a fuzzy semi-connected fuzzy Čech closure space, which is a contradiction. Hence, the only fuzzy subsets of Y which are both fuzzy semi-open and fuzzy semi-closed are 0_Y and 1_Y.

[2] ⇒ [3]: Suppose the only fuzzy subsets of 1_Y which are both fuzzy semi-open and fuzzy semi-closed are 0_Y and 1_Y. Let $f: Y \to \{0, 1\}$ be *Fs*-continuous and surjective. Then $f^{-1}\{0\} \neq 0_Y$, $f^{-1}\{0\} \neq 1_Y$. But $\{0\}$ is both fuzzy open and fuzzy closed in $\{0, 1\}$. Hence $f^{-1}\{0\}$ is fuzzy semi-open and fuzzy semi-closed in 1_Y. This is a contradiction to our assumption. Hence no *Fs*-continuous mapping $f: Y \to \{0, 1\}$ is surjective.

[3]⇒ [1]: Let no *Fs*-continuous mapping $f: Y \to \{0, 1\}$ be surjective. If possible let the fuzzy Čech closure space (Y, k) be not a fuzzy semi-connected fuzzy Čech closure space. So, $Y= A \vee B$, A and B are also fuzzy semi-closed sets.

Let $\boldsymbol{X}_A(\boldsymbol{x}) = \begin{cases} 1; x \in A \\ 0; x \notin A \end{cases}$

Then $X_A(x)$ is *Fs*-continuous surjection which is a contradiction. Hence fuzzy Čech closure space (Y, k) is a fuzzy semi-connected fuzzy Čech closure space.

Theorem 3.10. The *Fs*-continuous image of a fuzzy semi-connected fuzzy Čech closure space is a fuzzy semi-connected fuzzy Čech closure space.

Proof: Let (X, k) be a fuzzy semi-connected fuzzy Čech closure space and consider an *F*-continuous mapping $f: X \to f(X)$ which is surjective. If $f(x)$ is not a fuzzy semi-connected fuzzy Čech closure space, then there would be an Fs-continuous surjection $g: f(x) \to \{0, 1\}$ so that the composite mapping $gof: X \to \{0, 1\}$ is also an *Fs*-continuous surjection, which is a contradiction to fuzzy semi-connectedness of fuzzy Čech closure space (X, k). Hence $f(x)$ is a fuzzy semi-connected fuzzy Čech closure space.

4 Fuzzy Pre-connectedness In Fuzzy Closure Space

Definition 4.1. Let (X, k) be a fuzzy Čech closure space. A fuzzy set A in a fuzzy Čech closure space (X, k) is called a fuzzy pre-open set if $A \leq int\,(k\,(A))$. The complement of a fuzzy pre-open set is called a fuzzy pre-closed set. The family of all fuzzy pre-open sets of X is denoted by $FPO(X, k)$.

Example 4.2. Let $X = \{a, b, c\}$ be a non-empty fuzzy set. Define a fuzzy Čech closure operator

$$k_1: I^X \to I^X \text{ such that } k_1(A) = \begin{cases} 0_X; & A = 0_X. \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b,c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{c\}} \\ 1_X; & \text{otherwise.} \end{cases}$$

Then (X, k_1) is called a fuzzy Čech closure space.
Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.
$FPO(X, k_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 0_X, 1_X\}$.

Definition 4.3. Let (X, k_1) and (Y, k_2) be two fuzzy Čech closure spaces. An *F*-mapping $f: X \to Y$ is pre-continuous if the inverse image of every fuzzy open set in Y is fuzzy pre-open in X.

Example 4.4. Let $X = \{a, b, c\}$ be a non-empty fuzzy set. Define a fuzzy Čech closure operator $k_1: I^X \to I^X$ such that

$$\boldsymbol{k_1(A)} = \begin{cases} 0_X; & A = 0_X. \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b,c\}} \\ 1_X; & \text{otherwise.} \end{cases}$$

Then (X, k_1) is called a fuzzy Čech closure space.
Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.
$FPO(X, k_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 0_X, 1_X\}$.
Let $Y=\{a, b, c\}$ be a non-empty fuzzy set. Define a fuzzy Čech closure operator $k_2: I^Y \to I^Y$ such that

$$k_2(A) = \begin{cases} 0_Y\ ; & A = 0_Y \\ 1_{\{a,b\}}; & \text{if } 0 \prec A \leq 1_{\{a\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b\}} \\ 1_{\{c,a\}}; & \text{if } 0A \leq 1_{\{c\}} \\ 1_Y\ ; & \text{otherwise.} \end{cases}$$

Then (Y, k_2) is called a fuzzy Čech closure space.
Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, 1_Y, 0_Y\}$.
$FPO(Y, k_2) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, 1_Y, 0_Y\}$.
There exists an F-mapping $f: X \to Y$ such that
$f^{1}\{a\} = \{a, b\}, f^{1}\{b\} = \{b\}, f^{1}\{c\} = \{a, c\}, f^{1}\{a, b\} = \{a\}$,
$f^{1}\{b, c\} = \{c\}, f^{1}\{a, c\} = X, f^{1}\{1_Y\} = 1_X, f^{1}\{0_Y\} = 0_X$.
Hence f is an Fp-continuous mapping.

Definition 4.5. A fuzzy Čech closure space (X, k) is called a fuzzy pre-connected fuzzy Čech closure space if and only if there exists a Fp-continuous map f from X to the fuzzy discrete space $\{0, 1\}$ which is constant. A fuzzy subset A of fuzzy pre-connected fuzzy Čech closure space (X, k) is said to be a fuzzy pre-connected fuzzy Čech closure space if A with the subspace topology is a fuzzy pre-connected fuzzy Čech closure space.

Example 4.6. Let $X =\{a, b, c\}$ be a non-empty fuzzy set. Define a fuzzy Čech closure operator

$$k_1: I^X \to I^X \text{ such that } k_1(A) = \begin{cases} 0_X\ ; & A = 0_X \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b,c\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{b\}} \\ 1_{\{b,c\}}; & \text{if } 0 \prec A \leq 1_{\{c\}} \\ 1_X; & \text{otherwise.} \end{cases}$$

Then (X, k_1) is called a fuzzy Čech closure space.
Fuzzy open sets = $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 1_X, 0_X\}$.
$FPO(X, k_1) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, 0_X, 1_X\}$.

Consider a fuzzy pre-continuous mapping $f: X \to \{0, 1\}$ such that
$f^{1}\{1\} = \{a\} = \{a, b\}=\{b, c\}=\{a, c\}= \{1_X\}=\{b\}=\{c\}, f^{1}\{0\}= 0_X$ is constant.
Hence, (X, k_1) is a fuzzy pre-connected fuzzy Čech closure space.

Definition 4.7. A fuzzy Čech closure space (X, k) is called a fuzzy pre-disconnected fuzzy Čech closure space if and only if every Fp-continuous map f from X to the fuzzy discrete space $\{0, 1\}$ is surjective.

Theorem 4.8. If $\{A_i : I \in \wedge\}$ is a family of fuzzy pre-connected fuzzy Čech closure subsets of a fuzzy pre-connected fuzzy Čech closure space (X, k) then $\vee A_i$ is also a fuzzy pre-connected fuzzy Čech closure subset of X, where $\wedge$ is any index set.

Proof: Each A_i, $i \in \wedge$ is a fuzzy pre-connected fuzzy Čech closure subset of X. So there exists an *Fp*-continuous mapping $f_i: A_i \to \{0, 1\}$ which is constant. Let an *Fp*-continuous mapping $f: \vee A_i \to \{0, 1\}$ be not constant. Then $f^{-1}\{1\} \neq A_i$ which is a contradiction to each A_i is fuzzy pre-connected subsets of X. That is, every *Fp*-continuous mapping f is constant. Hence $\vee A_i$ is a fuzzy pre-connected fuzzy Čech closure space.

Theorem 4.9. Let (X, k_1) and (Y, k_2) be two fuzzy Čech closure spaces and F-mapping $f: (X, k_1) \to (Y, k_2)$ be bijective. Then,

(i) if f is an *Fp*-continuous mapping and X is a fuzzy pre-connected fuzzy Čech closure space then Y is a fuzzy connected fuzzy Čech closure space.

(ii) if f is an *F*-continuous mapping and X is fuzzy pre-connected fuzzy Čech closure space then Y is fuzzy connected fuzzy Čech closure space.

(iii) if f is an *Fp*-open mapping and Y is a fuzzy pre-connected fuzzy Čech closure space then X is a fuzzy connected fuzzy Čech closure space.

(iv) if f is an *F*-open mapping and X is a fuzzy connected fuzzy Čech closure space then Y is a fuzzy pre-connected fuzzy Čech closure space.

Proof: (i) Let (Y, k_2) be a fuzzy Čech closure space and X be a fuzzy pre-connected fuzzy Čech closure space. Then there exists an *Fp*-continuous mapping $fog: X \to \{0, 1\}$ which is constant. Consider an *Fp*-continuous mapping $g: Y \to \{0, 1\}$, given that $f: X \to Y$ is *Fp*-continuous and bijective so that g is also a constant mapping. Hence, Y is a fuzzy connected fuzzy Čech closure space.

(ii) Given that X is a fuzzy pre-connected fuzzy Čech closure space, that is, every *Fp*-continuous mapping $g: X \to \{0, 1\}$ is constant. Since $f^{-1}: Y \to X$ is F-continuous bijection, so that *F*-continuous mapping $f^{-1} og: Y \to \{0, 1\}$ is constant. Hence Y is a fuzzy connected fuzzy Čech closure space.

(iii) Given that Y is a fuzzy pre-connected fuzzy Čech closure space, that is, every *Fp*-continuous mapping $g: Y \to \{0, 1\}$ is constant. Since $f: X \to Y$ is Fp-open and bijective, we have *F*-continuous mapping $fog: X \to \{0, 1\}$ is constant. Hence X is fuzzy connected fuzzy Čech closure space.

(iv) Given that X is a fuzzy connected fuzzy Čech closure space, that is, an *F*-continuous mapping $g: X \to \{0, 1\}$ is constant and $f^{-1}: Y \to X$ is *F*-open mapping so that it is an *Fp*-open mapping then *Fp*-continuous mapping $f^{-1}og: Y \to \{0, 1\}$ is constant. Hence Y is a fuzzy pre-connected fuzzy Čech closure space.

Theorem 4.10. A fuzzy Čech closure space (X, k) is fuzzy pre-disconnected if and only if there exists a surjective *Fp*-continuous map f from X to a fuzzy discrete space $Y = \{0, 1\}$.

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