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Total edge Fibonacci irregular labeling of some star graphs

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Abstract

A total edge Fibonacci irregular labeling $f : V(G) \cup E(G) \to \{1, 2, \ldots, K\}$ of a graph $G = (V, E)$ is a labeling of vertices and edges of G in such a way that for any different edges xy and $x^{'}y^{'}$ their weights $f(x) + f(xy) + f(y)$ and $f(x^{'}) + f(x^{'}y^{'}) + f(y^{'})$ are distinct Fibonacci numbers. The total edge Fibonacci irregularity strength, tefs(G) is defined as the minimum K for which G has a total edge Fibonacci irregular labeling. If a graph has a total edge Fibonacci irregular labeling, then it is called a total edge Fibonacci irregular graph. In this paper, we prove $K_{1,n}$, bistar $<(B_{n,n})>$, subdivision of bistar $<(B_{n,n}; W)>$ and $<(B_{2,n}; W_i)>$ $(1 \leq i \leq n)$ are total edge Fibonacci irregular graphs.

Keywords: Total vertex irregular labeling, edge irregular total K-labeling, total edge Fibonacci irregular labeling.
AMS Subject Classification(2010): 05C78.

1 Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges. For terms not defined here, we refer to Harary [2]. A total vertex irregular labeling on a graph G with v vertices and e edges is an assignment of integer labels to both vertices and edges so that the weights calculated at vertices are distinct. The weight of a vertex v in G is defined as the sum of the label of v and the labels of all the edges incident with v, that is $wt(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv)$. The total vertex irregularity strength of G, denoted by tvs(G), is the minimum value of the largest label over all such irregular assignments. For a graph $G = (V, E)$, define a labeling $f : V(G) \cup E(G) \to \{1, 2, \ldots, K\}$ to be an edge irregular total K-labeling of the graph G if for every two different edges xy and $x^{'}y^{'}$ of G the edge weights $wt(\text{xy}) \neq wt(x^{'}y^{'})$. The total edge irregularity strength, tes(G), is defined as the minimum K for which G has an edge irregular total K-labeling. The notion of a total vertex irregular labeling and total edge irregular labeling are introduced by Baca et al [1]

Definition 1.1. The Fibonacci numbers can be defined by the linear recurrence relation

$$F_n = \begin{cases} 0 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ F_{n-1} + F_{n-2} & \text{if } n > 1. \end{cases}$$

This generates the infinite sequence of integers 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

2 Main Results

S. Amutha and K M.Kathiresan introduced the notion of total edge Fibonacci irregular labeling and also they proved that graphs like P_n, C_n and book with(3 and 4 sides) are total edge Fibonacci irregular graphs.

Definition 2.1. A total edge Fibonacci irregular labeling $f : V(G) \cup E(G) \to \{1, 2, \ldots, K\}$ of a graph $G = (V, E)$ is a labeling of vertices and edges of G in such a way that for any different edges xy and $x^{'}y^{'}$ their weights $f(x) + f(xy) + f(y)$ and $f(x^{'}) + f(x^{'}y^{'}) + f(y^{'})$ are distinct Fibonacci numbers.

The total edge Fibonacci irregularity strength, tefs(G) is defined as the minimum K for which G has total edge Fibonacci irregular labeling.

Note that if f is a total edge Fibonacci irregular labeling of $G = (V, E)$ with $|V(G)| = p$ and $|E(G)| = q$ then $F_4(= 3) \leq wt(xy) \leq F_{q+3}$ which implies that tefs $\geq \lceil \frac{F_{q+3}}{3} \rceil$.

Example 2.2. For the cycle C_4, $n = 4$, tefs $= \lceil \frac{F_{n+3}}{3} \rceil$. Therefore, tefs $= \lceil \frac{F_7}{3} \rceil$ =5.

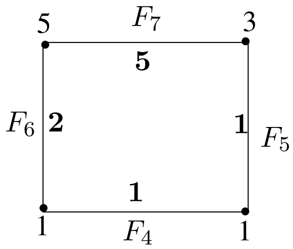

Figure 1

Theorem 2.3. The star graph $K_{1,n}$ has a total edge Fibonacci irregular labeling and tefs $(K_{1,n}) \leq \lfloor \frac{F_{n+3}}{2} \rfloor$, for any n.

Proof: Let $V = \{v, v_1, v_2, \ldots, v_n\}$ be the vertex set and $E = \{e_i = vv_i : i = 1, 2, \ldots, n\}$ be the edge set of $K_{1,n}$. Then $|V| = n + 1$ and $|E| = n$.
Define $f : V \cup E \to \{1, 2, \ldots, \lfloor \frac{F_{n+3}}{2} \rfloor\}$ by $f(v) = 1$, $f(v_i) = \lfloor \frac{F_{i+3}}{2} \rfloor$; $i = 1, 2, \ldots, n$ and $f(e_i) = F_{i+3} - \lfloor \frac{F_{i+3}}{2} \rfloor - 1$; $i = 1, 2, \ldots, n$.
By this labeling, $wt(e_i) = f(v) + f(e_i) + f(v_i)$; $i = 1, 2, \ldots, n$.

$$= 1 + (F_{i+3} - \lfloor \tfrac{F_{i+3}}{2} \rfloor - 1) + \lfloor \tfrac{F_{i+3}}{2} \rfloor$$
$$= F_{i+3}$$

Thus, the weights of $e_1, e_2, \ldots, e_n$ are $F_4, F_5, \ldots, F_{n+3}$ respectively. Also, tefs $(K_{1,n}) \leq \lfloor \frac{F_{i+3}}{2} \rfloor$, for any n. ∎

Example 2.4. The graph $(K_{1,8})$ is total edge Fibonacci irregular labeling and tefs $(K_{1,8}) \leq \lfloor \frac{F_{11}}{2} \rfloor = 44$.

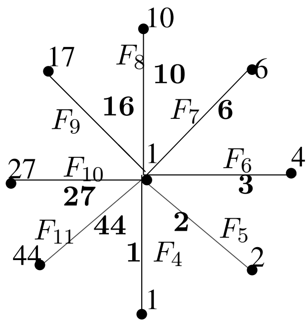

Figure 2: Total edge Fibonacci irregular labeling of $(K_{1,8})$.

Theorem 2.5. The bistar graph $B_{n,n}$ for $n \geq 2$ has a total edge Fibonacci irregular labeling and tefs $(B_{n,n}) \leq \lfloor \frac{F_{2n+4}}{2} \rfloor - 1$.

Proof: Let $V = \{u, v, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the vertex set and $E = \{e = uv, x_i = uu_i, y_i = vv_i \; ; \; i = 1, 2, \ldots, n\}$ be the edge set. Then $|V| = 2n + 2$ and $|E| = 2n + 1$.

Define $f : V \cup E \to \{1, 2, \ldots, \lfloor \frac{F_{2n+4}}{2} \rfloor - 1\}$ by $f(u) = 1$, $f(v) = 3$, $f(u_1) = 1$, $f(v_1) = 4$, $f(u_i) = \lfloor \frac{F_{2i+4}}{2} \rfloor - 1 \; ; \; i = 2, 3, \ldots, n$, $f(v_i) = \lfloor \frac{F_{2i+4}}{2} \rfloor - 1 \; ; \; i = 2, 3, \ldots, n$ and $f(e) = 1$, $f(x_1) = 1$, $f(x_i) = F_{2i+3} - \lfloor \frac{F_{2i+4}}{2} \rfloor \; ; \; i = 2, 3, \ldots, n$, $f(y_1) = 1$, $f(y_i) = F_{2i+4} - \lfloor \frac{F_{2i+4}}{2} \rfloor - 2 \; ;$ $i = 2, 3, \ldots, n$.

By this labeling,
$$
\begin{aligned}
wt(e) &= f(u) + f(e) + f(v) \\
&= 1 + 1 + 3 = 5 = F_5 \\
wt(x_1) &= f(u) + f(x_1) + f(u_1) \\
&= 1 + 1 + 1 = 3 = F_4 \\
wt(y_1) &= f(v) + f(y_1) + f(v_1) \\
&= 3 + 1 + 4 = 8 = F_6 \\
wt(x_i) &= f(u) + f(x_i) + f(u_i) \; ; \; i = 2, 3, \ldots, n. \\
&= 1 + (F_{2i+3} - \lfloor \tfrac{F_{2i+4}}{2} \rfloor) + (\lfloor \tfrac{F_{2i+4}}{2} \rfloor - 1) \\
&= F_{2i+3}
\end{aligned}
$$

Thus, the weights of $x_2, x_3, \ldots, x_n$ are $F_7, F_9, \ldots, F_{2n+3}$.

$$
\begin{aligned}
wt(y_i) &= f(v) + f(y_i) + f(v_i) \; ; \; i = 2, 3, \ldots, n. \\
&= 3 + (F_{2i+4} - \lfloor \tfrac{F_{2i+4}}{2} \rfloor - 2) + (\lfloor \tfrac{F_{2i+4}}{2} \rfloor - 1) \\
&= F_{2i+4}
\end{aligned}
$$

That is, weights of $y_2, y_3, \ldots, y_n$ are $F_8, F_{10}, \ldots, F_{2n+4}$.

Thus, the weights of edges $e, x_1, y_1, x_2, y_2, \ldots, x_n, y_n$ are $F_4, F_5, F_6, \ldots, F_{2n+3}, F_{2n+4}$ respectively and tefs $(B_{n,n}) \leq \lfloor \frac{F_{2n+4}}{2} \rfloor - 1$. ∎

Example 2.6. The graph $(B_{5,5})$ is total edge Fibonacci irregular labeling and tefs $(B_{5,5}) \leq \lfloor \frac{F_{14}}{2} \rfloor - 1 = 187$.

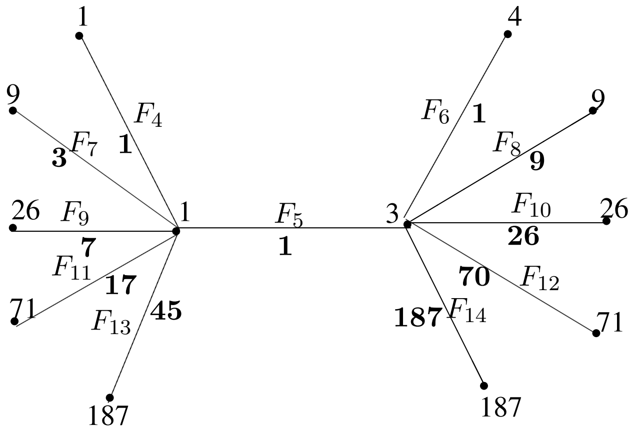

Figure 3: Total edge Fibonacci irregular labeling of $(B_{5,5})$.

Theorem 2.7. The Bistar graph $(B_{n,n}; W)$ for $n \geq 2$ has a total edge Fibonacci irregular labeling and tefs $(B_{n,n}; W) \leq \lfloor \frac{F_{2n+5}}{2} \rfloor - 1$.

Proof: Let $V = \{u, v, w, u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the vertex set and $E = \{x = uw, y = wv, x_i = uu_i, y_i = vv_i \ ; \ i = 1, 2, \ldots, n\}$ be the edge set. Then $|V| = 2n + 3$ and $|E| = 2n + 2$.

Define $f : V \cup E \to \{1, 2, \ldots, \lfloor \frac{F_{2n+5}}{2} \rfloor - 1\}$ by $f(u) = 1$, $f(w) = 2$, $f(v) = 3$, $f(u_1) = 1$, $f(v_1) = 5$, $f(u_i) = \lfloor \frac{F_{2i+5}}{2} \rfloor - 1$; $i = 2, 3, \ldots, n$, $f(v_i) = \lfloor \frac{F_{2i+5}}{2} \rfloor - 1$; $i = 2, 3, \ldots, n$ and $f(x) = 2$, $f(x_1) = 1$, $f(x_i) = F_{2i+4} - \lfloor \frac{F_{2i+5}}{2} \rfloor$; $i = 2, 3, \ldots, n$, $f(y) = 1$, $f(y_1) = 5$, $f(y_i) = F_{2i+5} - \lfloor \frac{F_{2i+5}}{2} \rfloor - 2$; $i = 2, 3, \ldots, n$.

By this labeling,
$$wt(x) = f(u) + f(x) + f(w)$$
$$= 1 + 2 + 2 = 5 = F_5$$
$$wt(y) = f(w) + f(y) + f(v)$$
$$= 2 + 3 + 3 = 8 = F_6$$
$$wt(x_1) = f(u) + f(x_1) + f(u_1)$$
$$= 1 + 1 + 1 = 3 = F_4$$
$$wt(y_1) = f(v) + f(y_1) + f(v_1)$$
$$= 3 + 5 + 5 = 13 = F_7$$
$$wt(x_i) = f(u) + f(x_i) + f(u_i) \ ; \ i = 2, 3, \ldots, n.$$
$$= 1 + (F_{2i+4} - \lfloor \tfrac{F_{2i+5}}{2} \rfloor) + (\lfloor \tfrac{F_{2i+5}}{2} \rfloor - 1)$$
$$= F_{2i+4}$$

That is, weights of $x_2, x_3, \ldots, x_n$ are $F_8, F_{10}, \ldots, F_{2n+4}$.

$$wt(y_i) = f(v) + f(y_i) + f(v_i) \ ; \ i = 2, 3, \ldots, n.$$
$$= 3 + (F_{2i+5} - \lfloor \tfrac{F_{2i+5}}{2} \rfloor - 2) + \lfloor \tfrac{F_{2i+5}}{2} \rfloor - 1)$$
$$= F_{2i+5}$$

That is, weights of $y_2, y_3, \dots, y_n$ are $F_9, F_{11}, \dots, F_{2n+5}$.

Thus, the weights of edges $x, y, x_1, y_1, x_2, y_2, \dots, x_n, y_n$ are $F_4, F_5, F_6, \dots, F_{2n+4}, F_{2n+5}$ respectively and tefs $\leq \lfloor \frac{F_{2n+5}}{2} \rfloor - 1$. ■

Example 2.8. The graph $(B_{6,6}; W)$ is total edge Fibonacci irregular labeling and tefs $(B_{6,6}; W) \leq \lfloor \frac{F_{17}}{2} \rfloor - 1 = 797$.

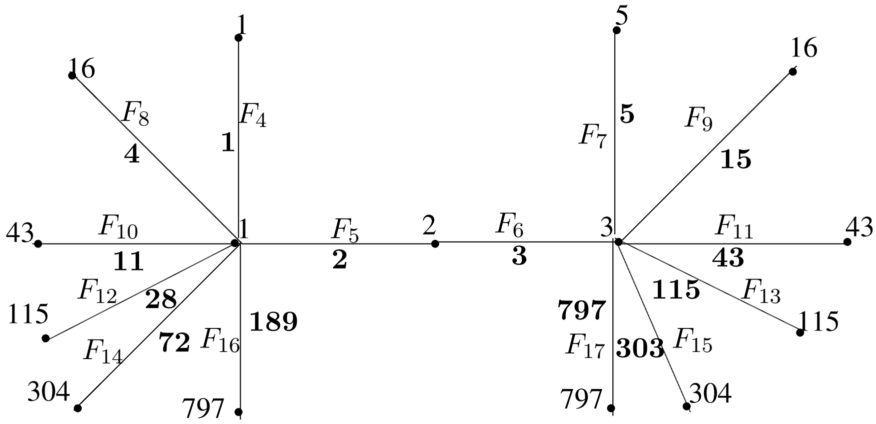

Figure 4: Total edge Fibonacci irregular labeling of $(B_{6,6}; W)$.

Definition 2.9. [3] Let u, v be the center vertices of $B_{2,n}$. Let u_1, u_2 be the vertices joined with u and $v_1, v_2, \dots, v_n$ be the vertices joined with v. Let $w_1, w_2, \dots, w_n$ be the vertices of the subdivision of edges vv_i $(1 \leq i \leq n)$ respectively and is denoted by $(B_{2,n}; W_i)$, $1 \leq i \leq n$.

Theorem 2.10. The graph $G = (B_{2,n}; W_i)$, $1 \leq i \leq n$, where $n \geq 2$ is a total edge Fibonacci irregular labeling and tefs = $\lceil \frac{F_{2n+6}}{3} \rceil$

Proof: Let $V = \{u, v, u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$ be the vertex set and $E = \{e = uv, x = uu_1, y = uu_2, x_i = vw_i, y_i = w_i v_i \ ; \ i = 1, 2, \dots, n\}$ be the edge set. Then $|V| = 2n + 4$ and $|E| = 2n + 3$.

Define $f : V \cup E \to \{1, 2, \dots, \lceil \frac{F_{2n+6}}{3} \rceil\}$ by $f(u) = 1$, $f(u_1) = 1$, $f(u_2) = 3$, $f(v) = 3$, $f(v_1) = 8$, $f(w_1) = 5$, $f(v_i) = \lceil \frac{F_{2i+6}}{3} \rceil$; $i = 2, 3, \dots, n$, $f(w_i) = F_{2i+6} - 2\lceil \frac{F_{2i+6}}{3} \rceil$; $i = 2, 3, \dots, n$ and $f(e) = 1$, $f(x) = 1$, $f(y) = 4$, $f(x_1) = 5$, $f(x_i) = F_{2i+5} + 2\lceil \frac{F_{2i+6}}{3} \rceil - F_{2i+6} - 3$; $i = 2, 3, \dots, n$, $f(y_1) = 8$, $f(y_i) = \lceil \frac{F_{2i+6}}{3} \rceil$; $i = 2, 3, \dots, n$.

By this labeling,
$$
\begin{aligned}
wt(e) &= f(u) + f(e) + f(v) \\
&= 1 + 1 + 3 = 5 = F_5 \\
wt(x) &= f(u) + f(x) + f(u_1) \\
&= 1 + 1 + 1 = 3 = F_4 \\
wt(y) &= f(u) + f(y) + f(u_2) \\
&= 1 + 4 + 3 = 8 = F_6 \\
wt(x_1) &= f(v) + f(x_1) + f(w_1)
\end{aligned}
$$

$$\begin{aligned}
&= 3+5+5 = 13 = F_7\\
wt(y_1) &= f(w_1)+f(y_1)+f(v_1)\\
&= 5+8+8 = 21 = F_8\\
wt(x_i) &= f(v)+f(x_i)+f(w_i)\ ;\ i=2,3,\ldots,n\,.\\
&= 3+(F_{2i+5}+2\lceil \tfrac{F_{2i+6}}{3}\rceil - F_{2i+6}-3)+(F_{2i+6}-2\lceil \tfrac{F_{2i+6}}{3}\rceil)\\
&= F_{2i+5}
\end{aligned}$$

Therefore, the weights of $x_2, x_3, \ldots, x_n$ are $F_9, F_{11}, \ldots, F_{2n+5}$.

$$\begin{aligned}
wt(y_i) &= f(w_i)+f(y_i)+f(v_i)\ ;\ i=2,3,\ldots,n\\
&= (F_{2i+6}-2\lceil \tfrac{F_{2i+6}}{3}\rceil)+\lceil \tfrac{F_{2i+6}}{3}\rceil+\lceil \tfrac{F_{2i+6}}{3}\rceil\\
&= F_{2i+6}
\end{aligned}$$

That is, weights of $y_2, y_3, \ldots, y_n$ are $F_8, F_{10}, F_{12}, \ldots, F_{2n+6}$.

Thus, the weights of edges $e, x, y, x_1, y_1, x_2, y_2, \ldots, x_n, y_n$ are $F_4, F_5, F_6, F_7, \ldots, F_{2n+5}, F_{2n+6}$ respectively and tefs $(B_{2,6}; W_6) = \lceil \frac{F_{2n+6}}{3}\rceil$. ∎

Example 2.11. The graph $(B_{2,6}; W_6)$ is total edge Fibonacci irregular labeling and tefs $(B_{2,6}; W_6) = \lceil \frac{F_{18}}{3}\rceil = 862$.

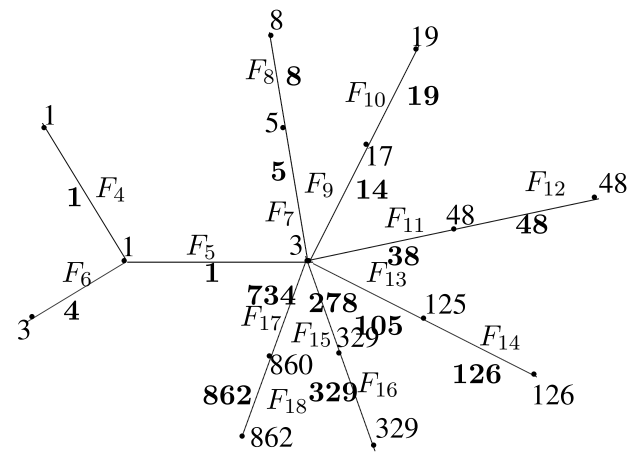

Figure 5: Total edge Fibonacci irregular labeling of $(B_{2,6}; W_6)$.

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