УДК 51(075)8

AN EXPLANATION OF G. GALILEI'S PARADOX AND THE ESTIMATE OF QUANTITIES OF BOTH RATIONAL AND PRIME NUMBERS

ОБЪЯСНЕНИЕ ПАРАДОКСА Г. ГАЛИЛЕЯ И ОЦЕНКА КОЛИЧЕСТВ РАЦИОНАЛЬНЫХ И ПРОСТЫХ ЧИСЕЛ

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Abstract. Let (k, A) and (m, B) be two natural variables that is $k \in A \subseteq \boldsymbol{N}$ and $m \in B \subseteq \boldsymbol{N}$. The pair $(k, m) \in (A, B)$ is said to be C-pair if $\exists\, C \in \boldsymbol{N}$: $\forall\, (k, m)$ which are as the neighboring elements in $E \triangleq A \cup B \subseteq \boldsymbol{N}, |k - m| < C$. Further we prove (Theorem 3) $\forall$ pair $(k, m) \in (A, B)\, \exists\, C \in \boldsymbol{N}$: this pair is C-pair. Let (k, A) be natural variable with unlimited step that is $\forall\, d>0$ $\exists n \in \boldsymbol{N}$: $k_{n+1} - k_n > d$. Theorem 3 implies that the (k, A) with unlimited step can be defined only some subset $N_A \subset \boldsymbol{N}$ and $J \triangleq \boldsymbol{N} \backslash N_A$ is any infinite set. That implies following conclusion (Statement 6). Let $\pi(n)$ be a set of all prime numbers p: $p \leq n$. If $\exists \lim_{n \to \infty} \pi(n) \triangleq \pi(\infty) \triangleq \Omega$ now it is obvious that $|\,\Omega\,| < |\,\boldsymbol{N}|$. This theorem was known still Euclid more two thousand years ago. In turn the set of primes is any sequence with unlimited step. Thus Theorem 3 proves an existence of infinite large number $\pi(\infty) = \Omega$. G. Galilei has (Example 1) paid his attention into the mapping $g: \boldsymbol{N} \to \boldsymbol{N}$, $g(n) = n^2$. In our time this fact is known as $Galilei's$ paradox. It is obvious that $g(\boldsymbol{N}) \triangleq \boldsymbol{N_g} \subset \boldsymbol{N}$. At the second hand, $\forall\, d>0\, \exists n \in \boldsymbol{N}$: $(n+1)^2 - n^2 > d$. Injective mapping $\varphi: \boldsymbol{N} \to \boldsymbol{N}$ with $\varphi(\boldsymbol{N}) = N_\varphi \subset \boldsymbol{N}$ *is said to be potentially antysurjective one* (Definition III). Let $Q(n)$ be (Example 2) square n-matrix (q_m^k), $q_m^k \triangleq k/m$ with $1 \leq k, m \leq n$. The $Q(n)$ contains n^2 of positive rational numbers q, with $1/n \leq q \leq n$. Everyone will easily believe that $|Q^+(n)| < n^2$, if we shall assume only distinct numbers in $Q^+(n)$. The $Q^+(n)$ depends essentially on values of the function $\pi(n)$, for example $Q^+(p) = Q^+(p-1) + 2(p-1)$. Now we accept $Q^+(n) = \mu(n) n^2$. If we assume a hypothesis that $\lim \mu(n) \approx 0{,}6$, then we have $|Q^+(N)| \approx 0{,}6 |N|^2$. (Example 3) Let $(\boldsymbol{A}) \triangleq \sum_{n=1}^{\infty} (n)^{-1}$ be a harmonic series (Example 3). We prove that $(\boldsymbol{A})$ is the convergent series in addition to it converges to any infinite large number Ω_h, though it is well known, its sum is not limited by any finite number. See, please, [1, 2].

Аннотация. Пусть (k, A) и (m, B) суть две натуральные переменные, так что $k \in A \subseteq \boldsymbol{N}$ и $m \in B \subseteq \boldsymbol{N}$. Пара $(k, m) \in (A, B)$ называется C-парой, если $\exists\, C \in \boldsymbol{N}$: $\forall\, (k, m)$, которые являются соседними элементами в $E \triangleq A \cup B \subseteq \boldsymbol{N}, |k - m| < C$. Далее мы доказываем (Теорема 3) $\forall$ пары $(k, m) \in (A, B)\, \exists\, C \in \boldsymbol{N}$ такое, что эта пара является C-парой. Пусть (k, A) будет натуральной переменной с неограниченным шагом, это означает по определению, что $\forall\, d>0$ $\exists n \in \boldsymbol{N}$: $k_{n+1} - k_n > d$. Теорема 3 утверждает, что натуральная переменная (k, A) с неограниченным шагом может быть определена только на некотором собственном подмножестве $N_A \subset \boldsymbol{N}$ и $J \triangleq \boldsymbol{N} \backslash N_A$ есть бесконечное множество, что влечёт следующее предложение (Утверждение 6). Пусть $\pi(n)$, по определению, означает множество всех простых чисел $p \leq n$. Тогда при предельном переходе мы получим, что $\exists \lim_{n \to \infty} \pi(n) \triangleq \pi(\infty) \triangleq \Omega$, где очевидно $|\,\Omega\,| < |\,\boldsymbol{N}|$. С другой стороны, давно известно, что множество простых чисел образует натуральную последовательность с неограниченным шагом и, по Теореме 3, эта последовательность не может быть определена на всём множестве $\boldsymbol{N}$. **Следовательно,** Теорема 3 определяет некоторое бесконечно большое число $\pi(\infty) = \Omega$. Г.Галилей обратил своё внимание на отображение $g: \boldsymbol{N} \to \boldsymbol{N}$, $g(n) = n^2$. В наше время этот факт известен как *парадокс Галилео Галилея.* Здесь очевидно, что $g(N) \triangleq \boldsymbol{N}_g \subset \boldsymbol{N}$. С другой стороны, $\forall\ d>0$ $\exists n \in \boldsymbol{N}$: $(n+1)^2 - n^2 > d$. Инъективное отображение $f: \boldsymbol{N} \to \boldsymbol{N}$, где $f(\boldsymbol{N}) = N_f \subset \boldsymbol{N}$ и подмножество N_f является бесконечным множеством, *называется потенциально антисюръективным отображением* (Определение III). Пусть $Q(n)$ будет (Пример 2) квадратной n-матрицей $(q_m^k), q_m^k \triangleq k/m$ и $1 \leq k, m \leq n$. Таблица $Q^+(n)$ содержит n^2 положительных рациональных чисел q, где $1/n \leq q \leq n$. Каждый может легко убедится в том, что $|Q^+(n)| < n^2$, если мы будем рассматривать только неравные числа в $Q^+(n)$. Множество чисел $Q^+(n)$ существенно зависит от значений функции $\pi(n)$, например, $Q^+(p) = Q^+(p-1) + 2(p-1)$. Теперь мы предположим, что $Q^+(n)=\mu(n)n^2$ и, кроме того, примем гипотезу, что $\lim\mu(n)\approx 0{,}6$. Тогда мы получим для множества $Q^+(N)$ следующую оценку $|Q^+(N)| \approx 0{,}6|N|^2$. Наконец, мы рассмотрим гармонический ряд $(\boldsymbol{A}) \triangleq \sum_{n=1}^{\infty}(n)^{-1}$ (Пример 3), где мы докажем, что этот ряд ($\boldsymbol{A}$) является сходящимся числовым рядом и сходящимся к некоторому бесконечно большому числу Ω_h, хотя с XV века много раз доказано, что сумма гармонического ряда не ограничена ни каким действительным числом. Некоторый материал этой статьи более (или менее) подробно изложен нами в [1] и (в [2]).

Keywords: natural variable, *C*-pair, Galilei's paradox, the prime numbers, the harmonious series convergence.

Ключевые слова: натуральная переменная, С-пара натуральных переменных, парадокс Г. Галилея, простые числа, сходимость гармонического ряда.

1. G. Galilei's paradox

Properties of infinity, surprising and not clear from the point of view of all final, were incentive motive of our research. Really, properties of infinity in the analysis: $a+\infty=\infty$, $a\times\infty=\infty$, $\infty+\infty=\infty$, $\infty\times\infty=\infty$, $\infty^{\infty}=\infty$ and others are not intelligible in the finite arithmetic. Moreover, the equalities $\sum(1)^n = \infty = \sum n^{-1}$ deprive concept of infinity of any definiteness and structure that increases a risk of any mistakes occurrence in proofs of statements about infinite. In the beginning of XVII century G. Galilei has opened as if quantities of natural numbers and their squares are equal. On this basis he approved, that «…properties of equality, and also greater and smaller size have no place there where it is a question of infinity, and they are employ only to finite quantities» [3, p. 140–146].
Below we follow this thesis and at the first we check a surjectivity of all injective mappings of set $\boldsymbol{N}$ of natural numbers and its infinite subsets which everyone accepted as obvious by default in the traditional analysis.

2. The properties of injective mappings N→N

Let $A \cap B \supseteq \varnothing$ and $E \triangleq A \cup B \subseteq \boldsymbol{N}$.

Definition 1. *The pair* (*m*, *k*) *of natural variables m∈A and k∈B is said to be C-pair if there exists a number C*>0 *and inequality*

$$|m\text{-}k\,| < C \tag{1}$$

is true for everyone pair (m, k) of elements m and k which are neighbouring ones in E.

The condition (1) of C-pair (m, k) is equivalent with $q(k), p(m) \in \boldsymbol{Z}, |q(k)| < C, |p(m)| < C$ to each of two following ones:

$$1)\ \forall m \in A\ \exists k \in B: m = k + q(k).\ 2)\ \forall k \in B\ \exists m \in A: k = m + p(m). \tag{2}$$

Below we prove Theorem 3: $\forall\ (m,k)\exists C^0$of this kind, that this pair (m, n) is C^0-pair at m, $n \to \infty$. This statement is one of constituent parts of alternative methodology. Any mapping $f: N \to N$ defines a sequence $\{a_n\}_{n=1}^{\infty} \triangleq (\boldsymbol{a}) \triangleq (a_n)$ of natural numbers a_n,where $\triangleq f(n), n \in N$. Now we consider the injective mapping $\varphi: N \to N$ by default. Let $\xi \triangleq (1, n_1, n_2, \ldots, n_i, \ \ldots)$ be a strictly monotonous sequence, $N(\xi) \triangleq \{i: \exists n_i \in \xi\} \subseteq N$ and $N_i \triangleq (1, n_1, n_2, \ldots, n_i)$. Further, let $\Delta_{i+1} \overset{\Delta}{=} N_{i+1} \backslash N_i$. The sequence ξ breaks up the set N into not crossed pieces: $N = \cup\ \Delta_i$, we shall name this partition by ξ–partition of set N. Sequence ξ and mapping $\varphi: N \to N$ define three sequences (d_i), (δ_i), $i \in N(\xi)$ and (φ_n) of natural (integer) numbers at $i \in N(\xi)$, $n \in \boldsymbol{N}$, by formulas:

$$d_i \overset{\Delta}{=} |D_i| \geq 0,\ D_i \overset{\Delta}{=} N_i \backslash \varphi(N_i),\quad \delta_i \overset{\Delta}{=} \max_{n \leq n_i}\{\varphi(n) - n_i\} \geq 0,\ \ \varphi_n \overset{\Delta}{=} \varphi(n) - n. \qquad (3)$$

In (3) symbol $|M|$ designates a quantity of elements of set *M* and, generally, $\varphi_n \in Z$. Let $D_i^- \overset{\Delta}{=} \varphi(N_i) \backslash N_i$ and $d_i^- \overset{\Delta}{=} |D_i^-| \geq 0$, then $d_i^- = d_i \leq \delta_i$. Really, $d_i^- = \delta_i$ if and only if $\{p:\ n_i < p < \delta_i,\ \forall n \leq n_i\ \ p \neq \varphi(n)\} = \emptyset$. Otherwise, $d_i^- < \delta_i$.

Figure 1 illustrates the mapping φ: $\boldsymbol{N} \to \boldsymbol{N}$ with $(m) \overset{\Delta}{=} p$.

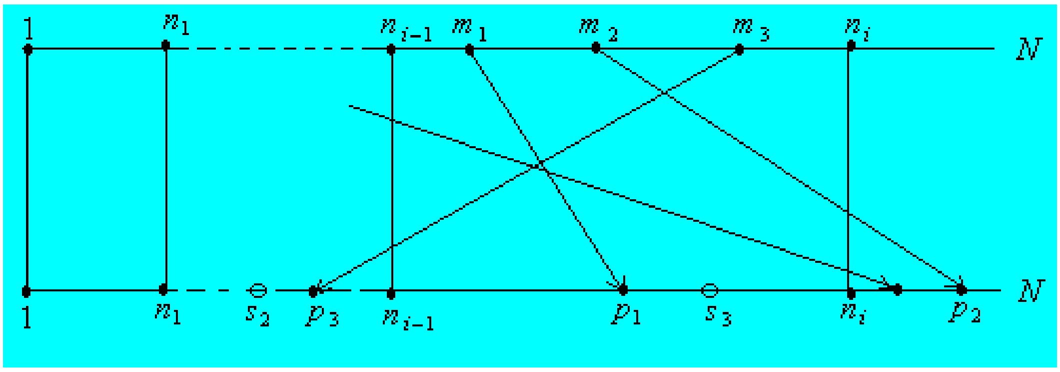

Figure 1. The mapping φ: $\boldsymbol{N} \to \boldsymbol{N}$ with $(m) \overset{\Delta}{=} p$.

Here $p_2 \in D_i^-$, s_2, $s_3 \in D_i$, $\delta_i = p_2 - n_i$. Now we emphasize, that $\forall i \in \boldsymbol{N}(\xi)$ the number $d_i = |D_i| = d_i$ defines a quantity of "holes" in the $\boldsymbol{N}_i \overset{\Delta}{=} \{1, 2, \ldots, n_i\}$, which is a quantity of those elements of a subset $N_i \triangleq (1, n_1, n_2, \ldots, n_i)$, everyone of them has no prototype on N_i. Now we formulate almost obvious fairly

Statement 1. If $\delta_\varphi \overset{\Delta}{=} \sup_{n \in \boldsymbol{N}}\{\varphi(n) - n\}$ and for any sequence ξ $\delta_\xi \overset{\Delta}{=} \sup_{i \in \boldsymbol{N}(\xi)}\{\delta_i\}$, *then* $\delta_\xi \leq \delta_\varphi$

.

However, there exists such **ξ-** partition of ***N*** with $\varphi: N \to N$ of this kind so we have

$$\delta_\xi = \delta_\varphi . \qquad (4)$$

Statement 2. Necessary condition of surjectivity for every injective mapping $\varphi: N \to N$ has following two equivalent forms

$$\forall\ i \in \boldsymbol{N}(\xi)\ \exists\ \ j \in \boldsymbol{N}:\ D_i\ \cap D_{i+j} = \varnothing\, and\ \boldsymbol{N}_i \subset \varphi(\boldsymbol{N}_{i+j}). \qquad (5)$$

• Let $\varphi(N) = N$. Then the condition $\exists j \in N: D_i \cap D_{i+j} = \emptyset$ and $N_i \subset \varphi(N_{i+j})$ follows from inclusion $N_i \subset \varphi(N)$ and finiteness of set $\boldsymbol{N_i}$. Further we shall prove implication $\left(D_i \cap D_{i+j} = \emptyset\right) \Rightarrow (N_i \subset \varphi(N_{i+j}))$. Really, inclusion $\varphi^{-1}(D_i) \subset N_{i+j}$ follows from both then $D_i \subset \varphi(N_{i+j})$ and definition of set D_{i+j}. Then $D_i \subset \varphi(N_{i+j})$. Besides by definition of set D_i we have inclusion $N_i \backslash D_i \subset \varphi(N_i) \subset \varphi(N_{i+j})$. Hence $N_i \subset \varphi(N_{i+j})$. Return implication $(N_i \subset \varphi(N_{i+j})) \Rightarrow (D_i \cap D_{i+j} = \emptyset)$ is proved similarly. ■

Below the phrase «for almost all *i*» designates «for exceptions of finite set of indexes *i*» and we write "$\widetilde{\forall} i$"by definition.

Sufficient condition of surjectivity (a) and antysurjectivity (b) of mapping φ are written in terms of sequence (d_i) below as consequence of both Statement 2 and definition of the (d_i).

Statement 3. Sufficient conditions of surjectivity (a) and antysurjectivity (b) of injective mapping $\varphi: N \to N$ have, accordingly, following form:

$$(a)\, \widetilde{\forall} i \in N(\xi) d_i = 0, (b)\ \forall C \exists i(C) \in N(\xi): d_{i(C)} > C. \quad (6)$$

•The condition (6*a*) guarantees an existence of number i_0 such so for mapping φ there exist the following chain of implications:

$\forall j > i_0\ \ d_j = 0 \Rightarrow D_j = \varnothing \Rightarrow \varphi(N_j) = N_j \Rightarrow \varphi(\boldsymbol{N}) = \boldsymbol{N}$. The condition (6*b*) approves limitlessness of sequence (d_i), $i \in \boldsymbol{N}(\xi)$, which contradicts to the surjectivity of injective mapping $\varphi: N \to N$, as each number d_i is equal by definition to quantity of elements *n* of set D_i each of them has no prototype $\varphi^{-1}(n)$ in N_i. ■

As show examples, conditions (6a) and (6b*)* are not necessary, accordingly, for surjectivity (a) and antysurjectivity (b) of mapping $\varphi: N \to N$. We say about the injective antysurjective mapping, that it is *potentially not realizable on all set* ***N*****.**

Theorem1. Sequences (d_i) and (δ_i), $i \in N(\xi)$, defined by the pair (ξ, φ), satisfy to one and only to one of three following conditions:

$$(a)\ \widetilde{\forall}\ i \in \boldsymbol{N}(\xi):\ (\delta_i = 0) \Leftarrow (d_i = 0), \quad (7a)$$

$$(b)\ (\exists C_1, C_2, C_2 \le C_1 \in \boldsymbol{N}):\ (\widetilde{\forall}\, i \in \boldsymbol{N}(\xi)\ (0 < \delta_i < C_1) \Leftarrow (0 < d_i < C_2)), \quad (7b)$$

$$(c)\ i \in \boldsymbol{N}(\xi)\ \ (d_i \to \infty) \Leftrightarrow (\delta_i \to \infty). \quad (7c)$$

A consequence of Statements 1–3 and Theorems 1 is written down below.

Statement 4. Necessary attribute of surjectivity of an injection $\varphi: N \to N$ has the following form in terms of sequence (δ_i):

$$(\forall \xi,\ \exists C_\xi):\ \forall i \in \boldsymbol{N}(\xi)\ \ 0 \le \delta_i < C_\xi. \quad (8)$$

One more necessary and more effective attribute of the surjectivity of injection $\varphi: N \to N$ in view of the equality (4) gives

Theorem 2. The boundedness of sequence (φ_n) of the integers $\varphi_n \triangleq \varphi(n) - n, n \in N$, is a necessary condition of the injective mapping $\varphi: N \to N$ surjectivity that has form $\varphi(N) = N$ and following limiting kind:

$$\lim_{n \to \infty} (\varphi(n) : n) = 1. \quad (9)$$

Existence of limit (9) follows from a necessary condition (5) of the surjectivity of injective mapping $\varphi: N \to N$. As show the examples, necessary conditions (8) and (9) of surjectivity of an injection φ are independent ones and, hence, any of these conditions cannot be sufficient. The sequence$\xi = (1,\ n_1, n_2, \cdots, n_i, \cdots)$ is said to be *the sequence with the limited step* if $\exists C>0$ such, so $\tilde{\forall} i,\ i \in N(\xi),\ \ n_{i+1} - n_i < C$.

Statement 5. Injective mapping $\varphi^*: N \to N$ is impracticable on all set ***N***, if it defines any sequence $\xi^* = (1, m_1, m_2, \ldots),\ m_{i+1} > m_i$, with unlimited step or, in other words, this mapping φ^* is antysurjective one.

The statement 5 implicates the following statement.

Theorem 3. *Let* $A \triangleq \{n\} \subseteq N$ *and* $B \triangleq \{m\} \subseteq N$ *be infinite subsets of set* **N**. *Then there is a number* $C \in N$ *such so the pair* (n, m) *of variables n and m is C-pair variables* (1).

Statement 6. Let $\pi(n)$ be a set all prime numbers p: $p \leq n$. If $\exists \lim_{n\to\infty} \pi(n) \triangleq \pi(\infty) \triangleq \Omega$ then $|\Omega| < |\boldsymbol{N}|$ that it is obvious. In turn it is well known the set of primes is any sequence with unlimited step, thus the function $\pi(n)$ does not defined on all set ***N.***

The following below the statement is consequence of all proved above propositions.

Theorem 4. *There does not exist any bijiction between set* **N** *of natural numbers and its own subset* $\boldsymbol{A} \subset \boldsymbol{N}$.

The proved above propositions allow us divide all injective mappings $\varphi: N \to N$ onto six not crossed classes.

Definition I. The injection $\varphi: N \to N$ *is said to be precisely surjective one* if there exists such ξ- partition of set ***N*** that $\tilde{\forall} i \in N(\xi)\ \delta_i = 0$.

Definition II. *The injection* $\varphi: N \to N$ *is said to be potentially surjective one* if it is satisfied following two conditions: for some sequence $\xi \triangleq (1, n_1, n_2, \ldots, n_i,\ \ldots)$ there is a number$C(\xi) > 0$ of this kind *a)* $\tilde{\forall} i \in N(\xi)\ 0 < \delta_i \leq C(\xi)$, *b)* $\exists j \in N(\xi): D_i \cap D_{i+j} = \emptyset$.

Definition III. The injective mapping $\varphi: N \to N$ *is said to be potentially antysurjective one* if following conditions are satisfied:*a)*$\exists\ \xi$- partitio*n* of set N: the sequence $(\delta_i), i \in N(\xi)$defined by the pair $(\xi, \varphi)$is unlimited**,** *b)* $\forall i \in N(\xi) \exists j \in N(\xi): D_i \cap D_{i+j} = \emptyset$.

Definition IV. The injective mapping $\varphi: N \to N$ *is said to be C-finite antysurjective one* if *a)* the sequence $(\delta_i), i \in N(\xi)$, defined by the pair (ξ, φ), is bounded one and *b)* $\exists (C, i_0, N_\varphi: C > 0, i_0 \in N(\xi), N_\varphi \subset N)$: $\tilde{\forall} i > i_0\ N_\varphi \subset D_i, |N_\varphi| \leq C$.

Definition V. The injection injective mapping $\varphi: N \to N$ *is said to be tw-antysurjective one* if *a)* the sequence $(\delta_i), i \in N(\xi)$, is unlimited one, and *b)* $\exists (C, i_0, N_\varphi: C > 0, i_0 \in N(\xi), N_\varphi \subset N)$: $\tilde{\forall} i > i_0\ N_\varphi \subset D_i, |N_\varphi| = C$.

It is obvious that $N_\varphi \cap \varphi(N) = \emptyset$.

Definition VI. The injective mapping $\varphi: N \to N$ *is said to be total antysurjective one* if the $N_\varphi = N \backslash \varphi(N)$ is an infinite set.

3. The examples

Example 1. (G. Galilei's paradox). It is obvious, the mapping $\varphi: N \to N$ with $\varphi(n) \triangleq n^2$ is *total antysurjective* one, that is there exists $N_\varphi: N_\varphi \cap \varphi(N) = \emptyset$ and N_φ is any infinite subset of set ***N.***

Example 2. Let Q_n be the square table-matrix (Table 1).

So we have both the size of matrix Q_n is $\langle \boldsymbol{Q_n} \rangle \triangleq \langle n, n \rangle$ and $q_m^i \triangleq i/m$, $1 \leq i \leq n$, $1 \leq m \leq n$.

Let $Q^+(n)$ be the quantity of various positive rational numbers $q \in Q_n$. It is obvious that $\forall n,\ 1 < n,\ n < Q^+(n) < n^2$. The $Q^+(n)$ is depended essentially on values of a function $\pi(n)$ which defines a quantity of primary numbers p, $p \leq n$. For example,

$$\forall p \in \pi(n)\ Q^+(p) = Q^+(p-1) + 2(p-1).$$

Table 1.

Q_n=

1	2	3	4	5	…	n
1/2	1	3/2	4/2	5/2	…	n/2
1/3	2/3	1	4/3	5/3	…	n/3
1/4	2/4	3/4	1	5/4	…	n/4
1/5	2/5	3/5	4/5	1	…	n/5
…	…	…	…	…	…	…
1/n	2/n	3/n	4/n	5/n	…	1

Let symbol $\left[\frac{n}{m}\right]$ be an integral part of n/m, $n/m \notin N$. Then we have both $\left[\frac{n}{p}\right]^2 > \left(\frac{n}{p}-1\right)^2 = \left(\frac{n}{p}\right)^2 - \frac{2n}{p} + 1$ and $\left[\frac{n}{p}\right]^2 < \left(\frac{n}{p}\right)^2$. By this way we obtain estimation of quantity $Q^+(n)$ of the positive rational numbers q in table Q_n in the following form:

$$n^2(1-\rho(n)-\lambda(n)+\Psi(n))+\pi(n)<Q^+(n)<n^2(1-\rho(n)+\Psi(n)). \qquad (10)$$

Here $\rho(n) \triangleq \sum_{p\in\pi(n)} p^{-2}$, $\lambda(n) \triangleq 2\cdot \sum_{p\in\pi(n)} (pn)^{-1}$. The series $\sum_{p\in\pi(\infty)} p^{-1}$ divergent by Euler (comp. Example 3). Also it is easy to prove, that $\sum_{p\in\pi(\infty)} p^{-2} < 0.5$.
The function $\Psi(n)$ is defined in an inequality (10) with following expression:

$$\Psi(n)=\frac{1}{n^2}\left\{-\sum\left[\frac{n}{p_{i_1}}\right]\left[\frac{n}{p_{i_2}}\right]+\ldots+(-1)^{k+1}\sum\left[\frac{n}{p_{i_1}}\right]\left[\frac{n}{p_{i_2}}\right]\ldots\left[\frac{n}{p_{i_k}}\right]\right\}.$$

The exact value of number $Q^+(n)$ is defined under the formula $Q^+(n)$=1+2$Q_1^+(n)$, here $Q_1^+(n)$ means a quantity of various rational numbers $q > 1$ in matrix Q_n. A number $Q_1^+(n)$ is calculated under the obvious recurrent formula $Q_1^+(n)$ =$Q_1^+(n-1) + \Delta Q_1^+(n)$ and

$$\Delta Q_1^+(n) = n - \frac{n}{p_1} - \frac{n}{p_2} - \cdots - \frac{n}{p_k} + \frac{n}{p_1p_2} + \frac{n}{p_2p_3} + \ldots + \frac{n}{p_{k-1}p_k} + \ldots + +(-1)^k \frac{n}{p_1p_2p_{k-1}p_k},\ n = p_1^{n_1}p_2^{n_1}\ldots p_{k-1}^{n_{k-1}}p_k^{n_k}. \quad (11)$$

There symbols $p_1, p_2, \ldots, p_{k-1}, p_k$ designate in the formula (11) various prime dividers of the number n. Let $Q^+(n) \triangleq \mu(n)n^2$. We shall note some of properties of the function μ: $\boldsymbol{N} \to \boldsymbol{R}$. The function μ not monotone decreases on the set $\boldsymbol{N}$: for all prime numbers p, $p \geq 3$, $\mu(p) = \mu_{max}$, the function μ strictly decreases almost on all set $\pi(n)$ without the second from each pair prime numbers-twins and without the any ones.

If n lays between consecutive prim numbers p_1 and p_2, $p_1 < n < p_2$, almost for all compound n except for degrees of some prime numbers, so we have

$\mu(p_1) > \mu(n) < \mu(p_2)$. Now we illustrate the properties of function μ: $\boldsymbol{N} \to \boldsymbol{R}$ comparative estimations of some values of this function:

$$\mu{:}0{,}629696<\mu(47)<0{,}629697,\ 0{,}62765<\mu(49)<0{,}62766,\ 0{,}627625<\mu(53)<0{,}627626, \text{ that is}$$
$$\mu(47)>\mu(49)=\mu(7^2)>\mu(53);$$

0,610<μ(58)<0,611, 0,623<μ(59)<0,624, 0,611<μ(60)<0,612, 0,624<μ(61)<0,625, 0,619<μ(62)<0,620, 0,618<μ(63)<0,619,

that is μ(58)<μ(59)>μ(60), μ(60)<μ(61)>μ(62) and μ(62)>μ(63);

0,619<μ(79)<0,620, 0,621<μ(83)<0,622, 0,620<μ(103)<0,621, that is

μ(79)<μ(83), but we have μ(83)>μ(103).

Now if we accept a hypothesis limμ(n)≈0,6 for function $\mu: N \to R$ then we have the approximate equality: $|Q^{+}| \approx 0{,}6|N|^{2}$ by means of limiting transition in (11). This equality is consistent with Theorems 2-4 and gives an explanation of Galilee's paradox.

Example 3. Let ($\boldsymbol{A}$)$\triangleq \sum_{n=1}^{\infty}(n)^{-1}$ be harmonic series. Then we have,

$S_m \triangleq \sum_{n=1}^{m} n^{-1} = lnm + C_e + \gamma_k$, $S_k \triangleq \sum_{n=1}^{k} n^{-1} = lnk + C_e + \gamma_k, +\gamma_n \to 0$

and $C_e = 0{,}57721566490 \ldots$ is Euler's constant. Let further, $m > k$, for example. Let $R_{k,m} \triangleq S_m - S_k = ln(m/k) + \gamma_m - \gamma_k$. Hence, the rest r_k of the ($\boldsymbol{A}$) is defined by following equality $r_k = \lim_{m\to\infty} R_{k,m}$. Now we have $\lim_{k\to\infty} r_k = \lim_{k\to\infty} (\lim_{m\to\infty} R_{k,m})$. Here the pair (k, m) is C-pair variables (see (2), so it is possible to accept $m = k + q(k), 0 \le q(k) < C$. Therefore

$$\lim_{k\to\infty} r_k = \lim_{k\to\infty} (\lim_{m\to\infty} R_{k,m}) = \lim_{k\to\infty} \left(ln((k + q(k))/k) + \gamma_{k+p} - \gamma_k \right) = 0.$$

Thus, the rest r_k of harmonious series aspires to zero, and, hence, a harmonious series converges, though, as is well known, its sum is not limited by any finite number. Therefore, a series $\sum_{p\in\pi(\infty)} p^{-1}$ from Example 2 converges also.

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Работа поступила в редакцию 19.09.2016 г.

Принята к публикации 22.09.2016 г.