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$3-$equitable labeling in context of the barycentric subdivision of some special graphs

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Abstract

A function from the vertex set of a graph G to the set $\{0,1,2\}$ is called $3-$*equitable labeling* if the induced edge labels are produced by the absolute difference of labels of end vertices such that the absolute difference of number of vertices of G labeled with 0, 1 and 2 differ by at most 1 and similarly the absolute difference of number of edges of G labeled with 0, 1 and 2 differ by at most 1. In this paper we discuss $3-$equitable labeling in context of barycentric subdivision of cycle with one chord, cycle with twin chords, cycle with triangle, shell graph and wheel graph.

Keywords: $3-$equitable labeling, Barycentric subdivision.
AMS Subject Classification(2010): 05C78.

1 Introduction

We consider simple, finite, undirected graph. If the vertices of the graph are assigned values subject to certain conditions then it is known as *graph labeling*. A survey on graph labeling is given by Gallian[2].

Let G be a graph. The vertex set and the edge set of graph G are denoted by $V(G)$ and $E(G)$ respectively. A mapping f from $V(G)$ to $\{0,1,2\}$ is called *ternary vertex labeling* of G. A ternary vertex labeling of a graph G is called $3-$*equitable labeling* if the induced edge labeling function f^* from $E(G)$ to the set $\{0,1,2\}$ is defined as $f^*(e = uv) = |f(u) - f(v)|$ such that the absolute difference of number of vertices of G with label 0, 1 and 2 differ by at most 1 and similarly absolute difference of number of edges of G with label 0, 1 and 2 differ by at most 1. A graph which admits $3-$equitable labeling is called a $3-$*equitable graph*. We follow Gross and Yellen[3] for the graph theoretical terminology and notations.

Definition 1.1. A chord of a cycle $C_n, n \geq 4$ is an edge joining two non-adjacent vertices of the cycle C_n.

Definition 1.2. Two chords of a cycle $C_n, n \geq 5$ are said to be twin chords if they form a triangle with an edge of cycle C_n.

For positive integers n and p with $3 \leq p \leq (n-2)$, $C_{n,p}$ is the graph consisting of a cycle C_n with a pair of twin chords with which the edges of C_n form cycles C_p, C_3 and C_{n+1-p} without chords.

Definition 1.3. The cycle with triangle is a cycle with three chords which by themselves form a triangle.

For positive integers p, q, r and $n \geq 6$ with $p+q+r+3 = n$, $C_n(p,q,r)$ denotes the cycle with triangle whose edges form the edges of cycles C_{p+2}, C_{q+2} and C_{r+2} without chords.

Definition 1.4. The shell S_n is the graph obtained by taking $n-3$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the apex vertex.

Definition 1.5. The wheel W_n is the join of the graphs C_n and K_1. That is, $W_n = C_n + K_1$. Here, the vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n while the vertex corresponding to K_1 is called *apex* vertex.

Definition 1.6. Let $e = uv$ be an edge of a graph G and w is not a vertex of G. Then edge e is said to be subdivided when it is replaced by edges $e' = uw$ and $e'' = wv$.

Definition 1.7. If every edge of a graph G is subdivided, then the resulting graph is called barycentric subdivision of the graph G. It is denoted by $S(G)$.

Vaidya et al.[4] proved that cycle with twin chords is cordial as well as $3-$equitable. In [5] Vaidya et al. proved that the barycentric subdivision of cycle with one chord, cycle with twin chords and cycle with triangle are cordial. Youssef[6] proved that W_n is $3-$equitable for all $n \leq 4$. In this paper we prove that the barycentric subdivision of cycle with one chord, cycle with twin chords, cycle with triangle, shell and wheel are $3-$equitable graphs.

2 Main Results

Theorem 2.1. The barycentric subdivision of cycle C_n with one chord is $3-$equitable for all n, where chord forms a triangle with two edges of C_n.

Proof: Let G be the cycle C_n with one chord and let $S(G)$ be the barycentric subdivision of G. Note that $|V(S(G))| = 2n+1$ and $|E(S(G))| = 2n+2$. Let $v_1, v_2, \ldots, v_{2n+1}$ be the successive vertices of $S(G)$. Let $e_1 = v_1v_5$ be the chord of C_n. Here $v_2, v_4, \ldots, v_{2n}$ are the vertices inserted due to the barycentric subdivision of edges of C_n and v_{2n+1} is the vertex inserted due to the barycentric subdivision of the chord e_1. That is, v_{2n+1} is adjacent to v_1 and v_5, edge e_1 is subdivided into two edges $e_1' = v_1v_{2n+1}$ and $e_1'' = v_{2n+1}v_5$. Note that $d(v_1) = 3$, $d(v_5) = 3$ and $d(v_i) = 2$, $2 \leq i \leq 2n+1, i \neq 5$. To define labeling function $f : V(G) \to \{0,1,2\}$, we consider the following cases.

Case 1: $n \equiv 0, 2, 3, 5 (mod 6)$.

$f(v_i) = 0$; if $i \equiv 2, 5 (mod 6)$

$= 1$; if $i \equiv 3, 4(mod6)$

$= 2$; if $i \equiv 0, 1(mod6), 1 \leq i \leq 2n + 1$.

Case 2: $n \equiv 1(mod6)$.

$f(v_{2n+1}) = 1$,

$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$

$= 1$; if $i \equiv 0, 5(mod6)$

$= 2$; if $i \equiv 2, 3(mod6), 1 \leq i \leq 2n$.

Case 3: $n \equiv 4(mod6)$.

$f(v_{2n+1}) = 2$,

$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$

$= 1$; if $i \equiv 2, 3(mod6)$

$= 2$; if $i \equiv 0, 5(mod6), 1 \leq i \leq 2n$.

Above defined labeling pattern satisfies the conditions of 3−equitable labeling as shown in Table 1.

Table 1: Vertex and edge conditions for the barycentric subdivision of cycle C_n, where $n = 6a + b, n \in N$.

b	Vertex Conditions	Edge Conditions
0,3	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) = e_f(1) + 1 = e_f(2)$
2,5	$v_f(0) = v_f(1) = v_f(2) + 1$	$e_f(0) = e_f(1) = e_f(2)$
1	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$
4	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$

Hence the barycentric subdivision of cycle with one chord is 3−equitable. ■

Example 2.2. 3−equitable labeling of the graph obtained by the barycentric subdivision of cycle C_4 with one chord is shown in Figure 1.

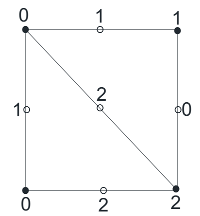

Figure 1: 3−equitable labeling of the barycentric subdivision of cycle C_4 with one chord.

Theorem 2.3. The barycentric subdivision of cycle with twin chords $(C_{n,3})$ is 3−equitable.

Proof: Let G be cycle C_n with twin chords and $S(G)$ denote the barycentric subdivision of G. Note that $|V(S(G))| = 2n + 2$ and $|E(S(G))| = 2n + 4$. Let $v_1, v_2, \ldots, v_{2n+2}$ be the successive vertices of $S(G)$. Let $e_1 = v_1v_5$ and $e_2 = v_1v_7$ be two chords of C_n. Here $v_2, v_4, \ldots, v_{2n}$ are the vertices inserted due to the barycentric subdivision of edges of C_n, v_{2n+1} and v_{2n+2} be the vertices inserted due to the barycentric subdivision of the chords e_1 and e_2 respectively. v_{2n+1} is adjacent to vertices v_1

and v_5, v_{2n+2} is adjacent to vertices v_1 and v_7. Edge e_1 is subdivided into two edges $e_1' = v_1v_{2n+1}$ and $e_1'' = v_{2n+1}v_5$, edge e_2 is subdivided into two edges $e_2' = v_1v_{2n+2}$ and $e_2'' = v_{2n+2}v_7$. Note that $d(v_1) = 4$, $d(v_5) = d(v_7) = 3$ and $d(v_i) = 2$, $2 \leq i \leq 2n+2$, $i \neq 5, i \neq 7$.
To define labeling function $f : V(G) \to \{0, 1, 2\}$, we consider the following cases.
Case 1: $n \equiv 0, 2, 3, 5(mod6)$.
$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$
$= 1$; if $i \equiv 0, 5(mod6)$
$= 2$; if $i \equiv 2, 3(mod6)$, $1 \leq i \leq 2n+2$.
Case 2: $n \equiv 1, 4(mod6)$.
$f(v_{2n+2}) = 2$,
$f(v_i) = 0$; if $i \equiv 2, 5(mod6)$
$= 1$; if $i \equiv 3, 4(mod6)$
$= 2$; if $i \equiv 0, 1(mod6)$, $1 \leq i \leq 2n+1$.
Above defined labeling pattern satisfies the conditions of $3-$equitable labeling as shown in Table 2.

Table 2: Vertex and edge conditions for the barycentric subdivision of cycle with twin chords $(C_{n,3})$, where $n = 6a + b$, $n \in N$.

b	Vertex Conditions	Edge Conditions
0,3	$v_f(0) = v_f(1) + 1 = v_f(2)$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$
1,4	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) = e_f(1) = e_f(2)$
2,5	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) = e_f(1) = e_f(2) + 1$

Hence the barycentric subdivision of cycle with twin chords is $3-$equitable. ∎

Example 2.4. $3-$equitable labeling of the graph obtained by the barycentric subdivision of cycle C_7 with twin chords is shown in Figure 2.

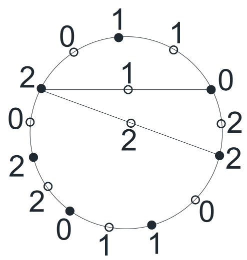

Figure 2: $3-$equitable labeling of the barycentric subdivision of cycle C_7 with twin chords.

Theorem 2.5. The barycentric subdivision of cycle with triangle $C_n(1, 1, n-5)$ is $3-$equitable.

Proof: Let G be cycle with triangle $C_n(1, 1, n-5)$. Let $S(G)$ be the barycentric subdivision of G and $v_1, v_2, \ldots, v_{2n+3}$ be the successive vertices of $S(G)$. Let $e_1 = v_1v_5$, $e_2 = v_5v_9$ and $e_3 = v_1v_9$ be three chords of C_n. Here $v_2, v_4, \ldots, v_{2n}$ are the vertices inserted due to the barycentric subdivision of the edges of C_n, v_{2n+1}, v_{2n+2} and v_{2n+3} be the vertices inserted due to the barycentric subdivision of

the chords e_1, e_2 and e_3 respectively. v_{2n+1} is adjacent to v_1 and v_5, v_{2n+2} is adjacent to v_5 and v_9, v_{2n+3} is adjacent to v_1 and v_9. Edge e_1 is subdivided into two edges $e_1' = v_1v_{2n+1}$ and $e_1'' = v_{2n+1}v_5$, e_2 is subdivided into two edges $e_2' = v_5v_{2n+2}$ and $e_2'' = v_{2n+2}v_9$, e_3 is subdivided into two edges $e_3' = v_1v_{2n+3}$ and $e_3'' = v_{2n+3}v_9$. Note that $d(v_1) = d(v_5) = d(v_9) = 4$ and $d(v_i) = 2$, $2 \leq i \leq 2n+3$, $i \neq 5, i \neq 9$. Here $|V(S(G))| = 2n + 3$ and $|E(S(G))| = 2n + 6$.
To define labeling function $f : V(G) \to \{0, 1, 2\}$, we consider the following cases.
Case 1: $n \equiv 0, 3(mod6)$.
$f(v_{2n+1}) = 1$,
$f(v_i) = 0$; if $i \equiv 0, 3(mod6)$
$= 1$; if $i \equiv 4, 5(mod6)$
$= 2$; if $i \equiv 1, 2(mod6)$, $1 \leq i \leq 2n + 3$, $i \neq 2n + 1$.
Case 2: $n \equiv 2, 5(mod6)$.
$f(v_i) = 0$; if $i \equiv 2, 5(mod6)$
$= 1$; if $i \equiv 3, 4(mod6)$
$= 2$; if $i \equiv 0, 1(mod6)$, $1 \leq i \leq 2n + 3$.
Case 3: $n \equiv 1, 4(mod6)$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod6)$
$= 1$; if $i \equiv 1, 2(mod6)$
$= 2$; if $i \equiv 4, 5(mod6)$, $1 \leq i \leq 2n + 3$.
Above defined labeling pattern satisfies the conditions of 3−equitable labeling as shown in Table 3.

Table 3: Vertex and edge conditions for the barycentric subdivision of cycle with triangle $C_n(1, 1, n - 5)$, where $n = 6a + b$, $n \in N$.

b	Vertex Conditions	Edge Conditions
0,3	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) = e_f(1) = e_f(2)$
1,4	$v_f(0) + 1 = v_f(1) = v_f(2)$	$e_f(0) = e_f(1) + 1 = e_f(2)$
2,5	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1$

Hence the barycentric subdivision of cycle with triangle $C_n(1, 1, n - 5)$ is 3−equitable. ■

Example 2.6. 3−equitable labeling of the graph obtained by the barycentric subdivision of cycle C_6 with triangle is shown in Figure 3. It is the case related to $n \equiv 0(mod6)$.

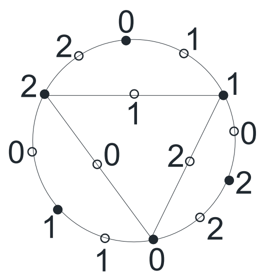

Figure 3: 3−equitable labeling of the barycentric subdivision of cycle C_6 with triangle.

Theorem 2.7. The barycentric subdivision of shell graph S_n is $3-$equitable.

Proof: Let $S(S_n)$ be the barycentric subdivision of shell S_n. Let v_0 be the apex vertex, $\{v_1, v_2, \dots, v_{2n-1}\}$ be the external vertices and $\{v'_1, v'_2, \dots, v'_{n-3}\}$ be the internal vertices in $S(S_n)$. Here the vertices $\{v_1, v_3, \dots, v_{2n-1}, v'_1, v'_2, \dots, v'_{n-3}\}$ are formed by the barycentric subdivision of shell graph S_n, where v'_j is the vertex which makes subdivision of the edge joining $v_{2(n-j-1)}$ and v_0, $j = 1, 2, 3, \dots, n-3$. Note that $|V(S(S_n))| = 3(n-1)$ and $|E(S(S_n))| = 4n-6$.
To define labeling function $f : V(S(S_n)) \to \{0, 1, 2\}$, we consider the following cases.
Case 1: $n \equiv 0(mod6)$.
$f(v_i) = 0$; if $i \equiv 2, 5(mod6)$
$= 1$; if $i \equiv 0, 1(mod6)$
$= 2$; if $i \equiv 3, 4(mod6), 0 \le i \le 2n-1$
$f(v'_j) = 0$; if $j \equiv 1, 4(mod6)$
$= 1$; if $j \equiv 2, 5(mod6)$
$= 2$; if $j \equiv 0, 3(mod6), 1 \le j \le n-3$
Case 2: $n \equiv 1(mod6)$.
$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$
$= 1$; if $i \equiv 0, 5(mod6)$
$= 2$; if $i \equiv 2, 3(mod6), 0 \le i \le 2n-1$
$f(v'_j) = 0$; if $j \equiv 1, 5(mod6)$
$= 1$; if $j \equiv 3, 4(mod6)$
$= 2$; if $j \equiv 0, 2(mod6), 1 \le j \le n-3$
Case 3: $n \equiv 2(mod6)$.
$f(v_i) = 0$; if $i \equiv 2, 5(mod6)$
$= 1$; if $i \equiv 3, 4(mod6)$
$= 2$; if $i \equiv 0, 1(mod6), 0 \le i \le 2n-1$
$f(v'_j) = 0$; if $j \equiv 3, 4(mod6)$
$= 1$; if $j \equiv 0, 5(mod6)$
$= 2$; if $j \equiv 1, 2(mod6), 1 \le j \le n-3$
Case 4: $n \equiv 3(mod6)$.
$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$
$= 1$; if $i \equiv 0, 5(mod6)$
$= 2$; if $i \equiv 2, 3(mod6), 0 \le i \le 2n-1$
$f(v'_j) = 0$; if $j \equiv 1, 5(mod6)$
$= 1$; if $j \equiv 3, 4(mod6)$
$= 2$; if $j \equiv 0, 2(mod6), 1 \le j \le n-3$
Case 5: $n \equiv 4(mod6)$.
$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$

$= 1$; if $i \equiv 0, 5(mod6)$
$= 2$; if $i \equiv 2, 3(mod6), 0 \leq i \leq 2n-1$
$f(v'_j) = 0$; if $j \equiv 0, 5(mod6)$
$= 1$; if $j \equiv 1, 2(mod6)$
$= 2$; if $j \equiv 3, 4(mod6), 1 \leq j \leq n-3$

Case 6: $n \equiv 5(mod6)$.
$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$
$= 1$; if $i \equiv 0, 5(mod6)$
$= 2$; if $i \equiv 2, 3(mod6), 0 \leq i \leq 2n-1$
$f(v'_j) = 0$; if $j \equiv 0, 5(mod6)$
$= 1$; if $j \equiv 1, 2(mod6)$
$= 2$; if $j \equiv 3, 4(mod6), 1 \leq j \leq n-3$

Above defined labeling pattern satisfies the conditions of 3−equitable labeling as shown in Table 4.

Table 4: Vertex and edge conditions for the barycentric subdivision of shell graph S_n, where $n = 6a + b$, $n \in N$.

b	Vertex Conditions	Edge Conditions
0,3		$e_f(0) = e_f(1) = e_f(2)$
1		$e_f(0) + 1 = e_f(1) = e_f(2) + 1$
2	$v_f(0) = v_f(1) = v_f(2)$	$e_f(0) + 1 = e_f(1) = e_f(2)$
4		$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$
5		$e_f(0) = e_f(1) = e_f(2) + 1$

Hence the barycentric subdivision of shell S_n is 3−equitable. ∎

Example 2.8. 3−equitable labeling of the graph obtained by the barycentric subdivision of shell S_9 is shown in *Figure 4*. It is the case related to $n \equiv 3(mod6)$.

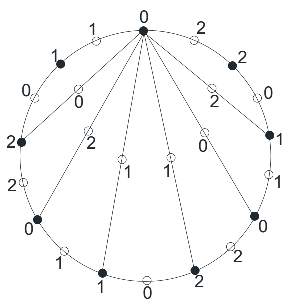

Figure 4: 3−equitable labeling of the barycentric subdivision of the shell S_9.

Theorem 2.9. The barycentric subdivision of wheel W_n is 3−equitable.

Proof: Let $S(W_n)$ be the barycentric subdivision of wheel W_n. Let $\{v_1, v_2, \ldots, v_{2n}\}$ be the rim vertices of G. Let $\{v'_1, v'_2, \ldots, v'_n\}$ be the internal vertices of $S(W_n)$ and v_0 be the apex vertex of G. Here, $v_2, v_4, \ldots, v_{2n}$ are the vertices inserted due to the barycentric subdivision of edges of C_n, where

v_j is the vertex which makes the subdivision of edge joining (v_{j-1}, v_{j+1}), $j = 2, 4, 6, \dots, 2n-2$, v_{2n} is adjacent to v_{2n-1} and v_i. v_i' is the vertex which makes the subdivision of edge joining (v_{2i-1}, v_0), $i = 1, 2, 3, \dots, n$. Note that $|V(S(W_n))| = 3n+1$ and $|E(S(W_n))| = 4n$.
To define labeling function $f : V(S(W_n)) \to \{0, 1, 2\}$ we consider the following cases.

Case 1: $n \equiv 0(mod6)$.
$f(v_0) = 2$.
$f(v_i) = 0$; if $i \equiv 2, 5(mod6)$
$= 1$; if $i \equiv 0, 1(mod6)$
$= 2$; if $i \equiv 3, 4(mod6)$, $1 \leq i \leq 2n$.
$f(v_i') = 0$; if $i \equiv 1, 2(mod6)$
$= 1$; if $i \equiv 3, 4(mod6)$
$= 2$; if $i \equiv 0, 5(mod6)$, $1 \leq i \leq n$.

Case 2: $n \equiv 1(mod6)$.
$f(v_0) = 2$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod6)$
$= 1$; if $i \equiv 1, 2(mod6)$
$= 2$; if $i \equiv 4, 5(mod6)$, $1 \leq i \leq 2n$.
$f(v_i') = 0$; if $i \equiv 0, 1(mod6)$
$= 1$; if $i \equiv 4, 5(mod6)$
$= 2$; if $i \equiv 2, 3(mod6)$, $1 \leq i \leq n$.

Case 3: $n \equiv 2(mod6)$.
$f(v_0) = 0$.
$f(v_i) = 0$; if $i \equiv 2, 5(mod6)$
$= 1$; if $i \equiv 0, 1(mod6)$
$= 2$; if $i \equiv 3, 4(mod6)$, $1 \leq i \leq 2n$.
$f(v_i') = 0$; if $i \equiv 4, 5(mod6)$
$= 1$; if $i \equiv 0, 1(mod6)$
$= 2$; if $i \equiv 2, 3(mod6)$, $1 \leq i \leq n$.

Case 4: $n \equiv 3(mod6)$.
$f(v_0) = 1$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod6)$
$= 1$; if $i \equiv 4, 5(mod6)$
$= 2$; if $i \equiv 1, 2(mod6)$, $1 \leq i \leq 2n$.
$f(v_i') = 0$; if $i \equiv 1, 4(mod6)$
$= 1$; if $i \equiv 0, 3(mod6)$
$= 2$; if $i \equiv 2, 5(mod6)$, $1 \leq i \leq n$.

Case 5: $n \equiv 4(mod6)$.
$f(v_0) = 2$.
$f(v_i) = 0$; if $i \equiv 1, 4(mod6)$
$= 1$; if $i \equiv 2, 3(mod6)$
$= 2$; if $i \equiv 0, 5(mod6)$, $1 \leq i \leq 2n$.
$f(v_i^{'}) = 0$; if $i \equiv 3, 4(mod6)$
$= 1$; if $i \equiv 0, 2(mod6)$
$= 2$; if $i \equiv 1, 5(mod6)$, $1 \leq i \leq n$.
Case 6: $n \equiv 5(mod6)$.
$f(v_0) = 0$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod6)$
$= 1$; if $i \equiv 1, 2(mod6)$
$= 2$; if $i \equiv 4, 5(mod6)$, $1 \leq i \leq 2n$.
$f(v_i^{'}) = 0$; if $i \equiv 3, 5(mod6)$
$= 1$; if $i \equiv 0, 1(mod6)$
$= 2$; if $i \equiv 2, 4(mod6)$, $1 \leq i \leq n$.
Above defined labeling pattern satisfies the conditions of 3−equitable labeling which is shown in Table 5.

Table 5: Vertex and edge conditions for the barycentric subdivision of the wheel W_n, where $n = 6a + b$, $n \in N$.

b	Vertex Conditions	Edge Conditions
0	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) = e_f(1) = e_f(2)$
1	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1$
2	$v_f(0) + 1 = v_f(1) + 1 = v_f(2)$	$e_f(0) = e_f(1) = e_f(2) + 1$
3	$v_f(0) + 1 = v_f(1) = v_f(2) + 1$	$e_f(0) = e_f(1) = e_f(2)$
4	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2)$
5	$v_f(0) = v_f(1) + 1 = v_f(2) + 1$	$e_f(0) = e_f(1) + 1 = e_f(2)$

Hence the barycentric subdivision of the wheel W_n is 3−equitable. ■

Example 2.10. 3−equitable labeling of the graph obtained by the barycentric subdivision of wheel W_6 is shown in Figure 5.

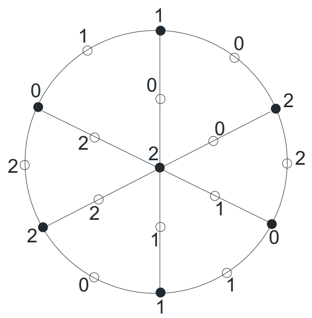

Figure 5: 3−equitable labeling of the barycentric subdivision of the wheel W_6.

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References

[1] I. Cahit, *On cordial and* $3-$*equitable labelings of graphs*, Util. Math., 37(1990), 189-198.

[2] J. A. Gallian, *A dynamic survey of graph labeling*, The Electronics Journal of Combinatorics, 16(2013), $\sharp DS6$ $1-308$.

[3] J. Gross and J. Yellen, Graph theory and its applications, CRC Press, 1999.

[4] S. K. Vaidya, S. Srivastav, V. J. Kaneria and G. V . Ghodasara, *Cordial and* $3-$*equitable of cycle with twin chords*, Proceedings of the First International Conference on Emerging Technologies and Applications in Engineering, Technology and Sciences, 1(2008), 905-907.

[5] S. K. Vaidya, K. K. Kanani, S. Srivastav and G. V. Ghodasara, *Barycentric subdivision and cordial labeling of some cycle related graphs*, Proceedings of the First International Conference on Emerging Technologies and Applications in Engineering, Technology and Sciences, 1(2008), 1081-1083.

[6] M. Z. Youssef, *A necessary condition on k-equitable labelings*, Util. Math., 64(2003), 193-195.