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Research Paper

Performance Based Plastic Design of Concentrically Braced Frame attuned with Indian Standard code and its Seismic Performance Evaluation

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ABSTRACT

In the Performance Based Plastic design method, the failure is predetermined; making it famous throughout the world. But due to lack of proper guidelines and simple stepwise methodology, it is not quite popular in India. In this paper, stepwise design procedure of Performance Based Plastic Design of Concentrically Braced frame attuned with the Indian Standard code has been presented. The comparative seismic performance evaluation of a six storey concentrically braced frame designed using the displacement based Performance Based Plastic Design (PBPD) method and currently used force based Limit State Design (LSD) method has also been carried out by nonlinear static pushover analysis and time history analysis under three different ground motions. Results show that Performance Based Plastic Design method is superior to the current design in terms of displacement and acceleration response. Also total collapse of the frame is prevented in the PBPD frame.

1 Introduction

Concentrically braced frames (CBF) are generally considered less ductile seismic resistant structures than other systems due to the brace buckling or fracture when subjected to large cyclic displacements. Concentrically braced frames when designed by the conventional force based methods, can undergo excessive storey drift after buckling of bracing members. The displacement based PBPD methodology to CBF with buckling type braces was proposed by [1]. In this method, the braces fail first due to post buckling and tension yielding. Due to this, hinges are formed in the predetermined locations in beams. This may result into local failure only, preventing total collapse of the frame. The PBPD method is based on “strong column weak beam principle” [2]. The main advantage of this method is that the failure occurs at predefined points only and at the same time the optimum utilization of section strength is attained [3]. The method has been successfully applied to steel Moment Frame [4], Eccentrically Braced Frame [5], Composite buckling restrained braced

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frame, [6], Concentrically Braced Frame [7] , Special Truss Moment Frame [8] as well as Reinforced Concrete Frames [9]. An excellent literature review of the method has been done in [10].

As on today, no guidelines or codes are available to execute the PBPD method for structures in India. In India, the force based design is used, in which the structures are designed to carry forces and checked for displacements. In the current Indian code of design of earthquake resistant structures, the following seismic design philosophy is adopted:

1. Minor and frequent earthquakes should not cause any damage to the structure.
2. Moderate earthquake should not cause significant structural damage but could have some non-structural damage.
3. Major and infrequent earthquakes should not cause collapse.

Whenever a structure is built having a life of usually 50-100 years it may or may not be subjected to large earthquakes during its lifetime. The frequency of large earthquakes to occur is around 1 in 500 years which can be found out from past records of large earthquakes that particular place has experienced. Therefore chances of the structure being hit by large earthquakes are very rare. Hence it is uneconomical to design the structure considering the actual seismic forces it may be subjected to.

Therefore, the current design code IS 1893:2002 [11] allows the structure to be damaged in case of severe ground motions. The code provides certain provisions to reduce the response of the structure by considering the effects of over strength, redundancy and ductility. The design force for an elastic structure can be reduced by using factor "R" which is known as the Response reduction factor, which depends on the ductile behaviour of the material used.

2 Lateral Force Calculation and Distribution in the Current Code Method.

The lateral seismic loads are calculated based on the Elastic Design Spectra prescribed in IS 1893:2002 which gives the design spectral acceleration, *"S_a"* of the structure having time period *"T_n"* due to elastic response ($\mu = 1$). The ductility factor *"μ"* is defined as the ratio between the maximum displacement and the yield displacement *(u_{max}/u_y)* or maximum drift and the yield drift *(θ_{max}/θ_y)*. This means, when μ=1, it will be an elastic response and when $\mu > 1$, it will be an inelastic response.

The inelastic activity is accounted in the current method by applying response reduction factors *"R"* (table 7, IS1893:2002), zone factors *"Z"* (table 2 clause 6.4.2, IS1893:2002) and importance factors *"I"* (table 6 clause 6.4.2, IS1893:2002) in calculation of the total base shear *"V_b"*. The seismic zone factor *(Z)* used to determine the value of design forces has been proposed after seismic zonation of our country and is based on the earthquake data available. The importance factor *(I),* depends on the functional use of the structure. From the above information it is very well observed that the design forces derived from the current code are quite assumptive as the factors governing the design of forces are mainly based on engineering judgments. Because of these reasons, the structures undergo large inelastic deformations during major earthquakes. Mathematically, the total design base shear is calculated as

$$V_b = A_h\, W \tag{1}$$

In which W is the total weight of the structure and A_h is the horizontal seismic coefficient.

$$A_h = \frac{Z\, I\left({S_a}/{g}\right)}{2\, R} \tag{2}$$

This base shear is distributed among the floors known as the lateral force distribution at each floor level, *"Q_i"* according to clause 7.7.1, IS1893:2002.

$$Q_i = V_b\, \frac{w_i\, h_i^{\,2}}{\sum_{j=1}^{n} w_i\, h_i^{\,2}} = V_b \beta_i \tag{3}$$

β_i =Shear distribution factor.

h_i = Height of level i measured from base

w_i =Seismic weight of level i

Note: The Shear distribution factor *"β_i"* can thus be defined as $\frac{w_i\, h_i^{\,2}}{\sum_{j=1}^{n} w_i\, h_i^{\,2}}$ based on above equation.

The frame is applied the above lateral loads along with gravity loads and then static analysis is carried out. This lateral load is applied at all the joints of all the bays at each floor. It means that the frame is assumed to remain connected as a continuous system. Based on the obtained values of axial force, shear force and bending moment, all the structural members of this frame are designed using the limit state design approach (IS800:2007 [12]).

3 Drawbacks of the Currently used design Procedure

The Response reduction factors do not predict the exact inelastic response of the structure because same value of *"R"* is assigned to all the systems falling under a defined category for available ductility [13]. Although two systems may have the same value of *"R"*, it may be appropriate to assign a higher reduction factor to a system with greater ductility [14]. Due to this, following drawbacks are observed:

1. The capacity of members in the inelastic zone can be utilized only after applying the Response reduction factors.
2. The structure actually does not always remain Elastic during ground excitation. Hence, the values of Lateral forces are at times exaggerated or sometimes underestimated.
3. Despite of being over safe in calculation of lateral forces as stated above, in case of a severe earthquake, the structure can collapse due to failure of columns.

The PBPD method counteracts these drawbacks. The members are designed for their full capacity by considering the ductility factor greater than one. The selection of *"μ"* in the PBPD method depends on the choice of the designer and hence it can be considered more rational than the currently used design process. For $\mu > 1$, the corresponding spectral acceleration which is due to inelastic response of the structure is designated as *"$S_{a\ inelastic}$"*. The *"$S_{a\ inelastic}$"* is calculated by applying reduction factors proposed by [15] to the elastic design spectrum of the IS1893:2002. As the value of *"μ"* increases, the value of *"$S_{a\text{-}inelastic}$"* decreases. Due to this, the total base shear also decreases (refer equation 4). Economy can be achieved if yielding members are designed for a greater ductility.

4 The PBPD Method

The Performance Based Plastic Design methodology is a displacement based design. The structure is designed to fail at predetermined locations only. Thus collapse mechanism (plastic theory) is defined in the beginning of the design.

The PBPD of concentrically braced frames can be adopted for the Indian codal provisions by following the analysis procedure as proposed by [4] because it is based on equilibrium equations available at the time of failure of the structure. Once the analysis is over and forces and moments are calculated, the design of the members is done satisfying the IS 800:2007.

In the PBPD method, the design base shear is calculated by equating the work needed to push the structure monotonically up to the target drift to the energy required by an equivalent Elasto Plastic Single Degree Of Freedom system to achieve the same state. A distribution of lateral design forces is used that is based on relative distribution of maximum storey shears consistent with inelastic dynamic response results [16]. Plastic design is then performed to detail the frame members and connections in order to achieve the intended yield mechanism and behaviour, hence the name Performance Based Plastic Design.

To implement the PBPD in conjunction with the Indian code, it is suggested that the design base shear and distribution of the lateral force can be done using the above stated method which is well established [17]. The calculation of the shear and moments in the PBPD method is based on the equilibrium equations and hence can be directly implemented. Once the analysis is over, the Plastic design of the members can be carried out as per the IS800:2007 code.

5 Lateral Force Calculation and Distribution in the PBPD Method

The total base shear *"V_b"* is calculated based on work energy equation and is expressed as

$$V_b = \left(\frac{-A_h + \sqrt{{A_h}^2 + 4\left(\frac{\gamma}{\eta}\right) {S_{a\ inelastic}}^2}}{2}\right) W \quad (4)$$

$$A_h = (\sum_{i=1}^{n}(\beta_i - \beta_{i+1})h_i)\left(\frac{w_n h_n}{\sum_{i=1}^{n} w_i h_i}\right)^{0.75T_n^{-0.2}}\left(\frac{\theta_p 8\pi^2}{T_n^2 g}\right) \quad (5)$$

The ductility factor is related to energy modification factor *"γ"* in the following way by applying the Energy balance concept as proposed by [4]

$$\gamma = \frac{2\mu-1}{R^2} \quad (6)$$

Where,
"R" is calculated as proposed by,[15] (figure 1) for different time periods *"T_n"*.
h_i (and h_n)= Height of level i (and roof) measured from base
w_i (and w_n)=Seismic weight of level i (and roof)
θ_p= the selected global inelastic drift ratio of the structure (θ_{max}-θ_y) i.e. predetermined target drift mechanism.

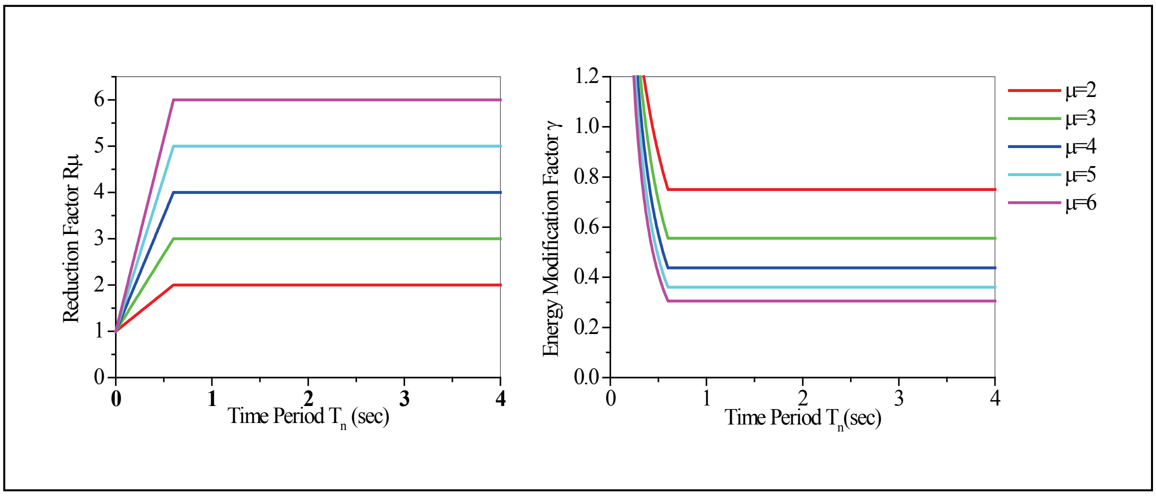

Fig. 1 Reduction Factor "R" and Energy Modification Factor "γ"

This *"γ"* is divided by a factor *"η"* (η=A_1/A_2 ,figure 2) for those systems who do not possess ideal Elasto Plastic force deformation behaviour and full hysteretic loops like steel braced frame and buckling type frames. For moment resisting frames it is taken as unity [2],[4]. The pin connected rigid beams and columns showed that the dissipated energy of a Concentrically Braced Frame is only 35% of the energy dissipated by corresponding frame with full elastic plastic hysteretic loop, with both frames having same strengths [7].

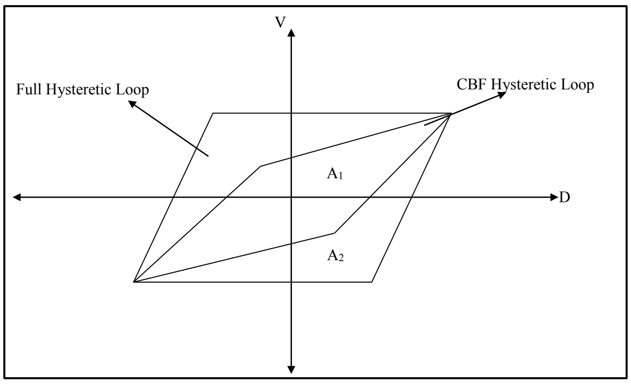

Fig. 2. Full Elasto Plastic Hysteretic Loop and CBF Hysteretic Loop

The base shear is distributed as suggested by [16]

$$Q_i = C'_{vi} V_b \quad (7)$$

Where

$$C'_{vi} = (\beta_i - \beta_{i+1}) \left(\frac{w_n h_n}{\sum_{i=1}^{n} w_i h_i}\right)^{0.75 T_n^{-0.2}} \quad \text{when i = n , } \beta_{i+1} = 0 \tag{8}$$

$$\beta_i = \left(\frac{\sum_{i=1}^{n} w_i h_i}{w_n h_n}\right)^{0.75 T_n^{-0.2}} \tag{9}$$

The base shear in the PBPD method is divided by the number of bays falling in the direction perpendicular to the lateral load. This is because in the PBPD method, mechanism is assumed to be formed during the extreme condition and so each bay behaves individually. Thus we get a term *"V'"* which is the total base shear *"V_b"* divided by number of bays. The lateral force of roof floor *"Q_n"* is first found and then it is distributed in other floors *"Q_i"*as shown below [2, 16]

$$Q_n = \frac{V'}{\sum(\beta_i - \beta_{i+1})} \tag{10}$$

$$Q_i = Q_n(\beta_i - \beta_{i+1}) \tag{11}$$

6 Stepwise PBPD Procedure

A predetermined failure pattern (figure 3) and target drift *(θ_u)* is selected. The seismic weight of the floors as well as the total design seismic load *"W"* and the natural time period is calculated as per the usual procedure. The lateral loads are calculated as explained above for the desired ductility factor based on the Inelastic Design Spectral acceleration *("$S_{ainelastic}$/g")*. Once the lateral loads are calculated, it is first distributed in bracings of each floor. While designing the bracing of a particular floor, the sum of design lateral forces at all levels above the storey under consideration (known as storey shear *"V_i"*) is used.

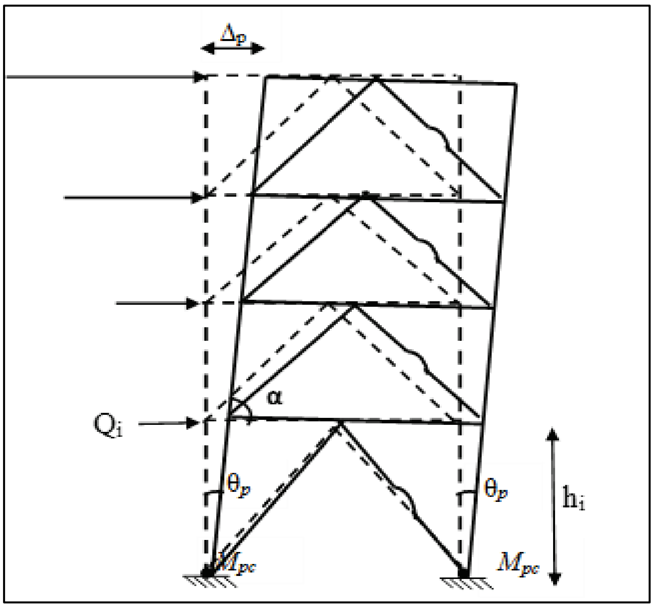

Fig. 3 Predetermined Failure Pattern and yield mechanism for the frame.

6.1 Design of Bracings

The bracings are designed such that following condition is satisfied.

$$V_i \leq \frac{(P_y + 0.5P_{cr})\cos\alpha_i}{\gamma_{mo}} \tag{12}$$

where

P_y = Tension yielding state load and $P_y = f_y A_g$

P_{cr} = Post buckling state load and $P_{cr} = f_{cr} A_g$

f_y = yield stress

A_g= Gross area of the section

α = Angle of bracing member with horizontal

γ_{mo}= Partial safety factor for material strength

In which

$$f_{cr} = 0.877 f_{cc}\text{ , When } f_{cc} < 0.44 f_y$$

$$f_{cr} = \left(0.658^{\frac{f_y}{f_{cc}}}\right) f_y\text{ , When } f_{cc} \geq 0.44 f_y$$

and

$$f_{cc} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

E = Modulus of Elasticity

KL/r = Effective Slenderness ratio as per clause 7.1.2.1 IS800:2007

6.2 Design of Beams

In the PBPD method, the failure is predetermined and will first occur at bracings. Once this happens, the forces are transferred to beams in the following way:

Axial force (horizontal)

$$F_h = \left(\gamma_{mo} P_y + 0.5 P_{cr}\right)\cos\alpha \tag{13}$$

Shear force (vertical)

$$F_v = \left(\gamma_{mo} P_y - 0.5 P_{cr}\right)\cos\alpha \tag{14}$$

Moments

$$M = \frac{wl^2}{8} + \frac{F_v\, l}{4} \tag{15}$$

The beams are designed for combined forces as per clause 9.3 (IS 800:2007)

$$\frac{N}{N_d} + \frac{M}{M_d} \leq 1.0 \tag{16}$$

Where

N = Axial force in the section = F_h

M_d = Design bending strength of section as per clause: 8.2, IS 800:2007

N_d = Design strength in tension (As per clause: 6, IS 800:2007) or in compression due to yielding.

6.3 Design of Columns

Only axial loads *"P"* are considered for column design, including the fixed base first story columns. Axial forces result primarily from the gravity loads and vertical component of the braces. Pre buckling and post buckling limit states are considered for the design of columns.

Axial force for typical exterior column in Pre buckling limit state is

$$P = (P_{transverse})_i + (P_{beam})_i + (P_{cr}\sin\alpha)_{i+1} \tag{17}$$

Axial force for typical interior column in Pre buckling limit state is,

$$P = (P_{transverse})_i + \sum (P_{beam})_i + (P_{cr}\sin\alpha)_{i+1} \tag{18}$$

Where

$(P_{transverse})_i$ = Tributary factored gravity load on columns from transverse direction at floor i = (1.2 times dead load +0.5 times imposed load)

$(P_{beam})_i$ = Tributary factored gravity load from the beam at floor i (=0.5W_i L)

$(P_{cr})_{i+1}$ = Post buckling state load of brace at floor i+1

Axial force for typical exterior column in Post buckling limit state is

$$P = (P_{transverse})_i + (P_{beam})_i + (P_{cr}\sin\alpha)_{i+1} + 0.5F_v \tag{19}$$

Axial force for typical interior column in Post buckling limit state is

$$P = (P_{transverse})_i + \sum(P_{beam})_i + (P_{cr}\sin\alpha)_{i+1} + 0.5F_v \tag{20}$$

The columns are to be designed as per clause: 7.1.2, IS800:2007 (such that the axial load coming on the section is less that the strength of the section) and checked for Clause 9.31.1 IS 800:2007 for combined axial force and bending moment. It should be noted that the Moment acts about the direction of lateral force

7 The Study Frame

In this section a study frame as shown in figure is selected and designed as per current codal provisions. The same frame is then redesigned by the PBPD method. The details of the frame are presented in table 1

Table 1 The Dimensions and Loading on the study frame

Parameter	Value
Number of bays in X direction	5
Number of bays in Y- direction	5
Bay Length in X direction	9 m
Bay Length in Y direction	9 m
Height of the floor	4 m
Number of Stories	6
Number of bracing at each floor in a frame	6
Height of the frame	24 m
Slab thickness	0.15 m
Live Load (on Floor)	3 kN/m^2
Live Load (on Roof)	1 kN/m^2

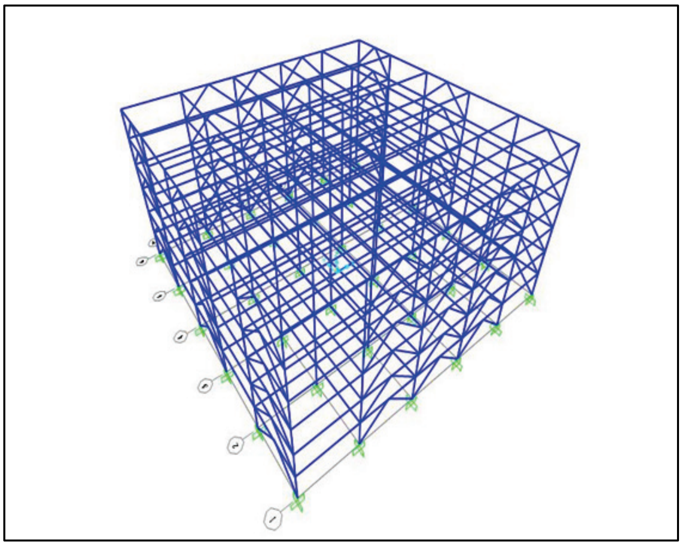

Fig. 4 The six storied study frame.

7.1 Design of the study frame by current code method

Seismic parameters required to analyse the problem by IS Code method are listed below (IS:1983:2002)

Table 2 Seismic design Parameters as per current code

Parameter	Value
Time Period of the Building (T_n)	0.921 sec
Spectral Acceleration (S_a/g)	1.47 g
Weight of the frame (W)	13149 kN
Response Reduction Factor (R)	3
Zone factor (Z)	0.36
Importance factor (I)	1.5
Soil Site	Medium
Horizontal seismic coefficient (A_h)	0.1323
Base Shear (V_b)	1739 kN
Time Period of the Building (T_n)	0.921 sec
Spectral Acceleration (S_a/g)	1.47 g

The analysis of this frame is done for above seismic parameters using standard design software. (SAP 2000) The design sections thus obtained are as follows.

Table 3 Section obtained as per IS code method

Floor	Brace Section	Beam Section	Column Section
6	ISMC 150	ISMB 600	ISB 450/450/16
5	ISMC 150	ISMB 600	ISB 450/450/16
4	ISMC 150	ISMB 600	ISB 450/450/16
3	ISMC 150	ISMB 600	ISB 450/450/16
2	ISMC 150	ISMB 600	ISB 450/450/16
1	ISMC 150	ISMB 600	ISB 450/450/20

7.2 Design of the study frame by PBPD method

The gravity loading and Seismic loading for the structure is calculated first. Then after, the desired target yield mechanism is selected. Calculation of the base shear distribution factor is carried out next to distribute the base shear at all the floor level. After calculating the lateral forces, they are transferred to the bracings. Then bracings design is done as the yielding member in the system, according the value of the force. Considering that section of the bracings and moment which directly comes through slab, the design of the beam is performed for axial force and bending moment. At the end, the column design is performed only for the axial force. The spectral acceleration in the PBPD method is calculated for μ =4.17 by applying the reduction factors as proposed by [15]. The base shear is calculated and distributed by the method as proposed by [16]. The critical parameters required for design of the sections are obtained using the equation of [7].Based on above calculated values, the design sections were provided satisfying the provisions of IS 800:2007. The design sections thus obtained are shown in table 5

Table 4 Seismic design parameters as per PBPD method

Parameter	Value
Time Period of the Building (T_n)	0.921 sec
Spectral Acceleration ($S_{a\ inelastic}$ /g)	0.5 g
Weight of the frame (W)	13149 kN
Yield Drift (θ_y)	0.3 %
Target Drift (θ_u)	1.25 %
Inelastic Drift (θ_p)	0.95%
Ductility factor (μ)	4.17
Energy modification factor (γ)	0.453
Horizontal seismic coefficient (A_h)	1.573
Base Shear (V_b)	1748 kN
Max. Storey Shear	291 kN
Max. Braced Strength	466 kN
Max. Moment in Beam	850 kNm
Max. Axial Force in Exterior Column (Pre-Buckling)	1814 kN
Max. Axial Force in Exterior Column (Post-Buckling)	2176 kN
Max. Axial Force in Interior Column (Pre-Buckling)	3384 kN
Max. Axial Force in Interior Column (Post-Buckling)	3747 kN

Table 5 Design Sections obtained as per PBPD method

Floor	Brace Section	Beam Section	Column Section
6	ISB 66/33/2.6	ISMB 500	ISHB 350
5	ISB 96/48/3.2	ISMB 600	ISHB 350
4	ISB 96/48/4.0	ISWB 600	ISHB 350
3	ISB 96/48/4.8	ISWB 600	ISWB 600
2	ISB 122/61/3.6	ISWB 600	ISWB 600
1	ISB 122/61/4.5	ISWB 600	ISHB 300 with 300/20 Cover Plate.

8 Seismic Performance Evaluation of Study Frames

8.1 Time History Analysis

The displacement and acceleration response (figure 5 and 6) of the study frames were observed by performing time history analysis under El Centro, Newhall and Bhuj ground motions (Table 6).

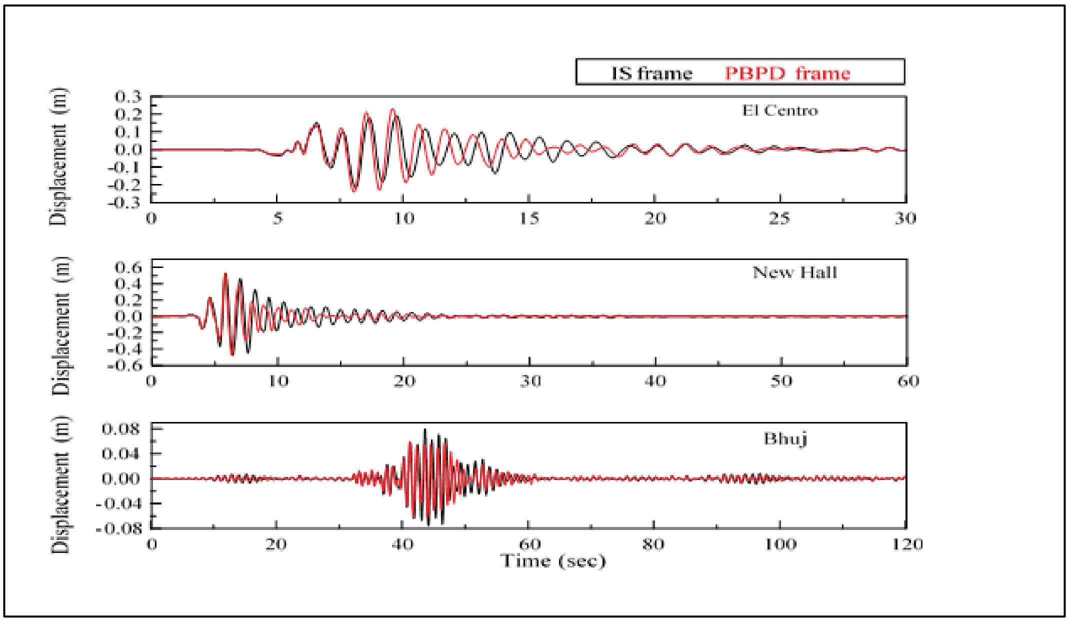

Fig. 5 The displacement response of the study frames

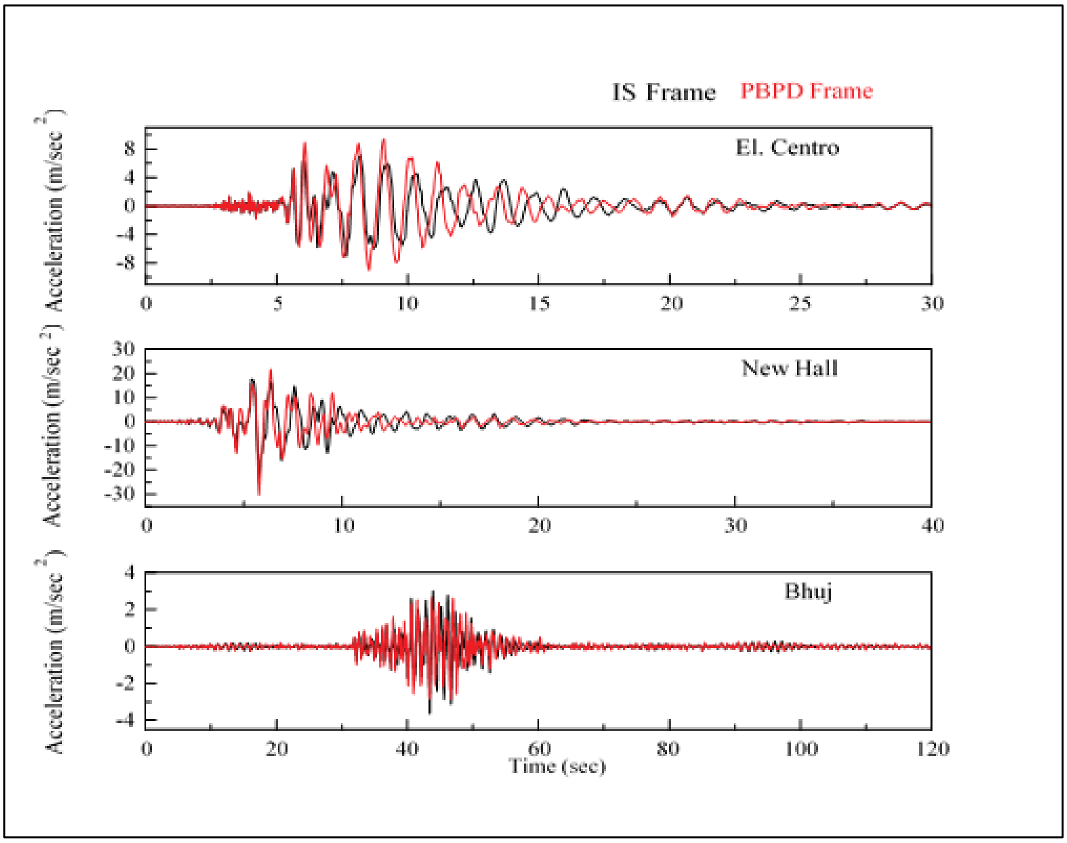

Fig. 6 The acceleration response of the study frames

Table 6 The Details of Ground Motions selected for Study

Earthquake Ground Motion	Recording Station	Duration (sec)	PGA(g)
1979 Imperial Valley, California	El Centro	39.420	0.37
1994 Northridge, California	Newhall	60.000	0.72
2001 Bhuj, India	Ahmedabad	133.53	0.11

8.2 Push Over Analysis

The failure pattern of selected study frame (figure 7 and 8) is observed by pushover analysis using SAP2000. The frame was pushed till maximum permissible drift of 0.004 times the total height.

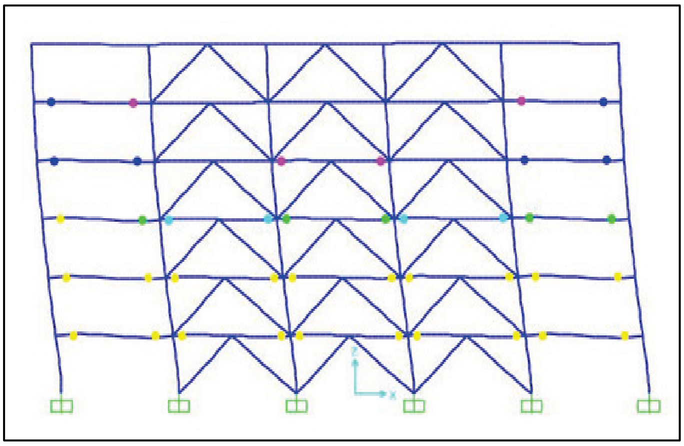

Fig. 7 The Failure pattern of the PBPD method

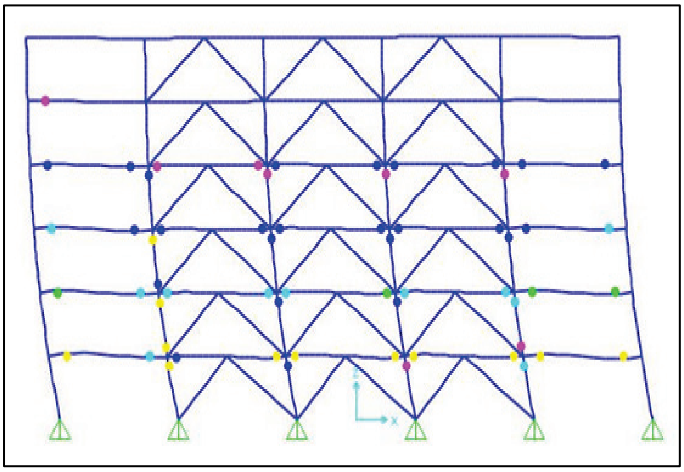

Fig.8 The Failure pattern of IS Code method

9 Discussion and Conclusion

The displacement response is higher in the PBPD method in compare to that of the IS Code method but PBPD gains stability earlier then IS code method.

Acceleration response indicates that the pounding effect will reduce in the PBPD frame. This leads to the construction of the closely spaced structure in dense populated area where land cost is high.

In the PBPD frame, the failure occurs at predetermined locations that is in the beams, which leads to repair at least cost. Whereas in the IS Code design frame, hinges were formed in the beams as well as in the columns. Formation of the hinge in columns may lead to the total collapse of the frame.

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