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## Review Paper

### Reinforced Concrete Corbels – State of the Art

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#### ABSTRACT

This paper represents a historical review, state-of-the-art review, on the experimental researches carried out on corbels in general and strengthening of corbels in particular. The corbels discussed are made of normal, high-strength and self-compacting concrete and subjected to two types of loading, monotonic and repeated loadings. Moreover, a review on the techniques of strengthening used to enhance the load carrying capacities of reinforced concrete corbels is presented too. Finally, directions for future work are proposed and a comprehensive list of references is provided.

## 1 Introduction

Brackets and corbels may be described as short cantilevers that project from the inner face of concrete columns or walls to support heavy concentrated loads of cranes and precast beams. The corbel is generally built monolithically with the column or wall. The shear span-to-depth is often less than unity.

Two design procedures are generally used for corbels, the first design following the ACI Code, and the other design uses the strut-and-tie models. Both design procedures give essentially the same results within the range of application of the ACI Code.

Brackets or corbels are designed mainly to provide for the vertical reaction ( $V_u$ ) at the end of the supported beam, but unless special precautions are taken to avoid horizontal forces caused by restrained shrinkage, creep or temperature change, they must also resist horizontal force ( $N_{uc}$ ). They are usually designed for direct shear using the shear friction theory.

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The nonlinear stress behavior of the short member is affected by the shear deformation in the elastic range and consequently the shear strength of the section becomes an important parameter for design consideration [1]. The principal failure mode for corbels reinforced with principal and secondary reinforcements (stirrups) is referred to as beam-shear failure, which is characterized by the opening of one or more diagonal cracks followed by shear failure in the compressed zone of the strut [2]. Moreover, corbels display several typical modes of failure, the most common of which are yielding of the tension tie; failure of the end anchorages of the tension tie, either under the load point or in the column; failure of the compression strut by crushing or shearing; and local failures under the bearing plate.

This paper represents a brief review on the experimental researches carried out on corbels in general and strengthening of corbels in particular to provide useful data in the development of a more general design criterion for determining the strength of reinforced concrete corbels. Moreover, directions for future work are proposed and a comprehensive list of references is provided for researchers interested in this subject.

## 2 Researches on Normal Strength Reinforced Concrete Corbels under Monotonic Loading:

Several researchers have studied the behavior and strength of reinforced concrete corbels with principal and secondary reinforcement.

One of the earliest researches in this domain was performed by Franz and Niedenhoff [3], who presented the simplest form of truss analogy for corbel design. This concept is illustrated in Fig. (1).

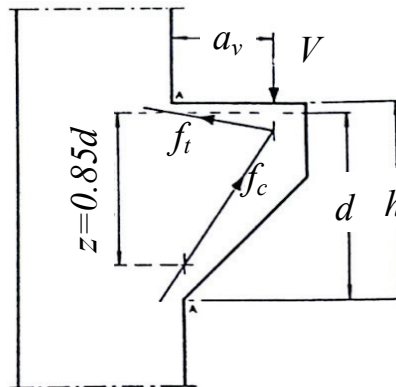
$$f_t = \frac{V \cdot a_v}{z} \quad (1)$$

The Area of main steel ( $A_{st}$ ) required for resisting force ( $V$ ) is:

$$A_{st} = \frac{f_t}{f_{adm}} \quad (2)$$

( $f_t$ ), is the tensile force, kN.

( $f_{adm}$ ), is the permissible tensile stress for the reinforcement, MPa.



**Fig. (1) – Simple Truss Analogy by Franz and Niedenhoff for the Design of Concrete Corbels [3]**

They showed that anchoring the main tension reinforcement at the outside face of the corbel may be done by providing horizontal loops. Moreover, they stated that the use of heavy bars inclined to the longitudinal axis of the supporting member is generally uneconomical and unsuitable except for the rare situation where the load is applied to the underside of the corbel. Although some diagonal ties remained in Franz and Niedenhoff's detailing recommendations, their proposal represented a big step forward from previous traditional methods which relied on inclined reinforcement.

Kriz and Rath [4], carried out three series of tests; the first set was exploratory tests (15 specimens), the second set on corbels subjected to vertical loads only, (114 specimens) and the third set on corbels subjected to combined vertical and

horizontal loads, (71 specimens). The exploratory tests involved testing procedures and reinforcing detailing. The other two series involved a systematic investigation of the effect of different variables on the strength and behavior of corbels.

The ultimate strength of a corbel, ( $V_u$ ), is a function of its width ( $b$ ), its effective depth ( $d$ ), the reinforcement ratio ( $\rho$ ), the concrete strength ( $f'_c$ ) and the ( $a_v/d$ ) ratio. The exploratory tests showed that the strength of corbels is not influenced by the additional load carried by the column and the arrangement and amount of reinforcement in the column, but it is proportional to  $\sqrt{f'_c}$ . The strength decreases as the ( $a_v/d$ ) ratio increases and increases as the main reinforcement ratio ( $\rho$ ) increases.

Their analysis of data from tests of corbels with horizontal reinforcement (stirrups) has shown that the stirrups are as effective in resisting vertical load as the main tension reinforcement.

Further tests were done by Kriz and Rath to study the effect of combined horizontal and vertical loading. It was found that the stirrups did not increase the resistance of a corbel to combined-loading by as large a proportion as was the case with a corbel subjected to vertical load only. Therefore, it was decided that any contribution from the stirrups should be regarded as reserve strength. For that reason it was concluded that a minimum amount of stirrups should always be provided.

The equation, proposed by them, for calculating the ultimate strength of corbels subjected to vertical or combined loading, based on fitting curves of their extensive test data, was as follows:

$$V_u = \phi \cdot b \cdot d \cdot \sqrt{f'_c} \cdot F_1 \cdot F_3 \quad (3)$$

Where:

$$F_1 = 6.5 \left( 1 - 0.5^{d/a} \right) \quad (3-a)$$

$$F_3 = \frac{(1000 \cdot \rho)^{\left( \frac{1}{3} + 0.4H/V \right)}}{10^{(0.8H/V)}} \quad (3-b)$$

$\rho$  is the reinforcement ratio at column face

$$\rho = \frac{A_s + A_h}{b \cdot d} \leq 0.02, \text{ for corbels subjected to vertical loads only}$$

$$\rho = \frac{A_s}{b \cdot d} \leq 0.013, \text{ for corbels subjected to combined loads.}$$

( $\phi$ ), is the capacity reduction factor; Corbel and bracket behavior is predominantly controlled by shear, therefore, a single value of ( $\phi = 0.85$ ) was used for all design conditions.

( $H/V$ ), is the ratio of horizontal to vertical load.

( $A_s$ ), is the area of tension reinforcement, mm<sup>2</sup>.

( $A_h$ ), is the total area of closed stirrups, mm<sup>2</sup>, ( $A_h$ ) must not be less than ( $A_s/2$ ). The design formula by Kriz and Rath have been adopted by ACI 318-71 [5] and PCI Design Handbook [6] for corbels.

Somerville, reported reviews to the available test data and previous design methods on corbels to determine the major parameters that influence its behavior [7]. He suggested an alternative method for enhancing the strength due to ( $a_v/d$ ) ratios. This method is illustrated in the form below:

$$\frac{V_{max}}{v_c} = \frac{3d}{a_v} < 6 \quad (4)$$

Where, ( $v_c$ ) is the shear stress permitted to be carried by the concrete alone at failure. He recommended that his method is valid for a range of ( $a_v/d$ ) ratios,  $\left( 0.6 < \frac{a_v}{d} < 1.5 \right)$  and suggested a practical range of (0.45-0.6) for the ratios of corbel

depth at the front face to the total depth to prevent secondary failure under the load. Somerville recommended that the depth at the front face of the corbel should be about one half of the total depth at the column face.

Some investigators indicated that truss analogy is not applicable to corbels with low values of  $(a_v/d)$  ratios. Somerville limits the use of truss analogy to values of  $(a_v/d)$  ratios equal or greater than (0.6). Therefore, for determining the shear strength of corbels having low values of  $(a_v/d)$  ratios, alternative approaches have been suggested.

Hermansen and Cowan [8], proposed the modified shear-friction theory. They assumed that when a crack forms on the shear plane the reinforcement crossing the crack is stressed to yield. The frictional resistance to concrete's movement along the crack is given by:

$$v = c + \rho \cdot f_y \cdot \tan \alpha \tag{5}$$

Where  $(c)$  is an apparent cohesive stress and  $(\tan \alpha)$  is the coefficient of friction.

They proposed the value of  $(c)$  to be (4 MPa) and  $(\tan \alpha)$  to be (0.8) for the purpose of obtaining a safe design.

The major conclusion reached by Hermansen and Cowan is that specimen with single corbel (exterior corbel) showed no significant difference in behavior in comparison to those with double corbels (interior corbel).

Mattock et al [9], carried out an experimental study on the behavior of reinforced concrete corbels subjected to vertical and horizontal loads. 28 specimens were tested, 26 of them contained horizontal stirrups; the variables studied were shear span to effective depth ratio,  $(a_v/d)$ , vertical to horizontal loads ratio,  $(V/H)$ , the amount of main reinforcement and secondary reinforcement (stirrups) and the type of aggregate. The failure of the specimens without stirrups was of splitting type. Two of the rest corbels had flexural failure. The remaining specimens failed as beam-shear failure, in which the flexural cracks remained fine and the failure was characterized by widening of one or more diagonal tension crack leading to shear-compression failure of the concrete near the intersection of the sloping face of the corbel and the column face. The failure was quite abrupt, but less brittle with more warning than the case of the diagonal tension failure of corbels without stirrups.

Based on these tests, they proposed the use of horizontal closed stirrups,  $(A_h)$ , parallel to the main reinforcement,  $(A_s)$ , such that  $(A_h \geq 0.5(A_s - N_u/f_y))$ . These bars were to be uniformly distributed within the upper two thirds of the total depth of the corbel. This amount of horizontal reinforcement will prevent premature diagonal tension failure of the corbel and permit yield strength of the main tension reinforcement to be developed. The failure of corbel may be either a flexural failure or a beam shear failure after the yielding of the flexural reinforcement. Either of these modes of failure is considered to be acceptable by the researchers, since the full strength of the main tensile reinforcement is attained.

In the same year, Mattock [10], proposed a simple design procedure for corbels with  $(a_v/d)$  ratio of unity or less, subjected to a combination of vertical and horizontal loads, based on a simple mechanical model (flexural model), which was adopted by the ACI 318 code. The main design procedure includes:

- 1) The ultimate shear stress  $(v_u)$  must not exceed the values indicated in equation (6).

$$v_u \leq \begin{cases} 0.2 \cdot f'_c \text{ MPa} \\ 5.5 \text{ MPa} \end{cases} \tag{6}$$

- 2) The area of reinforcement,  $A_{vf}$ , crossing the shear plane, need to be calculated according to equation (7):

$$A_{vf} = \frac{V_u}{\phi \cdot f_y \cdot \mu} \tag{7}$$

Where:  $\phi = 0.85$ ,  $\mu = 1.4$  for corbels cast monolithically with the column.

- 3) The ultimate moment need to be calculated, the corbel-column interface must resist the moment calculated by equation (8):

$$M_u = V_u \cdot a_v + N_u \cdot (h - d) \tag{8}$$

- 4) The area of reinfortment,  $A_f$ , necessary to resist the ultimate moment calculated in the previous item need to be calculated.

$$A_f = \frac{M_u}{\phi \cdot f_y \cdot (d - a/2)} \quad (9)$$

- 5) The area of reinfortment,  $A_t$ , necessary to resist the horizontal force,  $N_u$ , need to be calculated.

$$A_t = \frac{N_u}{\phi \cdot f_y} \quad (10)$$

- 6) The area of main tension reinfortment,  $A_s$ , will be the greater value of either of the following:

$$A_s = \begin{cases} A_f + A_t \\ \frac{2}{3} A_f + A_t \end{cases} \quad (11)$$

Closed Stirrups with area,  $A_h$ , parallel to the main tension reinfortment,  $A_s$ , must be uniformly distributed within two-thirds of the effective depth, and calculated as shown in Eq. (12):

$$A_h \geq 0.5 \cdot (A_s + A_t) \quad (12)$$

The steel ratio,  $\left(\rho = \frac{A_s}{b \cdot d}\right)$ , shall not be less than  $\left(0.04 \cdot \frac{f'_c}{f_y}\right)$ .

Hagberg [11], demonstrated the application of the truss analogy by describing a mathematical model to determine the capacity, that may be applied to all types of reinforcement (main and secondary), covering the practical range of (av/d) ratio from 0.15 to 1.5 and for any combination of horizontal and vertical loads. Hagberg proposed the following formulas:

$$\left(1 - \frac{2 \cdot f'_c \cdot b \cdot d}{F_s}\right) \cdot \tan^2 \beta + \left(\frac{2 \cdot f'_c \cdot b \cdot a}{F_s}\right) \cdot \tan \beta + 1 = 0 \quad (13)$$

Where:

$(\beta)$ , is the inclination compression strut with the vertical.

$(f'_c)$ , is the concrete cylinder strength, MPa.

$$F_s = F_{s1} + F_{s2} \quad (13-a)$$

$$F_{s1} = A_s \cdot f_y \quad (13-b)$$

$$F_{s2} = A_h \cdot f_{vy} \quad (13-c)$$

$A_s$  and  $A_h$  are the main and secondary reinforcement respectively, mm<sup>2</sup>.

$F_y$  and  $F_{vy}$  are the yield strength of main and secondary reinforcements respectively, MPa.

$$d = \frac{d_1 \cdot F_{s1} + d_2 \cdot F_{s2}}{F_s} \quad (13-d)$$

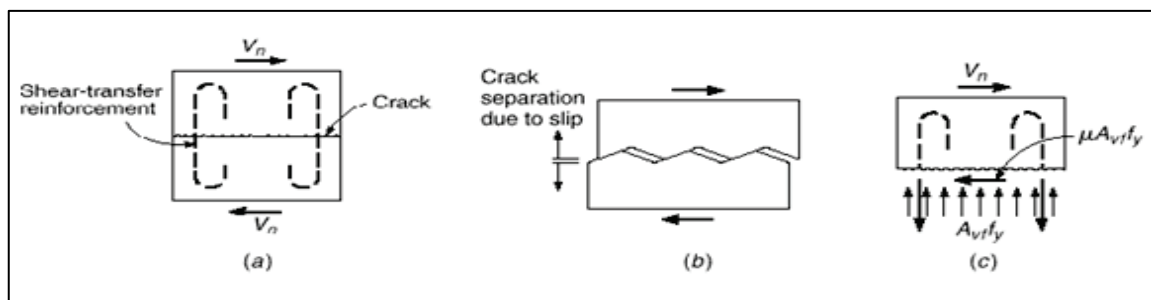
Where,  $d_1$  and  $d_2$  are the distance of main reinforcement and the center of gravity of the secondary reinforcement, respectively, mm.

Yong et al [12], performed a study to check the applicability of the ACI 318 Code and the truss analogy theory, proposed by Hagberg, on reinforced concrete corbels with concrete strength greater than 41.4 MPa. A total of eight high strength concrete corbels, divided into four series with concrete strength ranging from 41.7MPa to 82.7 MPa and the shear

span to depth ratio,  $(a_v/d)$ , was equal to 0.393. The corbels were loaded monotonically to failure. The researchers in this paper concluded that the ACI 318 code provisions are conservative for high strength concrete. In addition to the above, the truss analogy model predicted values were safe and less conservative than the ACI Code. But they concluded that this conservatism of the ACI Code may not necessarily be found in corbels with larger  $(a_v/d)$  ratios or subjected to horizontal loads in addition to the vertical loads.

Mast [13], introduced a technique to develop a simple rational approach, based on physical models (Shear friction hypotheses), that may be used for designing of different types of concrete connections. At first he applied the method on the design of the interface connection in composite beams, and then extended it to deal with concrete corbels using data from the tests of Kriz and Rath.

The shear-friction theory is very simple, and the behavior is easily visualized as shown in Fig. (2); the hypotheses developed by Mast was by considering a cracked concrete specimen, which is subjected to a normal compression across the crack and a shearing force along the crack.



**Figure (2) – Basis of Shear-Friction Design Method: a) Applied Shear; b) Enlarged Representation of Crack Surface; c) Free-Body Sketch of Concrete above Crack<sup>[14]</sup>**

The shearing force may be resisted by friction along the crack. If reinforcement is present normal to the crack, then slippage and subsequent separation of the concrete will stress the steel tension. The reinforcement will act as a tension member rather than as a dowel. The resulting tensile force will provide an equal and opposite pressure between the concrete faces on either side of the crack. As shown in the free-body diagram of Fig. (2-c), the maximum value of this interface pressure is  $(A_{vf}f_y)$ , where  $(A_{vf})$  is the total steel area crossing the crack and  $(f_y)$  is the yield strength. The concrete resistance to sliding may be expressed in terms of the normal force times a coefficient of friction  $(\mu)$ . By setting the summation of horizontal forces equal to zero:

$$V_n = A_{vf} \cdot f_y \cdot \mu \tag{14}$$

Where  $\mu = \tan \alpha$ ,  $\alpha$  is the angle of internal friction

Defining the reinforcement ratio  $(\rho = A_{vf} / A_c)$ , where  $(A_c)$  is the area of the cracked surface, equation (14) may be written as:

$$v_n = \rho \cdot f_y \cdot \tan \alpha \tag{15}$$

Where,

$$v_n = \frac{V_n}{A_c} \tag{15-a}$$

$V_n$ , is the shear force at ultimate load, kN.

$v_n$ , is the shear stress at ultimate load, MPa.

$A_{vf}$ , is the total steel area crossing the crack having yield strength of  $(f_y)$ .

Values of  $(\mu = \tan \alpha)$  are determined from tests. Mast, gave the following values for the design purposes:

Type of Surface	$\tan\alpha$
Concrete to concrete, rough surface, (crack in monolithic concrete) .....	1.4
Concrete to steel, composite beams .....	1.0
Concrete to concrete, smooth surface .....	0.7

These values were applied to the test data obtained by Kriz and Raths. The specimens were considered to be having  $[(a_v/d) \leq 0.7]$  and yielding steel, on the assumption that for higher  $(a_v/d)$  ratios, the quantity of steel required would be controlled by flexure rather than shear. When combined loading was used, the nominal shear stress was computed as:

$$v_n = \left( \rho \cdot f_y - \frac{H}{b \cdot h} \right) \cdot \tan \alpha \tag{16}$$

Where: H is the external horizontal force at ultimate load.

Both Hermansen and Cowan [8], Mast [12], proposed the modified shear-friction theory, but their values for  $(\tan \alpha)$  was different, therefore, for small amount of steel crossing the direct shear plane, where  $(\rho \cdot f_y)$  is low, and assuming  $(c=0)$ , Mast’s proposal will be more conservative than Eq. (5) proposed by Hermansen and Cowan. However for larger values of  $(\rho \cdot f_y)$  this trend will be reversed.

Zeller [15], concluded, from previous tests done on corbels, that for double corbels, having depth of 690mm and width 300mm, vertical stirrups would be more sufficient when the compression strut have an angle less than  $45^\circ$  with the horizontal. On the other hand, horizontal stirrups would be sufficient when that angle exceeds  $45^\circ$ , as in the case of corbels having  $(a_v/d)$  ratios less than (0.7-0.9).

Prasad et al. [16], analyzed reinforced concrete corbels using the nonlinear finite element method. The analysis indicated that the method may be used to predict the response of reinforced concrete structures. Details regarding orientation and distribution of cracks as well as stresses in concrete and steel at various locations may be obtained and studied. The finite element approach is regarded as a feasible method of analysis and design especially with the increasing of experimental costs and the decreasing of computational costs.

Siao [17], described a method for analyzing the shear strength of short shear-walls with height to length ratio of unity or less, top-loaded deep beams and corbels. A compression strut is present in all three elements, where shear force is transmitted to the support via a strut-and-tie system. By using the refined strut-and-tie system, shown in Fig. (3).

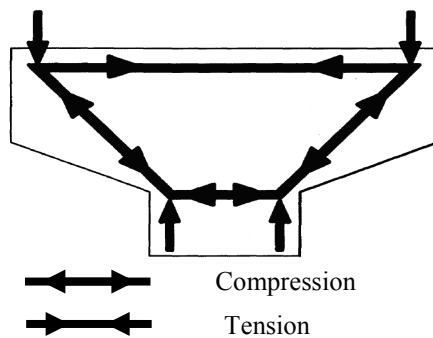


Figure (3) – Refined Strut and Tie Model [17]

The ultimate shear capacity may be calculated as follows:

$$V_u = 1.8 \cdot f_t \cdot b \cdot d \tag{16}$$

Where:  $(f_t)$ , is the allowable tensile strength of tension tie in refined compression strut; calculated by Eq. (16-a) prior to cracking, and by Eq. (16-b) for cracked concrete, where all stresses will be resisted by steel reinforcement.

$$f_t = 7 \cdot \sqrt{f'_c} \cdot \left[ 1 + n(\rho_h \cdot \sin^2 \theta + \rho_v \cdot \cos^2 \theta) \right] \quad (16-a)$$

$$f_t = (\rho_h \cdot \sin^2 \theta + \rho_v \cdot \cos^2 \theta) \cdot f_y \quad (16-b)$$

$(f_y)$ , is the steel yield strength.

$(n)$ , is the ratio of the elastic moduli of steel and concrete,  $(n = E_s/E_c)$

$(\rho_h \text{ and } \rho_v)$ , steel reinforcement ratio of horizontal and vertical bars, respectively.

$(\theta)$ , is the inclination of compression strut to tension tie.

Hwang et al. [18], presented a softened strut-and-tie model for determining the shear strength in corbels. 178 test specimens from literature were used for comparison with the proposed method. The studied corbels have various parameters, such as  $(a_v/d)$  ratios, different strength categories and horizontal reinforcement. The predictions of the ACI empirical equations were found to be conservative for the selected test data and more pronounced conservatism was found for corbels with low  $(a_v/d)$  or those made with high-strength concrete. It is thus confirmed that vertical stirrups are not useful for shear strength of a corbel with  $a/d \leq 1$ .

In the model proposed by Hwang et al., it was found that the web reinforcement was efficient in two ways; the first was to form tension ties and provide shear transferring paths, the other was to control the crack widths and retard the softening process of the cracked concrete. The following parameters were taken to cover the common range in practice:  $(\rho_h f_{yh})$  between 0 and 8 MPa,  $(a_v/d)$  between 1/4 and 1 and  $(f'_c)$  was equal to 30 MPa for normal strength concrete and 70 MPa for high-strength concrete. The concrete strength appears to set an upper limit value of  $(\rho_h f_{yh})$ , this limit is increased with the increasing of the concrete strength.

Aziz [19], studied the effect of crushed stone and their contribution to the shear strength of reinforced concrete corbels. 11 specimens were cast for this purpose, having the same dimensions, the variables studied were the type of aggregate used (normal gravel or crushed stones), the amount of longitudinal reinforcement, the amount of shear reinforcement, the compressive strength of concrete and the shear span to depth ratio,  $(a_v/d)$ . The researcher found that for the same proportion, workability curing and testing conditions, the crushed stone concrete shows higher values of compressive strength, tensile strength, shear stress when compared with the natural gravel concrete. The shear stress was increased with the increasing of the amount of longitudinal, Shear reinforcement and compressive strength of concrete, and it was decreased with the increasing of the shear span to depth ratio  $(a_v/d)$ .

According to the results obtained from this test and other results of 168 reinforced concrete corbels failing in shear, taken from literature, an equation for shear was proposed as follows:

$$V_u = 2.38 \times \left( \frac{f'_c \times k / d \times (\rho_w + \rho_h)}{a / d} \right)^{0.175} \quad (17)$$

Where:

$(V_u)$  is the ultimate shear stress of reinforced concrete corbels, N/mm<sup>2</sup>

$(f'_c)$  is the compressive strength of concrete, N/mm<sup>2</sup>

$(k, d)$  are properties of the section,  $k = 150$  mm

$(a / d)$  is the shear span to depth ratio

$(\rho_w, \rho_h)$  are the longitudinal and shear reinforcement ratios.

Good agreement was achieved when comparing the experimental results of this test with the results calculated by Eq. (17).



Singh et al. [20], presented a complete example on the analysis and design of a double corbel using the strut-and-tie method. The purpose was to amplify the application of the practical recommendations given by the ACI 318-02 Code regarding the design of structural members using the strut-and-tie method. The strut-and-tie method of design is based on the assumption that appropriate regions in concrete structures can be analyzed and designed using hypothetical pin-jointed trusses consisting of struts and ties connected at nodes.

A study performed by Russo et al. [21], to predict the shear strength of corbels by means of single accurate expression to avoid time consuming computing procedures. The model presented was obtained by superimposing the shear strength contribution of the strut-and-tie mechanism due to cracked concrete and main reinforcement, and the shear strength contribution due to horizontal stirrups. The first contribution was derived from a limiting shear expression, while the second contribution was derived from the equilibrium of the strut-and-tie mechanism in presence of stirrups.

The formula for computing the shear strength of corbel was obtained based on 243 test data available in literature; that formula is illustrated in Eq. (18).

$$v_u = 0.5 \cdot (k \cdot \chi \cdot f'_c \cdot \cos \theta + 0.65 \cdot \rho_h \cdot f_{yh} \cdot \cot \theta) \quad (18)$$

Where:

$(k)$ , is obtained from the classical bending theory of reinforced concrete beams with tensile reinforcement only.

$$k = \sqrt{(n \cdot \rho_f)^2 + (2 \cdot n \cdot \rho_f)} - (n \cdot \rho_f) \quad (18-a)$$

$(n)$ , is the ratio of the elastic moduli of steel and concrete,  $(n = E_s / E_c)$

$(\rho_f)$ , is the flexural reinforcement ratio.

$$\rho_f = \frac{A_s - A_n}{b \cdot d} \quad (18-b)$$

$$A_n = \frac{N_u}{f_{ys}} \quad (18-c)$$

$(f_{ys})$ , is the yielding strength of the main reinforcement.

$(f_{yh})$ , is the yielding strength of the stirrups.

$(\rho_h)$ , is the stirrups ratio at column-corbel interface.

$$\rho_h = \frac{A_h}{b \cdot d} \quad (18-d)$$

$(\theta)$ , is the angle between the compressed concrete strut and the vertical direction, and is provided by Eq. (18-e).

$$\theta = 2 \cdot \arctan \left( \frac{-1 + \sqrt{\left(\frac{a}{d}\right)^2 + 0.22 \left(1 - \frac{k^2}{4}\right)}}{\frac{a}{d} - \frac{k}{2}} \right) \quad (18-e)$$

$(\chi)$ , is provided by Eq. (18-f) with  $(10 \leq f'_c \leq 105, \text{ MPa})$ .

$$\chi = \left[ 0.74 \left( \frac{f'_c}{105} \right)^3 - 1.28 \left( \frac{f'_c}{105} \right)^2 + 0.22 \left( \frac{f'_c}{105} \right) + 0.87 \right] \quad (18-f)$$

The authors have excluded the corbels with a flexural reinforcement amount,  $(\rho_f)$ , lower than the minimum provided by the ACI 318-02,  $(\rho_{f_{min}} = 1.4/f_{ys})$ . The formula proposed for design was found adequately conservative and reliable. It led to an almost constant safety factor  $(v_{Experimental} = v_{Calculated})$ .

Rezaei et al. [22] investigated the effect of column-load on corbels. Single and double corbels with different levels of axial column load were modeled and analyzed by means of finite element software computer package (LUSAS version 14.1). The axial load capacity of the column was first determined, and then the load was applied on top of the column increasing gradually until reaching the predetermined level, after that, the load was applied on the corbels until failure was achieved.

It was found that the strength of double corbels was more influenced than that of single ones due to the axial column load. The latter was found to enhance the stiffness of corbels.

He et al. [23], proposed theoretical models and explicit equations regarding the main shear-resisting mechanisms in deep beams and corbels to provide a better understanding of the shear behavior of concrete structures. Deep beams transfer shear through two principal mechanisms: the direct strut mechanism, in which the load is transferred directly from the load point to the support, and the truss mechanism, which is the main shear-resistance mechanism in slender beams. The fraction of the total shear resistance associated with each mechanism depends on the  $(a_v/d)$  ratio of the member; the truss mechanism governs in members with  $(a_v/d) \geq 2$  whereas the direct strut mechanism is dominant in members with  $(a_v/d) \leq 1$ . Members with  $(a_v/d)$  ratios between 1 and 2, transfer load via a combination of the two mechanisms. The fraction of load transferred by both mechanisms was derived on the basis of maximum strength criterion, and since deep beams are very similar to double corbels due to their geometry, loading arrangement and failure modes, the strength of concrete corbels has been taken as an example of the methodology.

The theoretical proposal for determining the load distribution between the direct strut mechanism and the truss mechanism in deep beams and corbels has been derived on the basis of the maximum strength criterion.

The ultimate shear capacity of concrete corbels suggested by the authors was:

$$\frac{1}{V_u} = \frac{1}{V_1^*} + \frac{1}{V_2^*} \quad (19)$$

$$V_u = \frac{k \cdot \tan \theta}{[1 - (\gamma_{ht} \cdot \sin^2 \theta) / 2]} \cdot v \cdot f'_c \cdot b \cdot d \quad (19-a)$$

$$V_1^* = \frac{\tan \theta}{[1 - (\gamma_{ht} \cdot \sin^2 \theta) / 2]} \cdot v \cdot f'_c \cdot b \cdot d \quad (19-b)$$

$$V_2^* = \frac{2n \cdot (1 - 0.6 \cdot \gamma_{ht}) \tan \theta}{[1 - (\gamma_{ht} \cdot \sin^2 \theta) / 2]^2} \cdot \frac{(v \cdot f'_c)^2 \cdot b \cdot d}{f_y} \quad (19-c)$$

Where:  $(V_u)$ , is the total ultimate shear capacity

$(V_1^*)$  and  $(V_2^*)$ , are terms associated with the concrete and steel contributions to shear resistance.

$$\gamma_{ht} = \frac{\left( 2 \cdot \frac{z}{a} - 1 \right)}{3} \quad \text{for } (0 \leq \gamma_{ht} \leq 1) \quad (19-d)$$

### 3 Researches on High Strength Reinforced Concrete Corbels under Monotonic Loading:

During the later decades of the last century, a new procedure was introduced to improve the load carrying capacity of corbels; this procedure was by replacing the conventional secondary reinforcement with steel fibers. The addition of steel fibers improves the bond between steel and concrete, as well as the compressive, tensile, flexural and toughness properties of the concrete, moreover, the addition of steel fibers may provide an effective reinforcement against shear failure [24, 25].

Fattuhi [26], carried out tests on 22 (150×150×200-mm) concrete corbels reinforced with steel fibers, under vertical loading only, to estimate the improvement in shear properties due to the addition of steel fibers. The parameter studied included the fiber content and ( $a_v/d$ ) ratio. The results obtained indicated that the shear strength was improved by the addition of fibers, and it was concluded that the use of fibers as secondary reinforcement could be an attractive alternative to stirrups.

Abdul-Wahab [27], investigated the effect of steel fiber on reinforced concrete corbels and the applicability to SFRC corbels, of the design formula of three methods, the ACI 318 code, the truss analogy proposed by Hagberg [11] and the modified shear-friction equation suggested by Fattuhi [28] as shown in Eq. (20).

$$V_u = \phi (\eta A_{vf} f_{fu} \mu) + \phi (A_v f_y \mu) \quad (20)$$

Where:

( $\phi$ ), is the strength reduction factor (assumed to be 0.85 for shear).

( $\eta$ ), is the overall fiber efficiency factor (with average value of 0.1 from test results).

( $A_{vf}$ ), is the total area of fibers at the critical section.

( $f_{fu}$ ), is the ultimate tensile strength of the fiber.

( $\mu$ ), is the coefficient of friction (assumed to be 1.4 for monolithic concrete).

( $A_v$ ), is the area of reinforcement extending across the critical section.

( $f_y$ ), is the yield strength of the reinforcement.

Six corbels, in two groups, were tested to failure under vertical loading. In the first group, the steel fiber ratio (by volume) was varied from 0, 0.5, and 1 percent, while the ( $a_v/d$ ) ratio was kept constant at 0.522. In the second group, the fiber content was kept constant at 1 percent while the ( $a_v/d$ ) ratio was varied from 0.30 to 0.57; the strength reduction factor ( $\phi$ ) was taken as 1.0. Using (Eq. 22), suggested by Fattuhi [28], the following equation was obtained:

$$V_u = \phi (V_{u1} + V_{uf}) \quad (21)$$

Where:

( $\phi$ ), is the strength reduction factor (assumed to be 0.85 for shear).

( $\eta$ ), is the fiber efficiency factor (assumed to be 0.1).

( $V_{u1}$ ), is the minimum value of the ultimate load for reinforced concrete corbels using the ACI Building Code method.

( $V_{uf}$ ), is ( $\eta \cdot A_{vf} \cdot f_{fu} \cdot \mu$ ).

A very good correlation, compared with the three other methods, was obtained using Eq. (21). The average ratios of the tested ultimate load to the calculated load for the selected corbels with fibers are shown in table (1).

The results obtained indicated that an increase in the ultimate shear strength of the corbels was achieved due to the addition of moderate amounts of steel fibers; an increase of 40% in the strength was obtained with the addition of 1% of fiber content. The test results showed that the method in the ACI 318 code and the truss analogy method become too conservative by neglecting the contribution of fibers to shear strength.

**Table (1) - Average Ratios of the Tested Ultimate Load to the Calculated Load**

	Mean ( $V_{u-test}/V_{u-cal}$ )	Standard Deviation
ACI Code method	1.546	0.171
Truss Analogy (Hagberg <sup>[11]</sup> )	1.298	0.217
Eq. 20, (Fattuhi <sup>[28]</sup> )	1.139	0.255
Proposed Eq. 21, (Abdul-Wahab <sup>[27]</sup> )	1.110	0.099

In the same year, Hughes and Fattuhi [29, 30], performed a series of tests on concrete corbels reinforced with either main bars only, fibers only or main bars plus fibers, the volume fraction of steel or polypropylene fibers, or main bars, was kept constant at nearly (0.7%). Plain concrete corbels were tested as well for comparative purposes. The test results indicated that the addition of 0.7% by volume of crimped steel fibers almost doubled the ultimate load. All plain concrete and fiber reinforced concrete corbels failed in flexure. It was concluded as well that fiber reinforcement was in general unlikely to be an economical alternative to the main bars for resisting flexure. Corbels with plain concrete failed in a brittle manner soon after reaching their ultimate loads, while corbels with fiber reinforced concrete failed more gradually, to provide, in some cases, an almost elastic-plastic behavior.

Other tests were carried out, by the same researchers, on concrete corbels reinforced with main bars only or with steel fibers in addition to the main bars, the volume of the main bars and the ( $a_v/d$ ) ratio were varied. The test results indicated that corbels reinforced with main bars only failed suddenly and catastrophically shortly after reaching their maximum loads, and the mode of failure was diagonal splitting or constrained shear. But with using steel fibers (as a secondary reinforcement), the strength and ductility were improved. The improvement was more obvious for corbels reinforced with low volumes of main bars and/or those tested at large ( $a_v/d$ ) ratios.

Fattuhi [31], carried out tests on 18 concrete corbels reinforced with main bars and steel fibers. The fibers were used as secondary reinforcement. The volumes of main and fibrous reinforcements and shear span-to-depth ratio were varied. A concentric load was applied on the column segment of eight specimens, while, unequal loads were applied on the corbel segments of four of the double corbels. Test results indicated that loading the column segment of the specimen or the left-side corbel did not have any significant effect on the load-carrying capacity of the corbels; however, the presence of column load delayed the occurrence of the first crack. It was concluded, as well, that the addition of steel fibers improved both the strength and ductility of the corbels.

Test performed by Fattuhi [32, 33], on normal and high-strength concrete corbels reinforced with main bars only, with main bars and steel or monofilament polypropylene fibers, or with main bars and plastic mesh. The fibers or strips of plastic mesh were used as secondary (shear) reinforcement. These tests confirmed the findings of earlier investigations about improving both the strength and ductility of reinforced concrete corbel by adding steel fibers to the concrete. It was concluded as well that steel fibers were the only form of secondary reinforcement that successfully reduced the widths of cracks in corbels. The improvement in strength due the addition of fibrous reinforcement was more significant in high-strength concrete corbels relative to low or moderate strength concrete corbels.

These tests also led to the finding of semi-empirical equations to estimate the flexural strength of reinforced concrete corbels subjected to vertical loading by using two methods: flexural and truss models. Comparison between experimental and calculated shear strength of corbels using the two methods showed a good agreement.

In the flexural model, the corbel was assumed to behave as a short cantilever beam. By knowing the properties of the corbel, the nominal strength may be calculated as shown below:

$$V_n = \frac{f_y \cdot A_s}{a} \left( d - \frac{a_1}{2} \right) + \frac{K_o \cdot f_{ct} \cdot b}{2a} \left( h - \frac{a_1}{\beta_1} \right) \cdot \left( h + \frac{a_1}{\beta_1} - a_1 \right) \quad (22)$$

Where:

$$\left( a_1 = \beta_1 \cdot c = \frac{f_y \cdot A_s + K_o \cdot f_{ct} \cdot h \cdot b}{0.85 \cdot f'_c \cdot b + K_o \cdot f_{ct} \cdot \frac{b}{\beta_1}} \right) \quad (22-a)$$

$(a_1)$ , is the depth of rectangular compression block in concrete.

$(a)$ , is the shear span.

$(\beta_1)$ , according to the ACI 318, varies between 0.65 and 0.85.

$(c)$ , is the depth of the neutral axis.

$(f_{ct})$ , is the tensile splitting strength, MPa.

$(K_o)$ , is the contribution of fibrous concrete in tension obtained from regression analysis of test results.

The value of  $(K_o)$  was found to range between (0.185-0.675) with an overall average of 0.353 and a coefficient of variation of 30.4%; the relationship between  $(K_o)$  and  $(f'_c)$  from regression analysis was as follows:

$$K_o = \frac{9.519}{(f'_c)^{0.957}} \quad (22-b)$$

In the truss model, the semi-empirical formula derived was based on the truss analogy theory proposed by **Hagberg** [11], this formula took into consideration the tensile strength of the fibrous concrete as suggested in the flexural model.

By knowing the properties of the corbel, and using the appropriate effective depth,  $(d_i)$  for each layer of stirrups, the capacity of the corbel may be calculated as shown below:

$$V_n = \frac{f_y \cdot A_s \cdot \left( d - \frac{l \cdot \sin \beta}{2} \right) + f_{yi} \cdot A_{si} \cdot \left( d_i - \frac{l \cdot \sin \beta}{2} \right) + 0.5 \cdot K_o \cdot f_{ct} \cdot b \cdot h \cdot (h - (l \cdot \sin \beta))}{a + 0.5 \cdot (l \cdot \sin \beta) \cdot \cot \beta} \quad (23)$$

Where:

$$(l \cdot \sin \beta) = \frac{f_y \cdot A_s + f_{yi} \cdot A_{si} + K_o \cdot f_{ct} \cdot b \cdot h}{0.85 \cdot f'_c \cdot b + K_o \cdot f_{ct} \cdot b} \quad (23-a)$$

$(d_i)$ , is the distance of (ith) layer of stirrups from extreme compression fiber.

$(l)$ , is the width of compression strut in concrete.

$(\beta)$ , is the inclination of resultant concrete compression strut with the vertical.

$(f_{yi})$ , is the yield stress of (ith) layer of stirrups, MPa.

$(A_{si})$ , is the area of stirrups in (ith).

$(a)$ , is the shear span.

$(f_{ct})$ , is the tensile splitting strength, MPa.

Foster et al. [34], tested thirty corbels under vertical loading. The main variables studied were concrete strength (45–105 MPa), (6500–15200 psi), shear span-to depth ratio, and the provision of secondary reinforcement. Particular attention was given to determining the concrete efficiency factor for members failing in compression. And a comparison was made for the corbel behavior with the ACI 318-89 design method and the plastic truss model. The results showed that good load predictions can be obtained using the plastic truss. It was concluded from the results that corbels made from high-strength concrete behaved in the same manner as those made from normal-strength concrete. Moreover, the first cracks detected were flexural cracks propagating from the corbel-column intersection, and that the flexural cracking load decreases with an increase in the shear span-to-depth ratio.

It was also concluded that the provision of secondary reinforcement reduces crack widths, improves ductility, and for beams failing in compression may change the failure mode from diagonal splitting to compression strut crushing. Therefore, they advised the provision of a minimum quantity of horizontal stirrups, similar to that for normal-strength concrete, when fabricating high-strength concrete corbels. It was also concluded that the ACI 318-89 design method is not recommended for use with corbels designed with high and very high-strength concretes.

Muhammad [35], studied the strength and deformation characteristics of 48-HSC corbels with main reinforcement and with either steel fibers or stirrups as secondary reinforcement, or the latter was a combination of steel fibers and stirrups. The parameters studied were volume fraction of steel fibers, the ( $a_v/d$ ) ratios and the horizontal stirrups ratio. 36 of the specimens were tested to failure under monotonic loading, while the remaining 12 specimens were tested under repeated loading.

The following was concluded from test results of corbels under monotonic loading:

1. The use of high-strength fiber reinforced concrete (HSFRC) in corbels improved the behavior and strength of the specimens.
2. Specimens with combined shear reinforcement, stirrups and fibers, exhibited higher first cracking load than those reinforced with fibers only. Moreover, under a specific volume fraction of fibers, higher first cracking load appeared in the corbels reinforced with the highest amount of secondary steel ratio.
3. The addition of steel fibers increased the stiffness of the corbels and reduced the deflection for a specific load. On the other hand the combination of fibers and stirrups lead to a higher reduction in deflection.
4. The presence of steel fibers increased the ultimate shear strength. This increase was found to be dependent on the fiber volume and ( $a_v/d$ ) ratio.

Bourget et al. [36], carried out tests on three different concrete compositions, established by previous researchers (De Larrard et al. [37], having a mean strength of 80, 100, and 120 MPa respectively. The test specimens were divided into two series. The first one consisted of high-level reinforcement corbels, (C2-120), and the second series consisted of specimens with lower-level reinforcement quantity, (C1-80), (C2-80) and (C2-100). An anchor bar was welded to primary and secondary steel. The aim of the experimental program was to study the influence of concrete strength. The experimental corbel failure mode was first defined with the evolution of the main reinforcement strain versus conventional stress: if the main reinforcement has yielded the failure mode would be considered as tension failure; if it has not then the crack pattern would indicate the failure mode by either concrete crushing or by diagonal splitting.

Campione et al. [38] carried out an experimental investigation on the flexural behavior of normal and high strength fibrous reinforced concrete corbels in the presence of main and secondary reinforcement. The parameters investigated were the fiber content and geometric ratio of main and secondary steel and the concrete strength, normal- or high- strength types; the test results showed that the use of fibers instead of transverse stirrups, or coupled with them, may reduce the brittleness of the mechanism involving crushing of compressed regions. Moreover, it was found that it is possible to activate flexural failure in the case of fibrous concrete improving the ductility.

A simple analytical expression, based on truss analogy, considering the presence of fibers was proposed, to determine the bearing capacity, taking into account the shear contribution due to steel reinforcement and fibers. This model was verified against experimental data of the authors and other data available in literature. The results obtained showed that the proposed model fits the experimental results with a good level of approximation.

Ridha [39], presented a nonlinear finite element investigation on the behavior of high strength fiber reinforced concrete corbels subjected to monotonic loading, in order to get a better understanding of their behavior throughout the entire

loading history. The main variables were the fiber content, grade of concrete, main reinforcement and ( $a_v/d$ ) ratio. It was found that the presence of fibers helped in enhancing the ductility and energy absorption capacity. The cracking load and post cracking stiffness was found to increase with the increasing of the compressive strength of concrete and the decreasing in the ( $a_v/d$ ) ratios.

Ahmad and Shah [40], have focused on the application of the strut-and-tie model (STM) for the design of high strength concrete double-corbels, in an attempt to verify that the STM is an alternative design method for non-flexural disturbed regions in concrete structures. Nine double-corbels of high strength concrete (45, 52 and 59 MPa) were cast, with the minimum area of steel allowed by the ACI 318 code; the corbels have been analyzed on the basis of STM, and then were tested in the laboratory under monotonic loading to compare the theoretical and actual failure loads, which were found to be quite close and realistic. The authors recommended that further research on the application of the STM for the design of high strength concrete double corbels would be needed. The dimensions of the studied corbels are shown in Fig. (4).

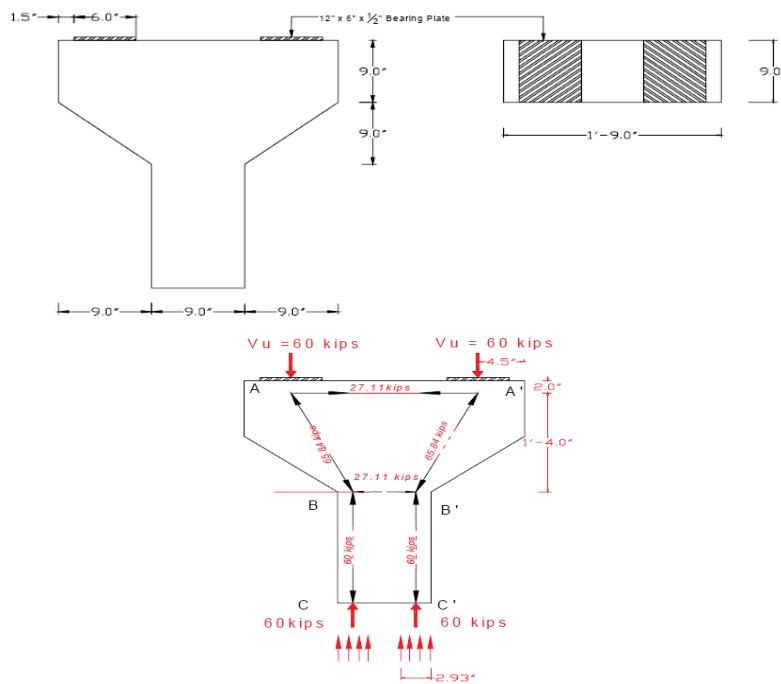


Figure (4) – Geometry of the Corbels (Truss and members) [40]

Campione [41], proposed a softened strut-and-tie- macro model in order to produce the load-deflection curves of corbels constructed with plain and fibrous concrete under both vertical and horizontal forces. An equivalent single or multiple truss structure was adopted for this purpose as shown in Fig. (5). the single truss represents corbels with main and

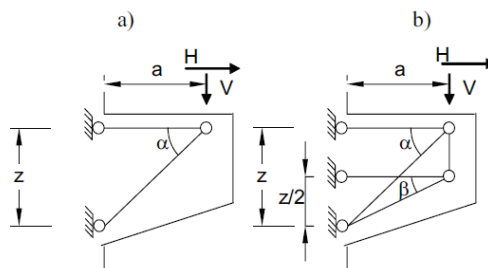


Figure (5) – Equivalent Truss Model: a) Single Truss. b) Multiple Truss [41]

A nonlinear finite element approach was implemented, in the same year, by Yousif [42] to study the behavior of high-strength reinforced concrete corbels using a simplified softened strut-and-tie model. The analysis was based on truss analogy, following the provisions of Appendix A of the ACI 318-05 Code. Different parameters such as  $(a_v/d)$  ratio, primary and secondary reinforcement ratios, and concrete compressive strength have been studied. The study showed that the finite element method was a suitable tool for the analysis of high-strength concrete corbels, and that the strut-and-tie model was a simple and effective design tool that may give reasonable predictions of the failure load capacity of high-strength reinforced concrete corbels.

Al-Zahawi [43], investigated the shear strength and the behavior of reinforced concrete corbels containing chopped carbon fibers subjected to concentrated vertical loads. His work was divided into two categories: experimental and theoretical. To perform the experimental part, 15 reinforced concrete corbels were cast with and without chopped carbon fibers, all specimens had the same dimensions and main reinforcement; the shear span to depth  $(a_v/d)$  ratios ranged from 0.3 to 0.6. The variables studied were the  $(a_v/d)$  ratios, the volume fraction of carbon fibers and the presence or absence of the secondary reinforcement. Most of the specimens failed in a diagonal splitting mode. Test results showed that the presence of carbon fibers in the concrete has enhanced the tensile strength of the corbels and delayed the formation of inclined diagonal shear cracks.

The presence of secondary reinforcement in addition to the main reinforcement and for the same values of volume fraction of fibers has increased the value of the ultimate load as the  $(a_v/d)$  ratio decreases, when the  $(a_v/d)$  ratio decreases from 0.6 to 0.3, the average increase in the ultimate load was found to be about 15.9%.

The theoretical part included the prediction of useful expressions to predict the nominal shear strength of fibrous and nonfibrous reinforced concrete corbels, using linear and nonlinear regression analysis of the results of the experimental test in addition to other tests from literature. The statistical results showed that the  $(a_v/d)$  ratio is inversely proportional to the ultimate shear capacity of fibrous and nonfibrous reinforced concrete corbel. It was found from the theoretical analysis that the nonlinear regression expression resulted in a lowest C.O.V. value, among the expressions available in literature, than that of the linear regression expression.

Khalifa [44], proposed a macro-mechanical strut and tie model to analyse fibrous high-strength concrete corbels. The fibers were applied as partial or full replacement of horizontal stirrups. The parameters studied were, the effect of fiber volume, fiber length, and fiber diameter, random distribution of fibers, fiber HSC interface, shear span to depth ratio and concrete strength. Results showed that the maximum vertical load carrying capacity applied on corbels was increased with the increasing of the fiber volume fraction, fiber aspect ratio and the increasing in concrete compressive strength.

For engineering requirement, the researcher recommended, for enhancing the load carrying capacity of corbels, the preference to increase the fiber volume and/or the fiber aspect ratio instead of the increase in the concrete compressive strength greater than 50 MPa.

Abass [45], carried out experimental and theoretical investigations to study the behavior and load carrying capacity of fibrous and non-fibrous self-compacting reinforced concrete corbels subjected to vertical loading. The experimental part studied both the material and structural properties of the self-compacting concrete used, this was performed by the cast of 29 fibrous and non-fibrous specimens; 8 non fibrous and 4 fibrous specimens were cast with normal strength self-compacting concrete, NSCC, and the same number of specimens were cast with high strength self-compacting concrete, HSCC; finally 5 specimens were cast with non-fibrous normal conventional reinforced concrete, NC, the latter was used for comparison purposes. All specimens had the same geometry; the variables studied were the  $(a_v/d)$  ratio, the amount of main reinforcement,  $(A_s)$ , the amount of secondary (horizontal) reinforcement,  $(A_h)$ , the volume fraction of fibers and the compressive strength of concrete,  $(f'_c)$ . Results from experimental tests showed that the cracking and ultimate loads have increased with the increase of the volume of steel fibers and the decrease of  $(a_v/d)$  ratio; the percentage of increase was greater in the HSCC specimens than those in the NSCC specimens. In addition to that and for the same amount of main reinforcement and  $(a_v/d)$  ratio, test results showed an increase in the cracking and ultimate loads with the increase of secondary (horizontal) reinforcement in both NSCC and HSCC types of specimens, but it was more pronounced in the latter type. Increasing the compressive strength led to an improvement in the cracking and ultimate loads; this improvement was more pronounced in fibrous specimens than that in non-fibrous ones.



The theoretical part included using nonlinear regression that led to the proposing of useful expressions to predict the nominal shear strength of fibrous and non-fibrous self-compacting reinforced concrete corbels based on the experimental data obtained from 18 specimens, these formulas were:

$$V_n = \left[ \left( \frac{1}{1193} \right) \right] \cdot \left[ \left( (f'_c)^2 + 665 \cdot (\rho_s \cdot f_y + \rho_h \cdot f_{yh}) \right) \cdot \left( \frac{d}{a} \right)^{\left( \frac{1}{3} \right)} \right] \cdot [(1 + 0.4F) \cdot b \cdot d] \quad (24)$$

$$V_n = \left[ \left( \frac{1}{590} \right) \right] \cdot \left[ \left( (f'_c)^2 + 695 \cdot (\rho_s \cdot f_y + \rho_h \cdot f_{yh}) \right) \cdot \left( \frac{1}{1 + 1.25 \cdot \left( \frac{a}{d} \right)} \right) \right] \cdot [(1 + 0.4F) \cdot b \cdot d] \quad (25)$$

$$V_n = \left[ \left( \frac{1}{200} \right) \right] \cdot \left[ \left( (f'_c)^{1.75} + 200 \cdot (\rho_s \cdot f_y + \rho_h \cdot f_{yh}) \right) \cdot (2.4)^{\left( \frac{a}{d} \right)} \right] \cdot [(1 + 0.4F) \cdot b \cdot d] \quad (26)$$

Comparisons between these three formulas and the expressions provided by the ACI 318M-11 and other six empirical expressions available in literature showed good agreement. The researcher recommended that using Eq. (26) in the design will lead to a safe and economic self-compacting reinforced concrete corbels.

#### 4 Researches on Reinforced Concrete Corbels under Repeated Loading:

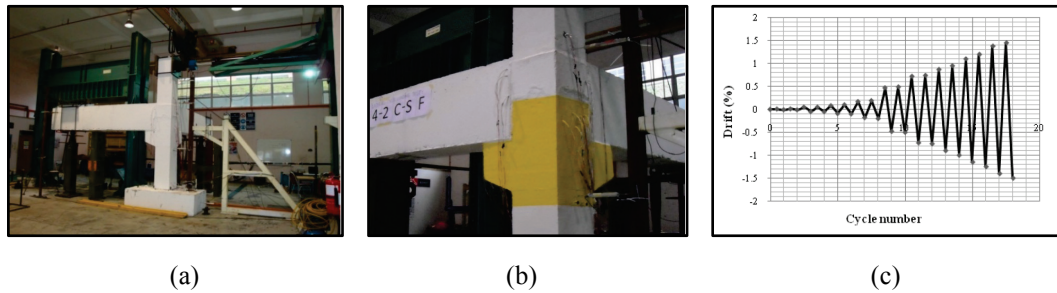
Most of the researches performed in the past decades on reinforced concrete corbels were concerned with their strength and ductility under monotonically increasing loads up to failure. Corbels subjected to other types of loading, such as repeated loadings, are very rare. Such loadings become significant in specimens with large live-to-dead loads ratios, as well as in zones of seismic activity or hurricane loads. However, under repeated or reversed loads larger deflections are produced and the shear distress becomes more evident.

Most of the researches available in literature deal with beam-column joint with or without corbels subjected to seismic or repeated loadings; researches on corbels under such loads are very rare.

During the eighties of the last century, there was a gradual transition from cast-in-place construction to precast construction. The form of precast construction mostly emulates monolithic (cast-in-place) construction. The use of precast concrete in seismic areas requires the development of adequate connectors between the precast members, which must exhibit high flexural strength, ductility and large energy absorbing capacities. Connections detailing and their location between precast members was one of the experimental and analytical investigation issues for researchers.

Muhammad [35] did some test on 12 specimens of corbels cast with high-strength fiber reinforced concrete subjected to repeated loading, in addition to other specimens subjected to monotonic loading. A quasi-static type of repeated loading was applied to the 12 specimens, the test began by applying load via a hydraulic machine with appropriate increment (4.9 kN) until the maximum deflection was read according to the specified value of  $(\Delta/\Delta_y)$  ratio, which were chosen by the researcher as 1.25 and 1.5. Unloading stage followed gradually by loosening the valve of the hydraulic machine. Then the second cycle followed. The loading and unloading stages were carried out in the same manner for six cycles after that the specimens were loaded to failure. Test results showed that specimens subjected to repeated loading failed in a more ductile manner and had a larger total deflection at failure than those subjected to monotonic loading. The shape of the hysteresis loops of the load-deflection curves had shown pinching near mid cycle, which was more pronounced as the number of cycles increased, at the same ductility  $(\Delta/\Delta_y)$ , the pinching was a sign of decreasing in energy dissipation.

Dora and Abdul Hamid [46], conducted an experimental study to investigate the seismic performance of full-scale precast beam-column corner joint with corbel mixed up together with steel fiber reinforced concrete (SFRC) as shown in Fig. (6- a and b).



**Figure (6) – Full scale beam-column joint sub-assembly [46] (a) Specimen ready for testing. (b) SFRC in concrete at beam-column joint with corbels painted with yellow which is ready for instrumentation. (c) Loading Regime Using Control Displacement.**

The sub-assembly of beam-column joint was subjected to reversible lateral cyclic loading up to  $\pm 1.5\%$  drift. The loading regime for the experimental work was performed using control displacement method. In this method, the target displacement is controlled in term of percentage drift. Drift is defined as the ratio of lateral displacement over height of the column multiply by one hundred. The loading regime is shown in Fig. (6-c).

The visual observation and experimental results showed that the cracks start to occur from  $+0.75\%$  drifts at the cast-in-place area of beam-column joints. The reinforcement bar in the beam-column joint yielded at  $+0.75\%$  drift with displacement of 26.39 mm in positive direction (loading).

The force-displacement response from experimental results shows agreement with prediction using moment-rotation analysis. The displacement ductility calculated showed that ductility increased by increasing the target drift with the maximum value of 2.09. Even though SFRC was provided at the beam-column joint and corbel areas, the result showed that the precast beam-column joint presented had low ductility and prone to severe damage when subjected to bigger drift or larger displacement due to lateral loading.

Farhan [47], carried out experimental and theoretical investigations to study the strength and deformation characteristics of reinforced concrete corbels tested under monotonic and repeated loading. For this purpose, 24 vibrated and self-compacting concrete (SCC) corbels with normal and high compressive strength were cast and tested under vertical loading. Test results indicated that corbels made of SCC showed an improvement in behavior and strength of the specimens from (8.2% to 14.2%).

## 5 Researches on Strengthened Reinforced Concrete Corbels:

The use of fiber reinforced polymer (FRP) materials for structural repair and strengthening has continuously increased during the past years, due to several advantages associated with these composites when compared to conventional materials like steel. These benefits include low weight, easy installation, high durability and tensile strength, electromagnetic neutrality and practically unlimited availability in size, geometry and dimension. FRP materials are lightweight, noncorrosive, and exhibit high tensile strength. Additionally, these materials are readily available in several forms ranging from factory-made laminates to dry fiber sheets that can be wrapped to conform to the geometry of a structure before adding the polymer resin [48].

Engindeniz et al. [49], presented a comprehensive literature search regarding the performance, repair and strengthening techniques for non-seismically designed reinforced concrete beam-column joints, reported between 1975 and 2003. These techniques included:

1. Epoxy repair.
2. Removal and replacement.
3. Concrete jacketing.
4. Concrete masonry unit jacketing.
5. Steel jacketing and addition of external steel elements.
6. Strengthening with fiber-reinforced polymeric (FRP) composite applications.

Each method of repair or strengthening was reviewed with emphasis on its application details, required labor, range of applicability, and performance; relative advantages and disadvantages of each method were discussed.

Test results reviewed showed that externally bonded FRP composites can eliminate some important limitations of other strengthening methods such as difficulties in construction and increases in member sizes. The shear strength of one-way exterior joints has been improved with (45°) fibers in the joint region; however, ductile beam failures were observed in only a few specimens, while in others, composite sheets were debonded from the concrete surface before a beam plastic hinge was formed. Reliable anchorage methods need to be developed to prevent debonding and to achieve full development of fiber strength within the small area of the joint. Most of the strengthening schemes developed have a limited range of applicability, either due to the unaccounted floor members (that is, transverse beams and floor slab) in real structures or to architectural restrictions. Experiments conducted to date have generally used only unidirectional load histories. Therefore, a significant amount of work is necessary to arrive at reliable, cost-effective, and applicable strengthening methods. In developing such methods, it is important that testing programs be extended to include critical joint types under bidirectional cyclic loads.

Published research work on the behavior of strengthened corbels with CFRP fabric is very limited. There were a series of experimental and theoretical tests carried out by Fattuhi and others [26, 28-32], Abdul-Wahab [27], Mohammad [35], Campion and others [37, 40], Ridha [38], Ahmad and Shah [39], Yousif [42], Al-Zahawi [43], Khalifa [44] and Abass [45], to investigate the effect of adding steel fibers and chopped carbon fibers to reinforced concrete corbels. Tests results showed that the addition of fibers improved the behavior of corbels. This procedure cannot be considered as a strengthening technique to corbels as much as it may be considered as an improvement to the behavior of those corbels.

Heidayet et al. [50], carried out an experimental investigation on 18 damaged reinforced concrete corbels repaired by external steel plates subjected to vertical loads. The parameters studied were the main and secondary reinforcements, the shear span to depth ratio, ( $a_v/d$ ), and the depth of the corbels. At first the corbels were loaded close to their ultimate capacity, and then repaired using externally steel plates, which were fixed by bolts, finally the specimens were loaded to failure. Test results showed that the ratio between the strength of the repaired corbel to that of original ones varied from 0.7 to 1.5; it was concluded from that investigation that this repair technique may be regarded as an effective and economical technique for strengthening existing structures.

Campione et al. [51], performed an analytical and experimental study on the flexural behavior of reinforced corbels made of plain and fibrous concrete with main and transverse steel bars or externally wrapped with carbon fiber reinforced polymers.

Twelve corbels, having the same dimensions but differs in the amount and type of reinforcement, were cast for this purpose. Figure (5.1) shows the geometry and steel reinforcement details.

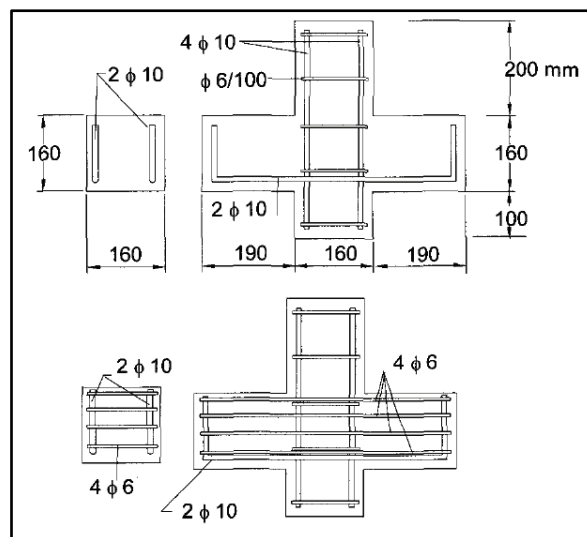
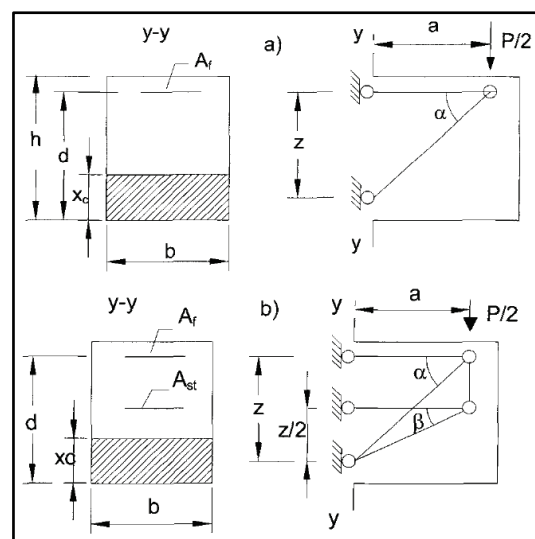


Figure (7) – Geometry of Corbels and Details of Steel Reinforcement<sup>[51]</sup>

The specimens, two for each Type, were made of the following:

1. Plain concrete.
2. Fiber reinforced concrete with hooked steel fiber at 1% by volume percentage.
3. Plain concrete reinforced with two longitudinal bars acting as main reinforcement, having a diameter of 10 mm, placed at the bottom of the beam.
4. Plain concrete reinforced with two longitudinal bars acting as main reinforcement, having a diameter of 10 mm, placed at the bottom of the beam and four longitudinal bars acting as secondary reinforcement, having a diameter of 6 mm uniformly distributed within the depth of corbel.
5. Fiber reinforced concrete with hooked steel fiber at 1% by volume percentage in addition to two longitudinal bars acting as main reinforcement, having a diameter of 10 mm.
6. Plain concrete reinforced with two longitudinal bars acting as main reinforcement, having a diameter of 10 mm, placed at the bottom of the beam and wrapped externally with one ply of flexible carbon fiber reinforced sheet (CFRP) having a thickness of 0.165 mm.

For corbels reinforced with main steel the rupture was related to the brittle failure of compressed zone arising after the yielding of steel bars occurs. For corbels externally wrapped with CFRP similar mode of failure was observed but the compressive rupture is consequent to the failure of CFRP wraps in tension. A more ductile behavior was observed when using secondary reinforcement, this may be referred to the bridging action of secondary reinforcement against the principal cracks, but the flexural capacity is not completely achieved. For FRC corbels, ductile balanced flexural failure is observed, which prove the effectiveness of using fibers as shear reinforcement and in improving the confinement of the compression zone of the beam. The analytical model used in this paper was performed with an equivalent single or multiple trusses, as shown in Fig. (8- a and b).



**Figure (8) – Equivalent truss model: a) single truss; b) multiple trusses.** <sup>[51]</sup>

The single truss refers to corbels detailed in 1, 2, 3 and 5, while the multiple trusses refer to corbels detailed in 4 and 6 listed above.

The bearing capacity of corbels for all cases could be determined by using this analytical model, based on the analysis at rupture of corbels by means of an equivalent multiple truss including all the possible modes of failure observed experimentally. The model showed good agreement with the experimental results and gave a physical interpretation of the behavior of corbels at rupture.

Elgwady et al. [52], carried out an experimental investigation on six corbels strengthened with carbon fiber reinforced plastic (CFRP) strips with different strengthening configurations subjected to monotonic loading. One specimen was considered as a control specimen (without strengthening); the parameters studied were the amount and orientation of CFRP

strips. Test results indicated that diagonal CFRP strips can enhance the corbel capacity by about 70% with relative to the control specimen. The other types of strengthening enhanced the ultimate load carrying capacity by values ranged from (8-30) %; the cracking loads of most strengthened specimens were about (70-80) % of their ultimate loads. It was observed that the specimen with mixed horizontal and diagonal strips showed an apparent delay of cracking. Most of the corbels showed a brittle mode of failure, the stiffness of all specimens was increased and they failed suddenly without adequate warning. The author recommended that the CFRP strips should be stopped before the end of the corbel, where the distance left may be considered as the weaker portion where cracks were propagated.

Ozden and Atalay [53], investigated the strength and post-peak performance of reinforced concrete corbels strengthened with GFRP overlays, which were bonded to the specimens using epoxy. 24 specimens of normal strength concrete having one-third scale were cast for this purpose. The specimens were tested to failure under quasi-static gravity loading.

The parameters studied were the strength of concrete, the main reinforcement, the shear span to depth ratio,  $a_v/d$ , and the GFRP layers and orientation, the GFRP wrapping configurations are shown in Fig. (9).

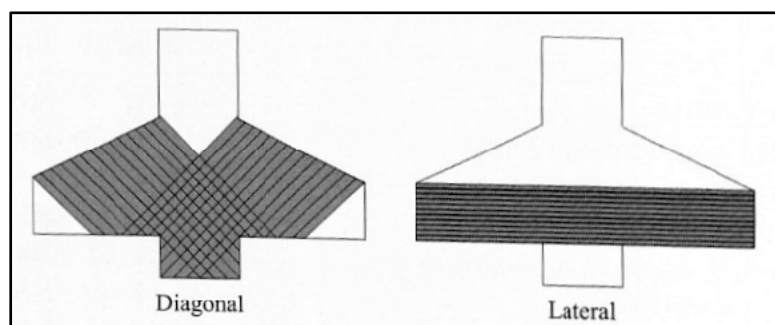


Figure (9) – GFRP Wrapping Configurations <sup>[53]</sup>

Test results concluded that GFRP wrapping with 45 degrees fiber orientation (diagonal) was more effective than lateral wrapping. The level of tension steel strain on the onset of failure depends on the type and number of GFRP layers and the tension reinforcement ratio. The results also revealed that the level of strengthening with GFRP wrapping ranges from (40-200) % depending on the reinforcement ratio,  $a_v/d$  ratio and the orientation and number of GFRP layers used.

Mohamad-Ali and Attiya [2], carried out an experimental investigation on the shear behavior of reinforced concrete corbels externally strengthened or retrofitted with carbon fiber reinforced polymer (CFRP) sheets. The variables studied were the width, number, length, location and direction of the CFRP sheets. Test results showed that corbels strengthened with inclined strips gave best results compared with specimens strengthened with horizontal strips; the enhancement in the ultimate shear strength of the corbels was about (45-60) % for inclined strips and about (15-31) % for horizontal strips compared with the control specimens. It was concluded as well that the external strengthening technique had a significant effect on the first cracking load, the percentage increase in cracking load for the inclined strips was 51%, and the percentage for the horizontal strips was 19%. For the repaired specimen, an increase in the ultimate load was detected; the amount of increase was 57% with respect to the control specimen. Finally, an average decrease in the width of crack was observed, this decrease was due to the presence of the CFRP strips. The percentage of decrease was 41% and 24% of the control corbels at ultimate load levels for the inclined and horizontal techniques respectively.

El-Maaddawy and El-Sayed [54], carried out experimental and theoretical tests to investigate the structural response of concrete corbels reinforced internally with steel rebars and externally with carbon fiber reinforced polymer (CFRP) composite sheets. Nine specimens were constructed and tested for this purpose. The specimens were divided into two groups based on whether it was reinforced with main reinforcement only or a combination of main and secondary reinforcement. In each group, one specimen was considered as a control specimen left without external composite reinforcement. The remaining specimens were reinforced externally with CFRP composite sheets, as shown in Fig. (10). The strengthening technique included primary longitudinal CFRP reinforcement (L1), parallel to the main reinforcement, secondary longitudinal CFRP sheets (L2), and diagonal CFRP sheets (D1) and (D2) oriented at 45°. Two U-shaped CFRP sheets were wrapped around the corbel cross section outside the test region, one at each end, to provide an anchorage for the longitudinal CFRP sheets.

For the theoretical analysis the FE software ATENA was used, all test specimens were represented as nine two-dimensional models, assuming a perfect bond between the CFRP and concrete, by using. Three additional FE models were developed for the specimens with the diagonal CFRP reinforcement. In these models, an interfacial bond stress-slip model was adopted between the diagonal CFRP reinforcement and concrete.

Test results concluded that the failure modes and predicted crack patterns observed demonstrated that the tested corbels resisted the applied load by two diagonal compressive struts feeding into the column segment, and the main reinforcement acted as a tie to resist the out-of-balance forces at the support points. The inclined shear cracks that developed in the corbels were more curved toward the outer surface of the corbels for both types of reinforcement.

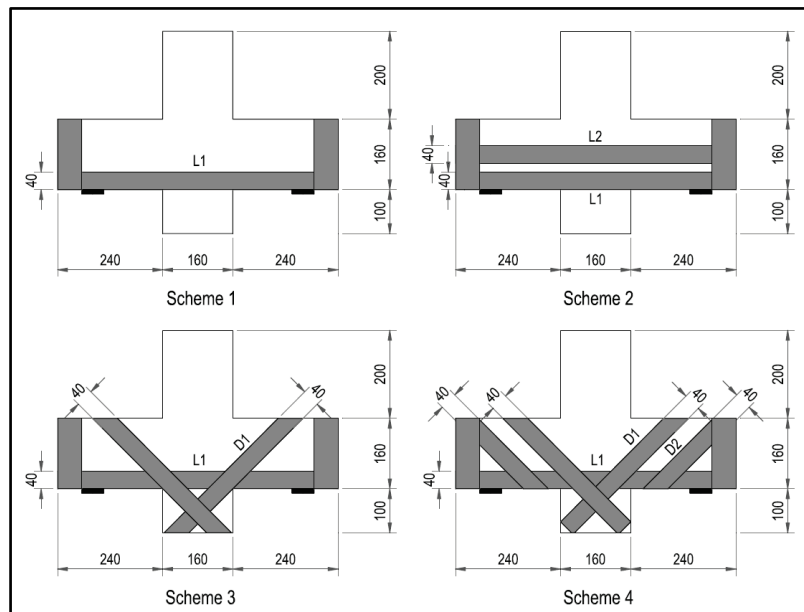


Figure (10) – Layout of the External Composite Reinforcements (in mm) <sup>[54]</sup>

It was also concluded that the load capacity of the corbels was improved by using CFRP as an external strengthening technique. This improvement was decreased with the increase in the amount of primary reinforcement. For the tested corbels reinforced with primary reinforcement only, a strength gain in the range of (21-40) % was recorded. Whereas, for other corbels with combined primary and secondary reinforcement, a (15-33) % strength gains were recorded. The numerical load capacity was within 10% error band.

Adding the primary longitudinal CFRP composite reinforcement reduced the increased the yield and ultimate loads of the corbels. But no significant improvement was observed by the addition of the secondary CFRP composite reinforcement, which was located at the mid height of the corbels. Moreover, it was observed that the diagonal CFRP composite reinforcement resulted in a significant increase in the load capacity.

## 6 Conclusions and Recommendations:

Experimental and theoretical test previously performed on reinforced concrete corbels were reviewed throughout this paper, this review showed that the ultimate strength of a corbel, ( $V_u$ ), is a function of its width ( $b$ ), its effective depth ( $d$ ), the reinforcement ratio ( $\rho$ ), the concrete strength ( $f'_c$ ) and the ( $av/d$ ) ratio. All tests showed that the strength of corbels increases with the increasing of the amount of longitudinal, shear reinforcement and compressive strength of concrete and decreases with the increasing of the shear span to depth ratio ( $av/d$ ).

Some investigators indicated that truss analogy is not applicable to corbels with low values of ( $av/d$ ) ratios. (Somerville, 1972), as an example, limits the use of truss analogy to values of ( $av/d$ ) ratios equal or greater than (0.6). Most researchers concluded that the ACI 318 code provisions for corbels are conservative for high strength concrete.

The horizontal reinforcement (stirrups) in corbels are as effective in resisting vertical loads as the main tension reinforcement, but they are not that effective with combined-loading, therefore, any contribution from the stirrups may be

regarded as reserve strength. For that reason a minimum amount of stirrups should always be provided. Moreover, vertical stirrups were found to be not useful for shear strength of a corbel with  $(a_v/d \leq 1)$ .

Replacing the conventional secondary reinforcement with other kinds of reinforcement such as steel or polypropylene fibers, or even plastic meshes was found effective in improving the properties of the concrete corbels.

Steel fibers, among many other kinds of fibers were the only form of secondary reinforcement that successfully reduced the widths of cracks in corbels. The presence of steel fibers increased the ultimate shear strength, the strength increases with the increase in the fiber volume and decreases with the increase of the  $(a_v/d)$  ratio. The role of fibers in combination with stirrups led to higher reduction in deflection, but it was found that fiber reinforcement is not an economical alternative to the main reinforcement for resisting flexure.

The improvement in strength due the addition of fibrous reinforcement was more significant in high-strength concrete corbels relative to low or moderate strength concrete corbels.

The finite element approach is regarded as a suitable tool for the analysis of normal and high-strength concrete corbels; the method may be used to predict the response of reinforced concrete structures in general. It is considered as a feasible method of analysis and design especially with the increasing of experimental costs and the decreasing of computational costs.

The strut-and-tie model is a simple and effective design tool that may give reasonable predictions of the failure load capacity of reinforced concrete corbels.

Using fibers as an external strengthening helped in enhancing the load carrying capacity of corbels subjected to monotonic loading; this enhancement depends on the orientation of the fibers, diagonal strips were found more effective than the horizontal strips. The enhancement in the corbel's capacity for diagonal orientation was about triple the value of that for the horizontal orientation.

Further studies are needed to investigate strengthened corbels subjected to repeated loading, since most of the corbels in practical life are under the effect of this kind of loading such as moving loads of vehicles on girders supported by corbels or corbels supporting cranes in warehouses and so on.

The provision of brackets and corbels in the ACI 318 has only a lower limit for the volume fraction of main bars; therefore further studies should be performed to establish a more precise upper limit for the main bars that can be used efficiently in corbels.

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