International Journal of Mathematics and Soft Computing
Vol.5, No.2. (2015), 51 - 57.

ISSN Print : 2249 – 3328
ISSN Online: 2319 – 5215

Total edge Lucas irregular labeling

A. Ponmoni[1], S. Navaneetha Krishnan[2], A. Nagarajan[3]

[1] Department of Mathematics
C.S.I.College of Engineering, Ketti – 643215
Tamil Nadu, India.
ponmonirpj@gmail.com

[2, 3] Department of Mathematics
V.O.C. College, Tuticorin-628008.
Tamilnadu, India.
snk.voc@gmail.com, nagarajan.voc@gmail.com

Abstract

For a graph $G = (V, G)$, total edge Lucas irregular labeling $f : V(G) \cup E(G) \rightarrow \{1,2,\ldots,K\}$ is defined as a labeling on $V(G)$ and $E(G)$ in such a way that for any two different edges xy and $x'y'$, their weights $f(x) + f(xy) + f(y)$ and $f(x') + f(x'y') + f(y')$ are distinct Lucas numbers.The total edge Lucas irregularity strength, tels(G), is defined as the minimum K for which G has total edge Lucas irregular labeling. In this paper we prove the graphs P_n, C_n, $K_{1,n}$ and Book (with 3 sides and 4 sides) admits total edge Lucas irregular labeling and we determine the total edge Lucas irregularity strength for those graphs.

Keywords: Total edge Lucas irregular labeling, prime labeling, prime graph, strongly prime graph.

AMS Subject Classification (2010): 05C78.

1 Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [3]. By labeling we mean any mapping that carries a set of graph elements to a set of numbers (usually positive integers), called labels. The concept of total vertex irregular labeling and Edge irregular total K-labeling were introduced by Baca et.al [2] and the definitions are given as follows:

A total vertex irregular labeling on a graph G with p vertices and q edges is an assignment of integer labels to both vertices and edges so that the weights calculated at vertices are distinct. The weight of a vertex v in G is defined as the sum of the label of v and the labels of all the edges incident with v, that is, $wt(v)=\lambda(v) + \sum_{uv \epsilon E} \lambda(uv)$.The total vertex irregularity strength of G, denoted by tvs(G), is the minimum value of the largest label over all such irregular assignments.

For a graph $G = (V,E)$, define a labeling $f: V(G) \cup E(G) \to \{1,2,\ldots,K\}$ to be an edge irregular total K-labeling of the graph G if for any two different edges xy and $x'y'$ of G the edge weights $wt(xy), wt(x'y')$ are distinct . The total edge irregularity strength, tels(G), is defined as the minimum K for which has an edge irregular total K- labeling.

Kristiana wijaya et al.[4] proved that tvs($k_{n,n+1}$) = 3for all n≥3, tvs($k_{n,n}$)=3 for all n≥3. Ali Ahmad and Martin Baca [1] proved that tes $(C_n \times P_m) = \left\lceil \frac{2n(m-1)+2}{3} \right\rceil$ for m ≥ 2, n ≥ 4 and m, n are even.

2 Total edge Lucas irregular labeling

Definition 2.1. A total edge Lucas irregular labeling $f: V(G) \cup E(G) \to \{1,2,\ldots,K\}$ of a graph $G = (V, G)$ is a labeling of vertices and edges of G in such a way that for any two different edges xy and $x'y'$ their weights $f(x) + f(xy) + f(y)$ and $f(x') + f(x'y') + f(y')$ are distinct Lucas numbers where the Lucas series is given by the recurrence relation $L_n = L_{n-1} + L_{n-2}, n > 1$, $L_1 = 1$, $L_2 = 3$, $L_3 = 4$, $L_4=7$ and so on.

The total edge Lucas irregularity strength, tels (G) is defined as the minimum K for which G has total edge Lucas irregular labeling.

Observation 2.2. Since every edge is incident with two vertices, $wt(e) \neq L_1$ for every edge $e \in E(G)$ and the weights of $E(G)$ are distinct Lucas numbers, $L_2 \leq wt(e) \leq L_{q+1}$ for every edge $e \in E(G)$. Also, $f(x) + f(xy) + f(y) \leq 3K$ this implies $K \geq \frac{1}{3} w(xy)$ for every $xy \in E(G)$. Therefore, $tels \geq \left\lceil \frac{L_{q+1}}{3} \right\rceil$.

Theorem 2.3. The path P_n of n vertices admits a total edge Lucas irregular labeling with $tels(P_n) = L_{n-2}$ if $n \geq 4$.

Proof: Consider a path P_n with n vertices.

Let $V(P_n) = \{v_1, v_2, v_3, \ldots, v_n\}$; $E(P_n) = \{e_1, e_2, e_3, \ldots, e_{n-1}\}$, Here, $q = n-1$.

Define $f: V(G) \cup E(G) \to \{1, 2, \ldots, L_{n-2}\}$ as follows:

$f(v_1) = 1$

$f(v_2) = 1$

$f(v_3) = 2$

$f(v_i) = L_{i-2} \quad , 4 \leq i \leq n$

$f(e_1) = 1$

$f(e_2) = 1$

$f(e_3) = 2$

$f(e_i) = L_{i-1} \quad , 4 \leq i \leq n-1$

By this labeling,

$wt(e_1) = f(v_1) + f(e_1) + f(v_2)$

$= 1 + 1 + 1 = 3$

$= L_2$

$wt(e_2) = f(v_2) + f(e_2) + f(v_3)$

$= 1 + 1 + 2 = 4$

$= L_3$

$wt(e_3) = f(v_3) + f(e_3) + f(v_4)$

$= 2 + 2 + 3 = 7$

$= L_4$

In general,

$wt(e_i) = f(v_i) + f(e_i) + f(v_{i+1})$

$= L_{i-2} + L_{i-1} + L_{i-1}$

$= L_i + L_{i-1}$

$=L_{(i+1)-2} + L_{(i+1)-1}$

$=L_{(i+1)}$, $4 \leq i \leq n-1$

Thus, the weights of $e_1, e_2, e_3, \ldots, e_{n-1}$ are $L_2, L_3, L_4, \ldots, L_n$ respectively.

Therefore, the path P_n of n vertices admits a total edge Lucas irregular labeling. Also, $tels(P_n) = L_{n-2}$ if $n \geq 4$. ■

Theorem 2.4. The cycle C_n of length n admits a total edge Lucas irregular labeling with $tels(C_n) = L_{n+1} - L_{n-2} - 1$ $if\ n \geq 4$.

Proof: Consider a cycle C_n of length n.

Let $V(C_n) = \{v_1, v_2, v_3, \ldots v_n\}$; $E(C_n) = \{e_1, e_2, e_3, \ldots e_n\}$.

Define f: V(G) ∪E (G) → {1, 2,…, $L_{n+1} - L_{n-2} - 1$}

$f(v_1) = 1$

$f(v_2) = 1$

$f(v_3) = 2$

$f(v_i) = L_{i-2}$, $4 \leq i \leq n$

$f(e_1) = 1$

$f(e_2) = 1$

$f(e_3) = 2$

$f(e_i) = L_{i-1}$, $4 \leq i \leq n$-1

$f(e_n) = L_{n+1} - L_{n-2} - 1$

By this labeling,

$wt(e_1) = f(v_1) + f(e_1) + f(v_2)$

$= 1 + 1 + 1 = 3$

$= L_2$

$wt(e_2) = f(v_2) + f(e_2) + f(v_3)$

$= 1 + 1 + 2 = 4$

$= L_3$

$wt(e_3) \; = f(v_3) + f(e_3) + f(v_4)$

$= \; 2 + 2 + 3 \; = \; 7$

$= \; L_4$

In general ,

$wt(e_i) \; = f(v_i) + f(e_i) + f(v_{i+1})$

$= \; L_{i-2} + L_{i-1} + L_{i-1}$

$= L_i + L_{i-1}$

$= L_{(i+1)-2} + L_{(i+1)-1}$

$= L_{(i+1)} \;, \quad 4 \le i \le n-1$

$wt(e_n) \; = f(v_1) + f(e_n) + f(v_n)$

$= 1 + L_{n+1} - L_{n-2} - 1 + L_{n-2}$

$= L_{n+1}$

Thus, the weights of $e_1, e_2, e_3, \ldots, e_n$ are $L_2, L_3, L_4, \ldots, L_{n+1}$ respectively. Therefore, the cycle C_n of length *n* admits a total edge Lucas irregular labeling.

Also, $tels(C_n) = L_{n+1} - L_{n-2} - 1$ if $n \ge 4$. ■

Theorem 2.5. The star $K_{1,n}$ of *n*+1 vertices admits a total edge Lucas irregular labeling with $tels(K_{1,n}) = \left\lceil \frac{L_{n+1}-1}{2} \right\rceil$ for all *n*.

Proof: Consider the star $K_{1,n}$ with n+ 1 vertex.

Let $V(K_{1,n}) = \{v_0, v_1, v_2, \ldots, v_n\}$; $E(K_{1,n}) = \{e_1, e_2, e_3, \ldots, e_n\}$

Here, $q = n$.

Define f: $V(G) \cup E(G) \to \{1, 2, \ldots, \left\lceil \frac{L_{n+1}-1}{2} \right\rceil\}$ as follows:

$f(v_0) = 1$

$f(v_i) = \left\lceil \frac{L_{i+1}-1}{2} \right\rceil \quad , 1 \le i \le n$

$f(e_i) = L_{i+1} - 1 - \left\lceil \frac{L_{i+1}-1}{2} \right\rceil \quad , 1 \le i \le n$

Now,

$wt(e_i) = f(v_0) + f(e_i) + f(v_i)$

$= 1 + L_{i+1} - 1 - \left\lceil \frac{L_{i+1}-1}{2} \right\rceil + \left\lceil \frac{L_{i+1}-1}{2} \right\rceil$

$= L_{(i+1)} \;, \quad 1 \le i \le n$

Thus, the weights of $e_1, e_2, e_3, \ldots, e_n$ are $L_2, L_3, L_4, \ldots, L_{n+1}$ respectively.

Therefore, the star $K_{1,n}$ of *n*+1 vertices admits a total edge Lucas irregular labeling. Also, $tels(K_{1,n}) = \left\lceil \frac{L_{n+1}-1}{2} \right\rceil$ for all *n*. ■

Theorem 2.6. Books with 3 sides (*n* copies of C_3 with an edge is common) admits a total edge Lucas irregular labeling and its total edge Lucas irregularity strength is $\le \left\lceil \frac{L_{2n+2}}{2} \right\rceil - 1$ for all *n*.

Proof: Consider a book with 3 sides (n copies of C_3 with an edge is common).

Let $V = \{u, v, u_1, u_2, \ldots\ldots, u_n\}$ be the vertex set and $E = \{e = uv, x_i = uu_i, y_i = vu_i, i = 1,2, \ldots\ldots n\}$ be the edge set.

Here, $|V| = n + 2$ and $|E| = 2n + 1$.

Define $f: V(G) \cup E(G) \longrightarrow \left\{1,2, \ldots\ldots, \left\lceil \frac{L_{2n+2}}{2} \right\rceil - 1\right\}$ as follows:

$f(\mathrm{u}) = 1$

$f(v) = 2$

$f(u_1) = 1$

$f(u_i) = \left\lceil \frac{L_{2n+2}}{2} \right\rceil - 1, \quad 2 \le i \le n$

$f(e) = 1$

$f(x_1) = 1$

$f(x_i) = L_{2i+1} - \left\lceil \frac{L_{2i+2}}{2} \right\rceil, \quad 2 \le i \le n$

$f(y_1) = 4$

$f(y_i) = L_{2i+2} - \left\lceil \frac{L_{2i+2}}{2} \right\rceil - 1, \quad 2 \le i \le n$

By this labeling,

$wt(e) = f(u) + f(e) + f(v)$

$= 1+1+2 = 4$

$= L_3$

$wt(x_1) = f(u) + f(x_1) + f(u_1)$

$= 1+1+1 = 3$

$= L_2$

$wt(x_i) = f(u) + f(x_i) + f(u_i)$

$= 1 + L_{2i+1} - \left\lceil \frac{L_{2i+2}}{2} \right\rceil + \left\lceil \frac{L_{2i+2}}{2} \right\rceil - 1$

$= L_{2i+1}, \quad 2 \le i \le n$

Thus, the weights of $x_2, x_3, x_4, \ldots\ldots, x_n$ are $L_5, L_7, L_9, \ldots\ldots, L_{2n+1}$.

$wt(y_1) = f(v) + f(y_1) + f(u_1)$

$= 2 + 4 + 7 = 13$

$= L_4$

$wt(y_i) = f(v) + f(y_i) + f(u_i)$

$= 2 + L_{2i+2} - \left\lceil \frac{L_{2i+2}}{2} \right\rceil - 1 + \left\lceil \frac{L_{2i+2}}{2} \right\rceil - 1$

$= L_{2i+2}, \quad 2 \le i \le n$

Thus, the weights of $y_2, y_3, y_4, \ldots\ldots, y_n$ are $L_6, L_8, L_{10}, \ldots\ldots, L_{2n+2}$.

Hence the weights of $x_1, e, y_1, x_2, y_2, x_3, y_3, \ldots x_n, y_n$ are $L_2, L_3, L_4, \ldots\ldots, L_{2n+1}, L_{2n+2}$ respectively.

Therefore, books with 3 sides (n copies of C_3 with an edge is common) admits a total edge Lucas irregular labeling.

Also, total edge Lucas irregularity strength is $\leq \left\lceil \frac{L_{2n+2}}{2} \right\rceil - 1$ for all n. ■

Theorem 2.7. Books with four sides (n copies of C_4 with an edge is common) admits a total edge Lucas irregular labeling and its total edge Lucas irregularity strength is $\left\lceil \frac{L_{3n+2}}{3} \right\rceil$ for all n.

Proof: Consider a book with four sides (n copies of C_4 with an edge is common).

Let $V = \{u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the vertex set and $= \{e = uv, e_i = u_i v_i, x_i = uu_i, y_i = vv_i, i = 1,2, \ldots, n\}$ be the edge set.

Here $|V| = 2n + 2$ and $|E| = 3n + 1$.

Define $f: V(G) \cup E(G) \longrightarrow \left\{1,2, \ldots, \left\lceil \frac{L_{3n+2}}{3} \right\rceil \right\}$ as follows:

$f(\mathrm{u}) = 5$

$f(v) = 1$

$f(u_1) = f(v_1) = 1$

$f(u_i) = f(v_i) = \left\lceil \frac{L_{3i+2}}{3} \right\rceil \quad 2 \leq i \leq n$

$f(e) = 5$

$f(e_1) = 1$

$f(e_i) = L_{3i+2} - 2\left\lceil \frac{L_{3i+2}}{3} \right\rceil, \quad 2 \leq i \leq n$

$f(x_1) = 1$

$f(x_i) = L_{3i+1} - 5 - \left\lceil \frac{L_{3i+2}}{3} \right\rceil, \quad 2 \leq i \leq n$

$f(y_1) = 2$

$f(y_i) = L_{3i} - 1 - \left\lceil \frac{L_{3i+2}}{3} \right\rceil, \quad 2 \leq i \leq n$

By this labeling,

$wt(e) = f(u) + f(e) + f(v)$

$= 5+5+1 = 11$

$= L_5$

$wt(e_1) = f(u_1) + f(e_1) + f(v_1)$

$= 1+1+1 = 3$

$= L_2$

$wt(e_i) = f(u_i) + f(e_i) + f(v_i)$

$= \left\lceil \frac{L_{3i+2}}{3} \right\rceil + L_{3i+2} - 2\left\lceil \frac{L_{3i+2}}{3} \right\rceil + \left\lceil \frac{L_{3i+2}}{3} \right\rceil$

$= L_{3i+2}, \qquad 2 \leq i \leq n$

Thus, the weights of $e_{2,}\ e_{3,}\ e_{4,} \ldots\ldots, e_{n,}$ are $L_{8,}\ L_{11,}\ L_{14,} \ldots\ldots, L_{3n+2}$.

$wt(x_1) = f(u) + f(x_1) + f(u_1)$

= 5+1+1 =7

$= L_4$

$wt(x_i) = f(u) + f(x_i) + f(u_i)$

$= 5 + L_{3i+1} - 5 - \left\lceil \frac{L_{3i+2}}{3} \right\rceil + \left\lceil \frac{L_{3i+2}}{3} \right\rceil$

$= L_{3i+1}, \quad 2 \le i \le n$

Thus the weights of $x_{2,}\ x_{3,} \ldots\ldots, x_{n,}$ are $L_{7,}\ L_{10,}\ L_{13,} \ldots\ldots, L_{3n+1}$

$wt(y_1) = f(v) + f(y_1) + f(v_1)$

= 1+2+1 = 4

$= L_3$

$wt(y_i) = f(v) + f(y_i) + f(v_i)$

$= 1 + L_{3i} - 1 - \left\lceil \frac{L_{3i+2}}{3} \right\rceil + \left\lceil \frac{L_{3i+2}}{3} \right\rceil$

$= L_{3i}, \quad 2 \le i \le n$

Thus, the weights of $y_{2,}\ y_{3,}\ y_{4,} \ldots\ldots, y_n$ are $L_{6,}\ L_{9,}\ L_{12,} \ldots\ldots, L_{3n}$.

Hence, the weights of $e_1, y_1, e, y_2, x_2, e_2, \ldots\ldots, x_n,\ e_n$ are $L_3,\ L_4,\ L_5,\ L_6,\ L_7,\ L_8 \ldots L_{3n},\ L_{3n+1},\ L_{3n+2}$ respectively. Therefore, books with four sides (n copies of C_4 with an edge is common) admits a total edge Lucas irregular labeling. Also, total edge Lucas irregularity strength $is \left\lceil \frac{L_{3n+2}}{3} \right\rceil$. ■

References

[1] Ali Ahmed and Martin Baca, *Edge irregular total labeling of certain family of graphs*, AKCE J. Combin, 6,No.(2009), 21-29.

[2] M. Baca, m Stanislav Jendrol, Mirka Miller and Joseph Ryan, *On irregular total labeling*, *Discrete Math*., 307(2007), 1378 -138.

[3] F. Harary, *Graph theory*, Addition Wesley reading, Mass, 1972.

[4] Kristinana Wijaya, Slamin, Surahmat and Stanislav Jendrol, *Total vertex irregular labeling of complete bipartite graphs*, JCMCC, 5(2007), 129 - 136.