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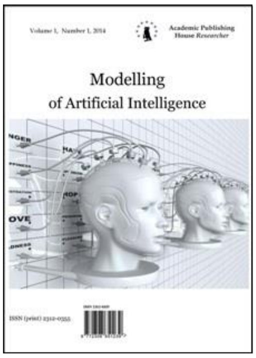

Articles and statements

UDC 004

Moments of the Distance between Two Random Points

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Abstract

Let D be a bounded convex domain in the Euclidean plane and we choose uniformly and independently points P_1 and P_2 from D. Denote by $\rho(P_1, P_2)$ the Euclidean distance between points P_1 and P_2 and by $F_\rho^D(x)$ the distribution function of $\rho(P_1, P_2)$. Using the explicit form of distribution function we obtain a formula for the calculation of moments of order k for any natural k. In particular, using the formula for $F_\rho^D(x)$, we derive the mean distance between the points P_1 and P_2 for a disc, a regular triangle, a rectangle, a regular hexagon and a rhombus (see Santalo, 2004; Burgstaller, Pillichshammer, 2009; Dunbar, 1997).

Keywords: chord length distribution function, mean distance, bounded convex domain.

1. Introduction

The paper continues the investigations in (Aharonyan, 2015). Let D be a bounded convex domain in the Euclidean plane R^2 (with the area $\| D \|$ and the perimeter $|\partial D|$) and we choose uniformly and independently two points P_1 and P_2 from D.

We denote by $F_D^\rho(x)$ the distribution function of the distance $\rho(P_1, P_2)$ between P_1 and P_2 , i.e.

$$F_\rho^D(x) = \frac{1}{||D||^2} \int_{A_D^x} dP_1 dP_2, \tag{1.1}$$

where $A_D^x = \{(P_1, P_2)\colon\ \rho(P_1, P_2) \le x\}$ is the set of pairs (P_1, P_2) with distance between them less or equal to then x, dP_i (i=1, 2) is the 2-dimensional Lebesgue measure (that is an area element in Cartesian coordinates).

Denote by $F_D(y)$ the distribution function of the length of random chord $\chi(g) = D \cap g$, where g is from the space G of all lines in the Euclidean plane R^2 (see Harutyunyan, 2007):

$$F_D(y) = \frac{1}{|\partial D|} \mu(B_D^y) = \frac{1}{|\partial D|} \iint_{B_D^y} dg, \tag{1.2}$$

where $B_D^y = \{g \in G\colon\ g \cap D \neq \emptyset,\ |\chi(g)| \le y\}, y \in R$ and $|\chi(g)|$ is the length of the chord $\chi(g)$, and μ is the Euclidean motion invariant measure in the space G.

It is well known that $\mu(g \in G\colon g \cap D \neq \emptyset) = \mu([D]) = |\partial D|$.

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We use the explicit formula for $F_D^\rho(x)$ obtained in (Santalo, 2004) (see formula (2.8)) which has the following form when $x \in [0, d]$ (d is the diameter of D):

$$F_D^\rho(x) = \frac{1}{||D||}\left[\pi x^2 - \frac{2|\partial D|}{3||D||}x^3 + \frac{|\partial D|}{||D||}\int_0^x (x^2 - t^2)F_D(t)dt\right]. \tag{1.3}$$

For the density function $f_\rho^D(x)$ of $\rho(P_1, P_2)$ (i.e. $(f_\rho^D(x))' = F_\rho^D(x)$) the following result is obtained:

$$f_D^\rho(x) = \frac{2}{||D||}\left[\pi x - \frac{|\partial D|}{||D||}x^2 + \int_0^x (1 - F_D(t))dt\right]. \tag{1.4}$$

These formulae allow to find an explicit forms of distribution and density functions of the distance between two points randomly and independently distributed in the bounded convex domain D when the chord length distribution function is known for that domain. In particular, using above formulae we obtain already known results for the mean distance between P_1 and P_2 when D is a disc, a regular triangle, a rectangle, a regular hexagon and a rhombus (see Santalo, 2004; Burgstaller, Pillichshammer, 2009; Dunbar, 1997).

2. Discussion

Moments of Distance Between Two Points In A Domain

One of the simplest applications of the formulae (1.3) and (1.4) is the calculation of the k-th moment between two points randomly and independently distributed on the bounded convex domain. To find the k-th moment between points P_1 and P_2 (we denote it by M_k^ρ) we need to calculate the following integral

$$M_k^\rho = \int_0^d x^k f_D^\rho(x)dx \quad .$$

Using (2.8) we rewrite the last equation in simpler form:

$$M_k^\rho = \int_0^d x^k f_D^\rho(x)dx = \frac{2\pi}{\|D\|}\int_0^d x^{k+1}dx - \frac{2|\partial D|}{\|D\|^2}\int_0^d x^{k+1}dx \int_0^x \big(1 - F_D(t)\big)dt = \frac{2\pi d^{k+2}}{(k+2)\|D\|} - \frac{2|\partial D|}{(k+2)\|D\|^2}\int_0^d dx^{k+2}\int_0^x \big(1 - F_D(t)\big)dt = \frac{2\pi d^{k+2}}{(k+2)\|D\|} - \frac{2|\partial D|}{(k+2)\|D\|^2}\left[d^{k+2}\int_0^d \big(1 - F_D(t)\big)dt - \int_0^d dx^{k+2}\big(1 - F_D\,(x)\big)dx\right]. \tag{2.1}$$

In (2.1) we can calculate the integral $\int_0^d (1 - F_D(t))dt$. We have

$$\pi \parallel D \parallel = \int_{|x(g)|<d} |x(g)|dg \quad = |\partial D|\int_0^d t dF_D(t) = -|\partial D|\int_0^d t d\big(1 - F_D(t)\big) = -|\partial D|\Big(d\big(1 - F_D(d)\big) - \int_0^d \big(1 - F_D(t)\big)dt\Big) = |\partial D|\int_0^d \big(1 - F_D(t)\big)dt \tag{2.2}$$

Putting (2.2) in (2.1) we obtain

$$M_k^\rho = \frac{2|\partial D|}{(k+2)\|D\|^2}\int_0^d x^{k+2}\big(1 - F_D(x)\big)dx\,. \tag{2.3}$$

(2.3) gives us a relationship between M_k^ρ and the k-th moment of random chord's length of the domain D. If denote by M_k the k-th moment of chord's length then it is not difficult to verify that

$$M_k = k\int_0^d x^{k-1}(1 - F_D(x))dx \tag{2.4}$$

Taking into notice (2.4) we can rewrite (2.3) in the following form:

$$M_k^\rho = \frac{2|\partial D|}{(k+2)(k+3)\|D\|^2}M_{k+3}. \tag{2.5}$$

Another interesting fact can be noted in (2.5). Denote

$$I_k = \int_{g \cap D \neq 0} |x(g)|^k dg \quad \text{and} \quad J_k = \int_{P_1, P_2 \in D} \rho^k(P_1, P_2)dP_1P_2.$$

Therefore, we obtain

$$I_k = \int_{g \cap D \neq 0} |x(g)|^k dg = |\partial D|\int_0^d x^k dF_D(x) = |\partial D|M_k\,. \tag{2.6}$$

Finally, we have

$$J_k = \| D \|^2 M_k^{\rho}. \qquad (2.7)$$

Due to (2.6) and (2.7), (2.5) gives us the following equality

$$J_k = \frac{2}{(k+2)(k+3)} I_{k+3}$$

which is proved in (Santalo, 2004) without mentioning the concepts of the k-th moments of the distance between two points and that of a chord length.

Mean Distance Between Two Points In A Domain

Using (2.3) for $k = 1$ we obtain a formula for calculating the mean distance between two points uniformly and independently distributed in a bounded convex domain D:

$$M_1^{\rho} = M(D) = \frac{2|\partial D|}{3\|D\|^2} \int_0^d x^3 (1 - F_D(x)) dx \qquad (3.1)$$

The case of a Disc

Consider a disc of radius $r, D = C_r$. In this case the chord length density function has the following form (see Stoyan, Stoyan, 1994):

$$F_{C_r}(x) = \begin{cases} 0, & if\ x \leq 0 \\ 1 - \sqrt{1 - \dfrac{x^2}{4r^2}} & if\ 0 \leq x \leq 2\mathrm{r} \\ 1, & if\ x \geq 2r. \end{cases}$$

Using (3.1) for this case we obtain

$$M(C_r) = \frac{4\pi r}{3\pi^2 r^4} \int_0^{2r} x^3 \sqrt{1 - \frac{x^2}{4r^2}}\, dx = \frac{32r}{3\pi} \int_0^1 t\sqrt{1-t}\, dt =$$
$$= \frac{32r}{3\pi} B\left(2, \frac{1}{2}\right) = \frac{32r}{3\pi} \cdot \frac{4}{15} = \frac{128r}{45}$$

where B(x, y) is Euler's Beta function.

The case of a Regular Triangle

For a regular triangle T_a with side a we have

$$F_{T_a}(x) = \begin{cases} 0, & if\quad x \leq 0 \\ \left(\dfrac{1}{2} + \dfrac{\pi}{3\sqrt{3}}\right)\dfrac{x}{a}, & if\ 0 \leq x \leq \dfrac{a\sqrt{3}}{2} \\ \dfrac{x}{2a} - \dfrac{2\pi}{3\sqrt{3}}\dfrac{y}{a} + \dfrac{2x}{a\sqrt{3}} \arcsin\dfrac{a\sqrt{3}}{2x} + \dfrac{\sqrt{4x^2 - 3a^2}}{2x} & if\quad \dfrac{a\sqrt{3}}{2} \leq x \leq a \\ 1 & if\ \ x \geq a \end{cases}$$

In this case (3.1) gives

$$M(T_a) = \frac{32}{3a^3}\left[\int_0^{\frac{a\sqrt{3}}{2}} x^3 \left(1 - \frac{x}{a}\left(\frac{1}{2} + \frac{\pi}{3\sqrt{3}}\right)\right) dx + \int_{\frac{a\sqrt{3}}{2}}^{a} x^3 \left(1 - \frac{x}{a}\left(\frac{1}{2} - \frac{2\pi}{3\sqrt{3}}\right) - \frac{2x}{a\sqrt{3}} \arcsin\frac{a\sqrt{3}}{2x} - \frac{\sqrt{4x^2-3a^2}}{2x}\right) dx\right] = \frac{32}{3a^3}\left[\int_0^a x^3 dx - \frac{1}{2a}\int_0^a x^4 dx + \frac{\pi}{3a\sqrt{3}}\left(2\int_{\frac{a\sqrt{3}}{2}}^{a} x^4 dx - \int_0^a x^4 dx\right) - \int_{\frac{a\sqrt{3}}{2}}^{a}\left(\frac{2x^4}{a\sqrt{3}} \arcsin\frac{a\sqrt{3}}{2x} + \frac{x^2\sqrt{4x^2-3a^2}}{2}\right) dx\right] = \frac{32}{3a^3}\left[\frac{a^4}{4} - \frac{a^4}{10} + \frac{(64\sqrt{3}-81)\pi a^4}{1440} - \frac{a^4(756 + 4(64\sqrt{3}-81)\pi - 81\ln 3)}{5760}\right] = \frac{32}{3a^3} * \frac{3a^4}{640}(4 + 3\ln 3) = a\left(\frac{1}{5} + \frac{3}{20}\ln 3\right).$$

The case of a Rectangle

The chord length distribution function for a rectangle $R_{a,b}$ with sides a and b has the following form

$$F_{R_{a,b}}(x)=\begin{cases}0, & if\ x\le 0\\ \dfrac{x}{a+b}, & if\quad 0\le x\le a\\ \dfrac{a}{a+b}+\dfrac{b}{a+b}\dfrac{\sqrt{x^2-a^2}}{x}, & if\ a\le x\le b\\ 1-\dfrac{x}{a+b}+\dfrac{1}{(a+b)x}\left(a\sqrt{x^2-b^2}+b\sqrt{x^2-a^2}\right), & if\ b\le x\le\sqrt{a^2+b^2}\\ 1, & if\quad x\ge\sqrt{a^2+b^2}\end{cases}$$

For this case (3.1) gives the following

$$M(R_{a,b})$$
$$=\frac{4(a+b)}{3a^2b^2}\left[\int_0^a x^3dx-\frac{1}{a+b}\int_0^a x^4dx+\left(1-\frac{a}{a+b}\right)\int_a^b x^3dx\right.$$
$$-\frac{b}{a+b}\int_a^b x^2\sqrt{x^2-a^2}dx+\frac{1}{a+b}\int_b^{\sqrt{a^2+b^2}}x^4dx$$
$$\left.-\frac{b}{a+b}\int_b^{\sqrt{a^2+b^2}}x^2\sqrt{x^2-a^2}dx-\frac{a}{a+b}\int_b^{\sqrt{a^2+b^2}}x^2\sqrt{x^2-b^2}dx\right]$$
$$=\frac{4(a+b)}{3a^2b^2}\left[\frac{a^5+b^5+a^2b^2\sqrt{a^2+b^2}\left(3-\frac{a^2}{b^2}-\frac{b^2}{a^2}\right)+\frac{5a^2b^2}{2}\left(\frac{b^2}{a}ln\frac{\sqrt{a^2+b^2}+a}{b}+\frac{a^2}{b}ln\frac{\sqrt{a^2+b^2}+b}{a}\right)}{20(a+b)}\right]$$

Finally, we obtain

$$M(R_{a,b})=\frac{1}{15}\left[\frac{a^3}{b^2}+\frac{b^3}{a^2}+\sqrt{a^2+b^2}\left(3-\frac{a^2}{b^2}-\frac{b^2}{a^2}\right)+\frac{5}{2}\left(\frac{b^2}{a}ln\frac{\sqrt{a^2+b^2}+a}{b}+\frac{a^2}{b}ln\frac{\sqrt{a^2+b^2}+b}{a}\right)\right]\qquad(3.2)$$

In particular, when we have square S_a with side a then putting $a=b$ in (3.2) we obtain

$$M(S_a)=\frac{a}{15}\left[2+\sqrt{2}+5\ln(1+\sqrt{2})\right].$$

The case of a Regular Hexagon

Denote by H_a the regular hexagon with side a. We have (see Harutyunyan, 2007)

$$F_{H_a}(x)=\begin{cases}0, & if\ x\le 0\\ \dfrac{x}{a}\left(\dfrac{1}{2}-\dfrac{\pi}{6\sqrt{3}}\right), & if\quad 0\le x\le a\\ 1+\dfrac{\pi x}{2a\sqrt{3}}-\dfrac{2x}{a\sqrt{3}}\arcsin\dfrac{a\sqrt{3}}{2x}-\dfrac{\sqrt{4x^2-3a^2}}{2x}, & if\ a\le x\le a\sqrt{3}\\ 1+\dfrac{x}{a}\left(\dfrac{\pi}{6\sqrt{3}}-\dfrac{1}{2}\right)+2\dfrac{\sqrt{x^2-3a^2}}{x}-\dfrac{x}{a\sqrt{3}}\arccos\dfrac{a\sqrt{3}}{x}, & if\quad a\sqrt{3}\le x\le 2a\\ 1, & if\quad x\ge 2a\end{cases}$$

For $D=H_a$ (3.1) gives

$$M(H_a)=\frac{32}{9a^3}\left[\int_0^a x^3dx-\frac{1}{a}\left(\frac{1}{2}-\frac{\pi}{6\sqrt{3}}\right)\int_0^a x^4dx-\frac{\pi}{2a\sqrt{3}}\int_a^{a\sqrt{3}}x^4dx\right.$$
$$+\frac{2}{a\sqrt{3}}\int_a^b x^4\arcsin\frac{a\sqrt{3}}{2x}dx+\int_a^{a\sqrt{3}}\frac{x^2\sqrt{4x^2-3a^2}}{2}dx$$
$$\left.-\frac{1}{a}\left(\frac{\pi}{6\sqrt{3}}-\frac{1}{2}\right)\int_{a\sqrt{3}}^{2a}x^4dx-2\int_{a\sqrt{3}}^{2a}x^2\sqrt{x^2-3a^2}dx+\frac{1}{a\sqrt{3}}\int_{a\sqrt{3}}^{2a}x^4\arccos\frac{a\sqrt{3}}{x}dx\right]$$

Finally, we obtain

$$M(H_a) = \frac{a}{30}\left(42\sqrt{3} - 14 + 84\ln 3 + 3\ln(2\sqrt{3} - 3)\right).$$

3.5 The case of a Rhombus

Consider a rhombus $RH_{a,\gamma}$ with side a and acute angle γ. In (Harutyunyan, Ohanyan, 2011) an explicit formula for the chord length distribution function for rhombus is obtained which for $\gamma \le \pi/3$ has the following form:

$$F_{Rh_{a,\gamma}}(x) = \begin{cases} 0, & if\ x \le 0 \\ \frac{x}{2a}\left(1 + \left(\frac{\pi}{2} - \gamma\right)\cot\gamma\right), & if\ \ x \in [0, a\sin\gamma] \\ \frac{x}{2a}\left(1 - \left(\frac{\pi}{2} + \gamma - 2arcin\ \frac{a\sin\gamma}{x}\right)\cot\gamma\right) + \frac{\sqrt{x^2 - a^2\sin^2\gamma}}{x}, & if\ x \in [a\sin\gamma, 2\,a\sin\frac{\gamma}{2}] \\ \frac{x}{4a}\left(3 + \left(2arcin\ \frac{a\sin\gamma}{x} - 3\gamma\right)\cot\gamma\right) + \frac{\sqrt{x^2 - a^2\sin^2\gamma}}{2x}, & if\ \ x \in [2a\sin\frac{\gamma}{2}, a] \\ 1 + \frac{x}{4a}\left(-1 + \left(\gamma - 2arcin\ \frac{a\sin\gamma}{x}\right)\cot\gamma\right) + \frac{\sqrt{x^2 - a^2\sin^2\gamma}}{2x}, & if\ \ x \in [a,\ 2a\cos\frac{\gamma}{2}] \\ 1, & if\ \ x \ge 2a\cos\frac{\gamma}{2} \end{cases}$$

Thus,

$$\begin{aligned} M(RH_{a,\gamma}) = \frac{8}{3a^3\sin^2\gamma} \Bigg[& \int_0^{a\sin\gamma} x^3 dx - \frac{1}{2a}\left(1 + (\frac{\pi}{2} - \gamma)\cot\gamma\right)\int_0^{a\sin\gamma} x^4 dx + \int_{a\sin\gamma}^{2\,a\sin\frac{\gamma}{2}} x^3 dx \\ & - \frac{1}{2a}\left(1 - (\frac{\pi}{2} + \gamma)\cot\gamma\right)\int_{a\sin\gamma}^{2a\sin\gamma/2} x^4 dx \\ & - \frac{\cot\gamma}{a}\int_{a\sin\gamma}^{2a\sin\gamma/2} x^4 \arcsin\frac{a\sin\gamma}{x} dx - \int_{a\sin\gamma}^{2a\sin\gamma/2} x^2\sqrt{x^2 - a^2\sin^2\gamma}\, dx \\ & + \int_{2a\sin\gamma/2}^{a} x^3 dx \\ & - \frac{3(1 - \gamma\cot\gamma)}{4a}\int_{2a\sin\gamma/2}^{a} x^4\, dx - \frac{\cot\gamma}{a}\int_{2a\sin\gamma/2}^{a} x^4 \arcsin\frac{a\sin\gamma}{x} dx \\ & - \frac{1}{2}\int_{2a\sin\gamma/2}^{a} x^2\sqrt{x^2 - a^2\sin^2\gamma}\, dx + \frac{1 - \gamma\cot\gamma}{4a}\int_a^{2a\cos\gamma/2} x^4 dx \\ & + \frac{\cot\gamma}{2a}\int_a^{2a\cos\gamma/2} x^4 \arcsin\frac{a\sin\gamma}{x} dx - \frac{1}{2}\int_a^{2a\cos\gamma/2} x^2\sqrt{x^2 - a^2\sin^2\gamma}\, dx \Bigg] \\ = \frac{a}{60}\Bigg[& 2 \\ & + 18\sqrt{2}\sin\left(\frac{\gamma}{2} + \frac{\pi}{4}\right) - 11\sqrt{2}\sin\left(\frac{3\gamma}{2} - \frac{\pi}{4}\right) \\ & + 3\sqrt{2}\sin\left(\frac{5\gamma}{2} + \frac{\pi}{4}\right) \\ & - 6\cos 2\gamma - 10\sin^2\gamma\log\left(\tan\frac{\gamma}{4}\cot\frac{\gamma + \pi}{4}\right) - 6\sin^2\gamma\cos\gamma\log\left(\tan\frac{\gamma}{4}\tan\frac{\gamma + \pi}{4}\cot^2\frac{\gamma}{2}\right)\Bigg]. \end{aligned}$$

When $\frac{\pi}{3} \le \gamma \le \pi/2$, $F_{Rh_{a,\gamma}}(x)$ has the following form:

$$F_{Rh_{a,\gamma}}(x) = \begin{cases} 0, & if\ x \le 0 \\ \frac{x}{2a}\left(1+\left(\frac{\pi}{2}-\gamma\right)\cot\gamma\right), & if\ \ x \in [0, a\sin\gamma] \\ \frac{x}{2a}\left(1-\left(\frac{\pi}{2}+\gamma-2arcin\ \frac{a\sin\gamma}{x}\right)\cot\gamma\right)+\frac{\sqrt{x^2-a^2\sin^2\gamma}}{x}, & if\ x \in [a\sin\gamma, a] \\ 1-\frac{x}{2a}\left(1+\left(\frac{\pi}{2}-\gamma\right)\cot\gamma\right)+\frac{\sqrt{x^2-a^2\sin^2\gamma}}{x}, & if\ \ x \in [a, 2a\sin\frac{\gamma}{2}] \\ 1+\frac{x}{4a}\left(-1+\left(\gamma-2arcin\ \frac{a\sin\gamma}{x}\right)\cot\gamma\right)+\frac{\sqrt{x^2-a^2\sin^2\gamma}}{2x}, & if\ \ x \in [2a\sin\frac{\gamma}{2},\ 2a\cos\frac{\gamma}{2}] \\ 1, & if\ \ x \ge 2a\cos\frac{\gamma}{2} \end{cases}$$

Therefore

$$M(RH_{a,\gamma}) = \frac{8}{3a^3\sin^2\gamma}\Bigg[\int_0^{a\sin\gamma} x^3dx - \frac{1}{2a}\left(1+(\frac{\pi}{2}-\gamma)\cot\gamma\right)\int_0^{a\sin\gamma} x^4dx + \int_{a\sin\gamma}^{0} x^3dx$$
$$-\frac{1}{2a}\left(1-(\frac{\pi}{2}+\gamma)\cot\gamma\right)\int_{a\sin\gamma}^{a} x^4dx$$
$$-\frac{\cot\gamma}{a}\int_{a\sin\gamma}^{a} x^4\arcsin\frac{a\sin\gamma}{x}dx - \int_{a\sin\gamma}^{a} x^2\sqrt{x^2-a^2\sin^2\gamma}\,dx$$
$$+\frac{1}{2a}\left(1+(\frac{\pi}{2}-\gamma)\cot\gamma\right)\int_a^{2a\sin\gamma/2} x^4dx$$
$$-\int_a^{2a\sin\gamma/2} x^2\sqrt{x^2-a^2\sin^2\gamma}\,dx$$
$$+\frac{1-\gamma\cot\gamma}{4a}\int_{2a\sin\gamma/2}^{2a\cos\gamma/2} x^4\,dx + \frac{\cot\gamma}{2a}\int_{2a\sin\gamma/2}^{2a\cos\gamma/2} x^4\ \arcsin\frac{a\sin\gamma}{x}dx$$
$$-\frac{1}{2}\int_{2a\sin\gamma/2}^{2a\cos\gamma/2} x^2\sqrt{x^2-a^2\sin^2\gamma}\,dx\Bigg]$$
$$=\frac{a}{60}\Bigg[2$$
$$+18\sqrt{2}\sin\left(\frac{\gamma}{2}+\frac{\pi}{4}\right)-11\sqrt{2}\sin\left(\frac{3\gamma}{2}-\frac{\pi}{4}\right)$$
$$+3\sqrt{2}\sin\left(\frac{5\gamma}{2}+\frac{\pi}{4}\right)$$
$$-6\cos 2\gamma - 10\sin^2\gamma\log\left(\tan\frac{\gamma}{4}\cot\frac{\gamma+\pi}{4}\right) - 6\sin^2\gamma\cos\gamma\log\left(\tan\frac{\gamma}{4}\tan\frac{\gamma+\pi}{4}\cot^2\frac{\gamma}{2}\right)\Bigg].$$

Thus, we get the same expression for both cases. It is not difficult to verify that putting $\gamma = \pi/2$ in the expression above we come to the result obtained in the case of a square.

3. Conclusion

We have derived a formula for the distribution and density functions of a distance between two points in a domain. We would like to stress that in (Harutyunyan, Ohanyan, 2009) there exist an explicit form for the chord length distribution function for any regular polygon, while in (Harutyunyan, Ohanyan, 2009) and (Harutyunyan, Ohanyan, 2011) an explicit formula for the chord length distribution function for any triangle is obtained. Thus it is possible to obtain explicit form for $F_D^{\rho}(x)$ and $f_D^{\rho}(x)$ for any regular polygon and a triangle. In particular we can find k-th moment of the distance between two points in a domain where the chord length distribution function is known.

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Моменты расстояния между двумя точками

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Аннотация. Пусть D ограниченная выпуклая область на евклидовой плоскости. Случайно выбираются независимо друг от друга две точки P_1 и P_2 равномерно распределенные в области D. Обозначим через $\rho(P_1, P_2)$ евклидово расстояние между точками P_1 и P_2, а через $F_\rho^D(x)$ функцию распределения расстояния $\rho(P_1, P_2)$. Используя явный вид функции распределения выводится формула для вычисления моментов произвольного порядка k. В частности, используя явный вид для $F_\rho^D(x)$, вычислены средние расстояния в случае, когда точки P_1 и P_2 случайно брошены в круг, в правильный треугольник, прямоугольник, в правильных шестиугольник и ромб (см. Santalo, 2004; Burgstaller, Pillichshammer, 2009; Dunbar, 1997).

Ключевые слова: функция распределения длины хорды, среднее расстояние, ограниченная выпуклая область.

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