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Analysis of Performance in Four Non-Preemptive Priority Fuzzy Queues by Centroid of Centroids Ranking Method

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Abstract:

Ranking techniques are very noteworthy in the fuzzy numbers system for defuzzification. Many authors have already proposed various types of techniques to find out the performance of fuzzy queues. In this paper, we are going to deal with a new methodology namely centroid of centroids ranking method to find out the performance measures of non-preemptive priority fuzzy queues with 4-priorities. It is possible to convert from fuzzy environment to crisp environment by our proposed ranking method in order to analyze the performance measures of fuzzy queues. Finally, the effectiveness and the accurate values of our proposed method have been successfully solved by an example.

Keywords- **Fuzzy sets, Fuzzy numbers, Centroid of centroids ranking, Non-preemptive priority fuzzy queues, Performance measures.**

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I. INTRODUCTION

Now-a-days, in real life circumstances, we face a lot of priority based problems in the queueing environment such as ATM points, Medical shops, Reservation centers, Ration shops, Hospitals, Making calls in Telecommunications, etc…, . Stress the importance of time management is the ultimate aim of the researcher. At this juncture, queueing models take a very prominent role. The basic preliminaries[1],[2],[3] and models of queueing are very essential for our research purpose. In our day to day life situation, most of the time we apply the Fuzzy logic and applications[4],[6]. Occasionally, we apply the priorities[5] in the queueing situations. In some situations the priorities are accepted immediately and in some other situations it takes too much of time. These performances can also be measured by fuzzy logic[7].

Generally, the class 1 customers (with priority) and class 2 customers (Without priority) are serviced by same server. But the cost measures are entirely different for these customers[8],[9]. In this day to day situation, the preemptive priority based customers are mostly considered as special class 1 customers. This is an added advantage of the above mentioned customers. But the non-preemptive priority based customers[10] are always normal in consideration. However, in general, these customers are far better than the class 2 customers.

Many authors have so far applied various ranking techniques to measure the performances of the fuzzy queues. Robust ranking technique[11] is considered as a well known ranking technique. But the best methods of ranking technique are the distance based[17] methods. That are the area between centroid and the original points[12],[13], circumcenter of centroids[14], and centroid of centroids ranking techniques[15].

We can also use the centroid of centroids[20] ranking technique to measure the performance of non-preemptive priority fuzzy queueing models. This is a very easy method to compute the actual crisp values of the queueing models.

II. PRELIMINARIES

Fuzzy Set: A Fuzzy Set $\widetilde{A}$ = {(x, $\phi_{\widetilde{A}}$ (x)); x Є U}is concluded by a membership function $\phi_{\widetilde{A}}$ mapping from elements of a universe of discourse U to the unit interval [0,1].

(i,e) $\phi_{\widetilde{A}}$: U →[0,1] is a mapping called the degree of membership function of the fuzzy set $\widetilde{A}$ and $\phi_{\widetilde{A}}$ (x)is called the membership value of x Є U in the fuzzy set $\widetilde{A}$.

Triangular Fuzzy Number: A Triangular Fuzzy Number $\widetilde{A}$(x) is represented by $\widetilde{A}$(a_1, a_2, a_3 ;1) with the membership function

$$\phi_{\widetilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1\quad, & x=a_2\\ \dfrac{x-a_3}{a_2-a_3}, & a_2\le x\le a_3\\ 0, otherwise\end{cases}$$

Trapezoidal Fuzzy Number: A Trapezoidal Fuzzy Number $\widetilde{A}$(x) is represented by $\widetilde{A}$(a_1, a_2, a_3, a_4 ;1) with the membership function

$$\phi_{\widetilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ 1\quad, & a_2\le x\le a_3\\ \dfrac{x-a_4}{a_3-a_4}, & a_3\le x\le a_4\\ 0, otherwise\end{cases}$$

III.NON-PREEMPTIVE PRIORITY FUZZY QUEUES

Let us consider the Kth priority customers arrive at single channel queue in respect of a Poisson process with the fuzzy rate $\widetilde{\lambda}_k$,

(k =1,2,3,…,r) and that they are waiting for service in FIFO discipline with the service time as an exponential distribution with fuzzy rate $\widetilde{\mu}$. On a FIFO served within their respective priorities. Let the service distribution for the Kth priority be exponentially with mean $1/\widetilde{\mu}_k$ unit that begins service and completes its service before another item is admitted, regardless of priorities. We begin with $\rho_k=\dfrac{\lambda_k}{\mu_k}$

(1 ≤ k < r) , $\sigma_k=\sum_{i=1}^{k}\rho_i(\sigma_0\approx 0,\sigma_r\approx\rho)$ .The system is stationary for σ_r = ρ < 1. Let $\phi_{\widetilde{\lambda}}(u)$, $\phi_{\widetilde{\mu}}(v)$ denote the membership functions of $\widetilde{\lambda}$, $\widetilde{\mu}$. Then we have the following fuzzy sets:

$$\widetilde{\lambda}_i=\left\{(u,\phi_{\widetilde{\lambda}}(u))\,\middle|\,u\in U\right\}$$

$\widetilde{\mu} = \left\{ (v, \phi_{\widetilde{\mu}}(v)) \,\middle|\, v \in V \right\}$, where U,V are the crisp universal sets of the arrival , service rates respectively. Let f(u,v) denote the system characteristic of interest. Since u,v are fuzzy numbers. f(u,v) is also a fuzzy number. Without loss of generality let us take the performance measures of Non-Preemptive of 4-Priority Queues. We use the queuing theory concepts under the steady-state conditions $\rho_k = \frac{\lambda_k}{\mu_k} < 1$,

The Expected Queue Size

$$L_q = \sum_{i=1}^{r} L_q^{(i)} = \sum_{i=1}^{r} \frac{\lambda_i \sum_{k=1}^{r} \frac{\rho_k}{\mu_k}}{(1-\sigma_{i-1})(1-\sigma_i)}$$

and by using little's formula ,

The waiting time in queue $W_q = \sum_{i=1}^{r} \frac{\lambda_i w_q^{(i)}}{\lambda}$,

where $w_q^{(i)} = \frac{\sum_{k=1}^{r} \frac{\rho_k}{\mu_k}}{(1-\sigma_{i-1})(1-\sigma_i)}$

Here we consider a system of single server with 4-priority queues (ie. The arrival rates $\widetilde{\lambda}_i$, i=1,2,3,4).

Using the concept of $FM/FM/1/L$ queue with 4-priority queues can be reduced to M/M/1 queue with equal service rates.

(ie) $\widetilde{\mu}_1 = \widetilde{\mu}_2 = \widetilde{\mu}_3 = \widetilde{\mu}_4 = \widetilde{\mu}$,

Now $\widetilde{\rho}_1 = \frac{\widetilde{\lambda}_1}{\widetilde{\mu}_1}$, $\widetilde{\rho}_2 = \frac{\widetilde{\lambda}_2}{\widetilde{\mu}_2}$, $\widetilde{\rho}_3 = \frac{\widetilde{\lambda}_3}{\widetilde{\mu}_3}$ and

$\widetilde{\rho}_4 = \frac{\widetilde{\lambda}_4}{\widetilde{\mu}_4}$

Since $\widetilde{\rho} = \widetilde{\rho}_1 + \widetilde{\rho}_2 + \widetilde{\rho}_3 + \widetilde{\rho}_4$,

$\widetilde{\lambda} = \widetilde{\lambda}_1 + \widetilde{\lambda}_2 + \widetilde{\lambda}_3 + \widetilde{\lambda}_4$

$$\widetilde{\rho} = \frac{\widetilde{\lambda}_1 + \widetilde{\lambda}_2 + \widetilde{\lambda}_3 + \widetilde{\lambda}_4}{\widetilde{\mu}}$$

Also $w_q^{(i)} = \frac{\frac{\widetilde{\rho}_1 + \widetilde{\rho}_2 + \widetilde{\rho}_3 + \widetilde{\rho}_4}{\widetilde{\mu}}}{(1-\sigma_{i-1})(1-\sigma_i)}$ and

$$L_q^{(i)} = \frac{(\widetilde{\rho}_1 + \widetilde{\rho}_2 + \widetilde{\rho}_3 + \widetilde{\rho}_4)\widetilde{\rho}_i}{(1-\sigma_{i-1})(1-\sigma_i)}$$

From which we deduce that

$$w_q^{(1)} = \frac{\widetilde{\rho}}{\widetilde{\mu} - \widetilde{\lambda}_1}$$

$$w_q^{(2)} = \frac{\widetilde{\rho}}{(1-\widetilde{\rho}_1 - \widetilde{\rho}_2)(\widetilde{\mu} - \widetilde{\lambda}_1)}$$

$$w_q^{(3)} = \frac{\widetilde{\rho}}{(1-\widetilde{\rho}_1 - \widetilde{\rho}_2 - \widetilde{\rho}_3)(\widetilde{\mu} - \widetilde{\lambda}_1 - \widetilde{\lambda}_2)}$$

$$w_q^{(4)} = \frac{\widetilde{\rho}}{(1-\widetilde{\rho}_1 - \widetilde{\rho}_2 - \widetilde{\rho}_3)(\widetilde{\mu} - \widetilde{\lambda})}$$

$$L_q^{(1)} = \frac{\widetilde{\rho}\ \widetilde{\rho}_1}{(1-\widetilde{\rho}_1)}$$

$$L_q^{(2)} = \frac{\widetilde{\rho}\ \widetilde{\rho}_2}{(1-\widetilde{\rho}_1 - \widetilde{\rho}_2)(1-\widetilde{\rho}_1)}$$

$$L_q^{(3)} = \frac{\widetilde{\rho}\ \widetilde{\rho}_3}{(1-\widetilde{\rho}_1 - \widetilde{\rho}_2 - \widetilde{\rho}_3)(1-\widetilde{\rho}_1 - \widetilde{\rho}_2)}$$

$$L_q^{(4)} = \frac{\widetilde{\rho}\ \widetilde{\rho}_4}{(1-\widetilde{\rho}_1 - \widetilde{\rho}_2 - \widetilde{\rho}_3)(1-\widetilde{\rho})}$$

IV.CENTROID Of CENTROIDS RANKING METHOD – Algorithm

The centroid of a trapezoid can be considered as the balancing point of the trapezoid (Fig.1).Divide the trapezoid into three plane figures. These three plane figures are a triangle (PAQ), a rectangle (QABR) and another a triangle(RBS) respectively. Let the centroids of the three plane figures are denoted by $C1$, $C2$ and $C3$ respectively. The centroid **C** of these centroids is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point $C1$, C2 and C3 are balancing points of each individual plane figure and the centroid of these centroid points is a much more balancing point for a generalized trapezoidal fuzzy number.

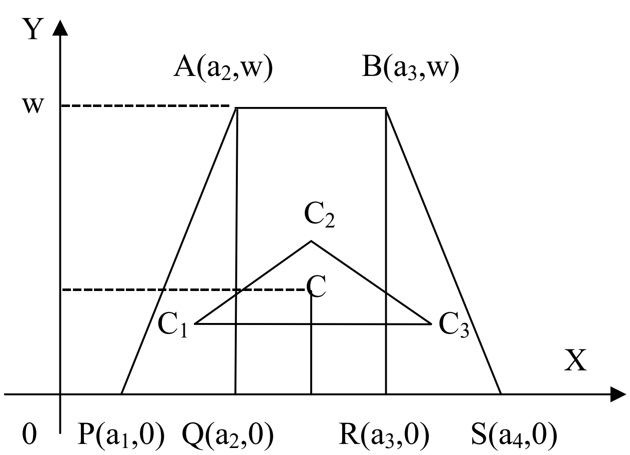

Figure 1. Centroid of centroids

Consider a Generalized Trapezoidal Fuzzy Number $\widetilde{A} = (a_1, a_2, a_3, a_4; w)$

The centroids of the three plane figures are $C_1 = \left(\frac{a_1 + 2a_2}{3}, \frac{w}{3}\right)$; $C_2 = \left(\frac{a_2 + a_3}{2}, \frac{w}{2}\right)$ and $C_3 = \left(\frac{2a_3 + a_4}{3}, \frac{w}{3}\right)$ respectively. Equation of the line $\overline{C_1C_3}$ is $y = \frac{w}{3}$ and C_2 does not lie on the Line $\overline{C_1C_3}$. So C_1,C_2 and C_3 are non-collinear and they form a triangle. We defined the centroid $C_{\widetilde{A}}(\bar{x}_0, \bar{y}_0)$ (ie C) of the triangle with centroids$C1$, $C2$ and $C3$ of the Generalized Trapezoidal Fuzzy Number $\widetilde{A} = (a_1, a_2, a_3, a_4; w)$ as

$$C_{\widetilde{A}}(\bar{x}_0, \bar{y}_0) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}, \frac{7w}{18}\right).$$

The centroid $C_{\widetilde{A}}(\bar{x}_0, \bar{y}_0)$ of Generalized Triangular Fuzzy Number $\widetilde{A} = (a_1, a_2, a_4; w)$ as $C_{\widetilde{A}}(\bar{x}_0, \bar{y}_0)$ $= \left(\frac{a_1 + 7a_2 + a_4}{9}, \frac{7w}{18}\right)$. (Here $a_2 = a_3$)

The ranking function of the Generalized Trapezoidal Fuzzy Number $\widetilde{A} = (a_1, a_2, a_3\ a_4; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\widetilde{A}) = \bar{x}_0 \times \bar{y}_0$

$$= \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}\right) \times \left(\frac{7w}{18}\right).$$

The ranking function of the Generalized Triangular Fuzzy Number $\widetilde{A} = (a_1, a_2\ , a_4; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R(\widetilde{A}) = \bar{x}_0 \times \bar{y}_0$

$$= \left(\frac{a_1 + 7a_2 + a_4}{9}\right) \times \left(\frac{7w}{18}\right).$$

V. NUMERICAL EXAMPLE

Let us imagine a critical situation takes place in Chennai Thulasi Medical Pharmacy Store where some of the customers are awaiting to get the medicines and to pay the bills. In this particular situation, every person is in an urgency to get the prescribed medicines. Suddenly some of the customers want to get the medicines especially for ICU patients. When the shop keeper allows (non-preemptive priority only) the persons who are in urgency to get the medicines, the situation is very critical to handle it. At this juncture, we should calculate the average waiting time and average queue length with four priorities. Moreover, we should analyse how the waiting time and queue length have been extended in this situation in our day to day life.

A: For Trapezoidal fuzzy number

Consider the arrival rates of 1st 2nd 3rd and 4th priority units are $\tilde{\lambda}_1 = [1,2,4,5:1]$, $\tilde{\lambda}_2$=[2,3,5,6:1], $\tilde{\lambda}_3$=[3,4,6,7:1], $\tilde{\lambda}_4$= [4,5,7,8:1] and the same service rate $\tilde{\mu}$ = [22,23,25,26:1] per hour respectively .

Now the membership function of the trapezoidal fuzzy number [1,2,4,5:1] is

$$\phi_{\tilde{\lambda}}(x) = \begin{cases} \dfrac{(x-1)}{(2-1)}, & 1 \le x \le 2 \\ 1 \quad , & 2 \le x \le 4 \\ \dfrac{(x-5)}{(4-5)}, & 4 \le x \le 5 \\ 0, otherwise \end{cases}$$

And similarly we can proceed for all remaining trapezoidal arrival rates in this same way.

Now we calculate Ranking by applying centroid of centroids ranking method.

$$\mathrm{R}(\tilde{\lambda}_1) = \mathrm{R}(1,2,4,5:1)$$

$$= \left(\frac{2(1)+7(2)+7(4)+2(5)}{18}\right) \times \left(\frac{7}{18}\right) = 1.16$$

$$\mathrm{R}(\tilde{\lambda}_2) = \mathrm{R}(2,3,5,6:1)$$

$$= \left(\frac{2(2)+7(3)+7(5)+2(6)}{18}\right) \times \left(\frac{7}{18}\right) = 1.55$$

$$\mathrm{R}(\tilde{\lambda}_3) = \mathrm{R}(3,4,6,7:1)$$

$$= \left(\frac{2(3)+7(4)+7(6)+2(7)}{18}\right) \times \left(\frac{7}{18}\right) = 1.94$$

$$\mathrm{R}(\tilde{\lambda}_4) = \mathrm{R}(4,5,7,8:1)$$

$$= \left(\frac{2(4)+7(5)+7(7)+2(8)}{18}\right) \times \left(\frac{7}{18}\right) = 2.46$$

$$\mathrm{R}(\tilde{\mu}) = \mathrm{R}(22,23,25,26:1)$$

$$= \left(\frac{2(22)+7(23)+7(25)+2(26)}{18}\right) \times \left(\frac{7}{18}\right)$$

= 9.33

And R($\tilde{\lambda}$) = 7.11 , R($\tilde{\rho}_1$) = 0.12 ,

R($\tilde{\rho}_2$) = 0.17 , R($\tilde{\rho}_3$) = 0.2, R($\tilde{\rho}_4$) = 0.26,
R($\tilde{\rho}$) = 0.76

From queuing theory formulas

Average waiting time of units of 1st priority in the queue is

$$w_q^{(1)} = \frac{\tilde{\rho}}{\tilde{\mu}-\tilde{\lambda}_1} = 0.093$$

Average waiting time of units of 2nd priority in the queue is

$$w_q^{(2)} = \frac{\tilde{\rho}}{(1-\tilde{\rho}_1-\tilde{\rho}_2)(\tilde{\mu}-\tilde{\lambda}_1)} = 0.13$$

Average waiting time of units of 3rd priority in the queue is

$$w_q^{(3)} = \frac{\tilde{\rho}}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(\tilde{\mu}-\tilde{\lambda}_1-\tilde{\lambda}_2)} = 0.22$$

Average waiting time of units of 4th priority in the queue is

$$w_q^{(4)} = \frac{\tilde{\rho}}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(\tilde{\mu}-\tilde{\lambda})} = 0.67$$

Average queue length of 1st priority is

$$L_q^{(1)} = \frac{\tilde{\rho}\ \tilde{\rho}_1}{(1-\tilde{\rho}_1)} = 0.1$$

Average queue length of 2nd priority is

$$L_q^{(2)} = \frac{\tilde{\rho}\ \tilde{\rho}_2}{(1-\tilde{\rho}_1-\tilde{\rho}_2)(1-\tilde{\rho}_1)} = 0.2$$

Average queue length of 3rd priority is

$$L_q^{(3)} = \frac{\tilde{\rho}\ \tilde{\rho}_3}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(1-\tilde{\rho}_1-\tilde{\rho}_2)} = 0.4$$

Average queue length of 4th priority is

$$L_q^{(4)} = \frac{\tilde{\rho}\ \tilde{\rho}_4}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(1-\tilde{\rho})} = 1.6$$

B: For Triangular fuzzy number

Consider the arrival rates of 1st2nd 3rd and 4th priority units are $\tilde{\lambda}_1$ = [1,4,5:1], $\tilde{\lambda}_2$ = [2,5,6:1], $\tilde{\lambda}_3$ = [3,6,7:1], $\tilde{\lambda}_4$ = [4,7,8:1] and the same service rate $\tilde{\mu}$ = [22,25,26:1] per hour respectively .

Now the membership function of the trapezoidal fuzzy number [1,4,5:1] is

$$\phi_{\tilde{\lambda}}(x) = \begin{cases} \dfrac{(x-1)}{(4-1)}, & 1 \le x \le 4 \\ 1 \quad , & x = 4 \\ \dfrac{(x-5)}{(4-5)}, & 4 \le x \le 5 \\ 0, otherwise \end{cases}$$

And similarly we can proceed for all remaining triangular arrival rates in this same way.

Now we calculate Ranking by applying centroid of centroids ranking method .

R($\tilde{\lambda}_1$) = R(1,4,5:1)

$$= \left(\frac{2+7(4)+5}{9}\right)\times\left(\frac{7}{18}\right) = 1.47$$

R($\tilde{\lambda}_2$) = R(2,5,6:1)

$= \left(\frac{2+7(5)+6}{9}\right)\times\left(\frac{7}{18}\right) = 1.85$

$R(\tilde{\lambda}_3) = R(3,6,7:1)$

$= \left(\frac{3+7(6)+7}{9}\right)\times\left(\frac{7}{18}\right) = 2.24$

$R(\tilde{\lambda}_4) = R(4,7,8:1)$

$= \left(\frac{4+7(7)+8}{9}\right)\times\left(\frac{7}{18}\right) = 2.63$

$R(\tilde{\mu}) = R(22,25,26:1)$

$= \left(\frac{22+7(25)+26}{9}\right)\times\left(\frac{7}{18}\right) = 9.63$

And $R(\tilde{\lambda}) = 8.19$, $R(\tilde{\rho}_1) = 0.152$,

$R(\tilde{\rho}_2) = 0.192$, $R(\tilde{\rho}_3) = 0.232$,

$R(\tilde{\rho}_4) = 0.273$ $R(\tilde{\rho}) = 0.85$

From queuing theory formulas

Average waiting time of units of 1st priority in the queue is

$$w_q^{(1)} = \frac{\tilde{\rho}}{\tilde{\mu}-\tilde{\lambda}_1} = 0.1$$

Average waiting time of units of 2nd priority in the queue is

$$w_q^{(2)} = \frac{\tilde{\rho}}{(1-\tilde{\rho}_1-\tilde{\rho}_2)(\tilde{\mu}-\tilde{\lambda}_1)} = 0.16$$

Average waiting time of units of 3rd priority in the queue is

$$w_q^{(3)} = \frac{\tilde{\rho}}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(\tilde{\mu}-\tilde{\lambda}_1-\tilde{\lambda}_2)} = 0.32$$

Average waiting time of units of 4th priority in the queue is

$$w_q^{(4)} = \frac{\tilde{\rho}}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(\tilde{\mu}-\tilde{\lambda})} = 1.39$$

Average queue length of 1st priority is

$$L_q^{(1)} = \frac{\tilde{\rho}\ \tilde{\rho}_1}{(1-\tilde{\rho}_1)} = 0.15$$

Average queue length of 2nd priority is

$$L_q^{(2)} = \frac{\tilde{\rho}\ \tilde{\rho}_2}{(1-\tilde{\rho}_1-\tilde{\rho}_2)(1-\tilde{\rho}_1)} = 0.29$$

Average queue length of 3rd priority is

$$L_q^{(3)} = \frac{\tilde{\rho}\ \tilde{\rho}_3}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(1-\tilde{\rho}_1-\tilde{\rho}_2)} = 0.71$$

Average queue length of 4th priority is

$$L_q^{(4)} = \frac{\tilde{\rho}\ \tilde{\rho}_4}{(1-\tilde{\rho}_1-\tilde{\rho}_2-\tilde{\rho}_3)(1-\tilde{\rho})} = 3.83$$

VI. CONCLUSION

In this paper, we have analysed the new methodology for finding the performance measures of Non-Preemptive Priority Fuzzy Queues. We may use this methodology for various Fuzzy Queues instead of using the existing methods. This methodology not only gives the crisp values but also gives more accuracy than the other values. This methodology will be useful and helpful to all the researchers and inventors in the days to come.

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