

Modified Korteweg-de Vries (mK-dV) Equation Describing Dust-ion-acoustic Solitary Waves in an Unmagnetized Dusty Plasma with Trapped Negative Ions

Obaydur Rahman and Md. Masum Haider

Department of Physics, Mawlana Bhashani Science and Technology University, Santosh, Tangail, Bangladesh
Email: armanphy203@gmail.com

Abstract The effects of vortex-like (trapped) negative ion, Maxwellian electron, arbitrarily charged stationary dust, and cold mobile inertial positive ion have been theoretically investigated in the study of nonlinear propagation of dust-ion-acoustic (DIA) solitary waves (SWs) in an unmagnetized dusty electronegative plasma (DENP). The properties of small but finite amplitude DIASWs are studied by employing the reductive perturbation technique (RPT). It has been found that owing to the departure from the Maxwellian negative ion distribution to a vortex-like one, the dynamics of such DIASWs is governed by a modified Korteweg-de Vries (mK-dV) equation which admits solitary wave solution under certain conditions. The basic properties (speed, amplitude, width, etc.) of such DIASWs are found to be significantly modified by the effects of trapped negative ions and arbitrarily charged stationary dust particles. The results should be useful for understanding the nonlinear propagation of DIASWs in laboratory and space plasmas (e.g., Earth's magnetosphere, auroral region, heliospheric environments etc.).

Keywords: Dust-ion-acoustic waves, solitary waves, electronegative dusty plasma, trapped negative ions.

1 Introduction

Nowadays, electronegative plasmas i.e, plasmas with a significant amount of negative ions [1], [2], [3], [4], which are highly chemically reactive have attracted a great deal of interest [5], [6], [7], [8], [9], [10], [11], [12] because of their wide technological applications, e.g., semiconductor materials processing [13], synthesis of nanomaterials [14], micro-electronic and photo-electronic industries [15] and they also play vital role in many astrophysical plasmas e.g., D region of the ionosphere, the mesosphere, cometary tails, planetary rings, interplanetary space, interstellar medium, the evolution of the solar system from its solar nebula to its present form, etc. These electronegative plasmas are also found in the laboratory and plasmas technology such as low temperature physics (radio frequency), plasma discharge [16] and dusty crystal [17],[18].

However, the electronegative plasmas are not pure in general because it is contaminated by the dust particles e.g., solid impurities. Such impurities are not practically neutral but are charged, which means this dust particles may be either positively or negatively charged [19] by absorbing electrons, positive as well as negative ions [10], [20], [21], [22], [23], [24], [25], [26]. For this reason the electronegative plasmas are sometimes called dirty or dusty electronegative plasma (DENP)[10], [20], [21], [22], [23], [24], [25], [26]. Therefore, a DENP [27], [28] is roughly defined as a normal electron-ion plasma with an additional charged component of micron/sub-micron sized dust particles which increases the complexity of the plasma system. For this reason a dusty plasma is also referred to as a "complex plasma" [29].

Cousens *et al.* [30] have studied the nonlinear features of dust-acoustic (DA) waves in a three component plasma consisting of strongly coupled dust particle and Maxwellian electrons and ions. El-Labany *et al.* [31] have studied the nonlinear features of the DA waves which are the low frequency electrostatic waves in a dusty plasma system with mobile dust, where restoring force is provided by the thermal pressure of electrons and ions, and inertia is provided by the mass of dust. The nonlinear features of the DIA waves in DENP have been analyzed by a number of authors [32], [33], [34], they have predicted that negative ions in such electronegative plasmas obey Maxwellian distribution. Ghim and Hershkowitz have conclusively

verified this prediction experimentally [35]. Kourakis *et al.* [36], Anowar *et al.* [37] and Moslem *et al.* [38] have studied the nonlinear features of the DIA which are also the low frequency electrostatic waves in a dusty plasma system with static dust in which restoring force is provided by the thermal pressure of electrons and inertia is provided by the mass of ion. The phase speed of such DIA waves is much smaller than the electron thermal speed but much larger than ion thermal speed, i.e, $v_{Ti} \ll v_p \ll v_{Te}$.

Recently, motivated by the laboratory experiment [35] Mamun *et al.* [20], [39], [40] have considered a dusty electronegative plasma containing Maxwellian electrons, Maxwellian negative ions, cold mobile positive ions, and negatively charged stationary dust, and have examined the possibility for the formation of IA and DIA solitary waves and double layers in a DENP. In the work of Mamun *et al.* [39], they have considered unmagnetized dusty plasma consisting the electrons and ions which are isothermal. Some numerical investigations on linear and nonlinear DA waves show a significant amount of ions trapping in the wave potential, indicating that there is a departure from Boltzmann ion distribution and one encounter vortex-like ion distribution in phase space [41], [42], [43], [44].

Very recently Du *et al.* [45] have studied experimentally a new bursting $m=1/n=1$ instability (m, n : poloidal and toroidal mode numbers) with rapid frequency chirping down in a helical plasma with intense perpendicular neutral beam which is destabilized by helically trapped energetic ions in a Large Helicon Device (LHD). It is also well known from [46], [47], [48], [49] that the electron and ion distribution functions can be significantly modified in the presence of large-amplitude waves that are excited by the two-stream instability [50]. Therefore, due to wave-particle interactions, the negative ions may not be isothermal (Maxwellian), but they follow other distributions such as non-thermal [51] Kappa [52], or trapped distribution [46], [47], [53], which plays an important role in modifying basic features of nonlinear electrostatic structures [54], [55], [56], [57], [58] in many space and laboratory plasmas. Recently, Rahman and Mamun [59] have studied the nonlinear propagation of DIA solitary waves in an unmagnetized three components dusty plasma consisting of trapped electrons, cold mobile ions, and arbitrarily charged stationary dust. They have not considered negative ion which plays very important role in dusty plasma system. Therefore, in our present work, we have considered a four component DENP containing (i) positively charged mobile ions, (ii) non-isothermal negatively charged ion which may not have sufficient energy to overcome the potential, as a result it is trapped and following vortex-like distribution, (iii) Maxwellian electrons and (iv) arbitrarily charged stationary dust. We have studied the basic properties (phase speed, amplitude, width) of the DIA solitary waves.

The manuscript is organized as follows. The governing equations for a small but finite amplitude DIA SWs in an unmagnetized collisionless dusty plasma with vortex-like negative ions are presented in section 2. The well known mK-dV equation is derived by employing the reductive perturbation method in section 3. The solitary wave solution of this mK-dV equation is derived in section 4. The numerical analysis of this work is presented in section 5. Finally, a brief summary of our results is discussed in section 6.

2 Governing Equations

A collisionless unmagnetized four components DENP containing Maxwellian electrons, trapped negative ions, cold mobile positive ions, and arbitrarily charged stationary dust is considered. Thus, at equilibrium, $n_{p0} - n_{e0} - n_{n0} + jZ_d n_{d0} = 0$, where n_{p0} , n_{e0} , n_{n0} , and n_{d0} are, respectively, positive ion, electron, negative ion, and dust number density at equilibrium. Z_d is the number of electrons residing onto the surface of a stationary dust, and $j = +1$ (-1) for positively (negatively) charged dust. We are interested in examining the nonlinear propagation of a low phase speed (in comparison with electron and negative ion thermal speeds), long wavelength (in comparison with Debye length) perturbation mode on the time scale of the DIA waves. The time scale of the DIA waves is much faster than the dust plasma period so that dust can be assumed stationary. The nonlinear dynamics of the low frequency electrostatic perturbation mode in such a DENP is described by

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p u_p) = 0 \quad (1)$$

$$\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu_e n_e - n_p + \mu_n n_n - j\mu_d \quad (3)$$

where n_p (n_n) is the positive (negative) ion number density normalized by n_{p0} (n_{n0}), u_p is the positive ion fluid speed normalized by the positive ion-acoustic speed $C_p = (k_B T_e / m_p)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_e / e$, $\mu_n = (1 - \mu_e + j\mu_d)$, $\mu_d = Z_d n_{d0} / n_{p0}$, $\mu_e = n_{e0} / n_{p0}$, and m_p is the mass of positive ion. The time variable t is normalized by the positive ion plasma period ω_{pp}^{-1} and the space variable is normalized by λ_D .

The inertialess negative ion have a non-Maxwellians characteristics, described by a vortex-like velocity distribution which leads to a negative ion number density of the form [46], [47], [53], [55], [58], [60],

$$n_n = 1 + \alpha\phi - \frac{4(1 - \beta_n)}{3\sqrt{\pi}} \alpha^{3/2} \phi^{3/2} + \frac{1}{2} \alpha^2 \phi^2 \quad (4)$$

where $\alpha = Z_n T_e / T_n$, T_e (T_n) is the electron (negative ion) temperature. β_n is a parameter which determines the number of trapped negative ions [61], [62] and it is defined as the ratio of the free negative ion temperature T_{if} to the trapped negative ion temperature T_{it} . We note that $\beta_n = 1$ ($\beta_n = 0$) represents a Maxwellian (flat-topped) negative ion distribution, whereas $\beta_n < 0$ represents a vortex-like excavated trapped negative ion distribution. We are interested in $\beta_n < 0$. On the other hand, the inertialess electron have a Maxwellian characteristics described by a Maxwellians Boltzmann velocity distribution which leads to a electron number density of the form

$$n_e = e^\phi = 1 + \phi + \frac{1}{2} \phi^2 \quad (5)$$

3 Derivation of Modified K-dV Equation

To investigate the behavior of the small, but finite amplitude electrostatic DIAWs in the DENP under consideration, we employ the standard reductive perturbation method (RPM) [63]. The RPM is a very powerful way of deriving simplified models describing nonlinear wave propagation and interaction and was first introduced around 50 years ago by Washimi and Taniuti [63] to the study of ion-acoustic waves. Later, many world renowned scientists [19], [20], [21], [22], [46], [47], [53], [63], [64], [65], [66], [67], [68] worked using this method. The most natural approach, which is also the most affordable technically, is to use a perturbation method. Typically the first order is the linear approximation and then the slowly varying envelope approximation. It can thus be qualified as “weakly nonlinear”, in accordance with the terminology of mathematicians. However, many phenomena, although they are weakly nonlinear in the sense that the frequency and so on still have a meaning, diverge in their results from the linear situation. This is due to the fact that small modifications of an evolution law can yield large variations, if the evolution time is long enough. In this frame, the multiscale analysis is fully justified: it is a perturbative approach, whose first order is a linear approximation, that allows us to write small corrections valid for the same evolution time as in the linear approximation, but their cumulative effect lasts for a very long evolution. This method rescales both space and time in the governing equations of the system to introduce space and time variables, which are appropriate for the description of long-wavelength phenomena. According to this method, the independent variables are stretched as [46], [47], [53], [63]

$$\left. \begin{aligned} \xi &= \epsilon^{1/4} (x - v_p t) \\ \tau &= \epsilon^{3/4} t \end{aligned} \right\} \quad (6)$$

where ϵ is a small dimensionless expansion parameter measuring the strength of nonlinearity and v_p is the unknown phase speed (to be determined later) normalized by the positive ion-acoustic speed (C_p). Furthermore, we can expand the physical quantities appearing in the basic equations (1-5) which are expanded as power series in ϵ about their equilibrium values [53], [64], [65], viz.

$$\left. \begin{aligned} n_p &= 1 + \epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + \dots \\ u_p &= 0 + \epsilon u_p^{(1)} + \epsilon^2 u_p^{(2)} + \dots \\ \phi &= 0 + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots \end{aligned} \right\} \quad (7)$$

Now, using Eqs. (4)-(7) into Eqs. (1)-(3) one can obtain the first order continuity equation, momentum equation and Poisson's equation which, after simplification, yield

$$n_p^{(1)} = \frac{\phi^{(1)}}{v_p^2} \quad (8)$$

$$u_p^{(1)} = \frac{\phi^{(1)}}{v_p} \quad (9)$$

$$v_p = \frac{1}{\sqrt{\mu_e + \alpha\mu_n}} \quad (10)$$

Equation (10) represents the linear dispersion relation for DIA waves. Putting the values of Eqs. (4)-(10) into Eqs. (1)-(3), we obtain the next higher order equations,

$$\frac{\partial n_p^{(1)}}{\partial \tau} - v_p \frac{\partial n_p^{(2)}}{\partial \xi} + \frac{\partial u_p^{(2)}}{\partial \xi} = 0 \quad (11)$$

$$\frac{\partial u_p^{(1)}}{\partial \tau} - v_p \frac{\partial u_p^{(2)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} = 0 \quad (12)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \frac{\phi^{(2)}}{v_p^2} - \frac{4(1 - \beta_n)}{3\sqrt{\pi}} \mu_n \alpha^{3/2} \left[\phi^{(1)} \right]^{\frac{3}{2}} - n_p^{(2)} \quad (13)$$

Now, Solving this system with the help of Eqs. (8)-(9) we can easily eliminate the second order perturbed quantities $n^{(2)}$, $u^{(2)}$ and $\phi^{(2)}$, and we finally obtain the mK-dV equations as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \sqrt{\phi^{(1)}} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \quad (14)$$

where,

$$A = \frac{(1 - \beta_n) \mu_n v_p^3}{\sqrt{\pi}} \alpha^{\frac{3}{2}} \quad (15)$$

$$B = \frac{v_p^3}{2}. \quad (16)$$

Equation (14) is a mK-dV equation, exhibiting a weakly nonlinearity, smaller width, and larger propagation velocity of the nonlinear wave.

4 SW Solution of mK-dV Equation

The stationary solution of this mK-dV equation can be obtained by transforming the independent variables ξ and τ to $X = \xi - u_0 \tau$, $\tau = \tau$, where u_0 is a constant velocity.

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial \tau} - u_0 \frac{\partial}{\partial X} \quad (17)$$

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial X} \quad (18)$$

Now substituting Eqs. (17)-(18) into Eq. (14) we get

$$\frac{\partial \phi^{(1)}}{\partial \tau} - u_0 \frac{\partial \phi^{(1)}}{\partial X} + A \sqrt{\phi^{(1)}} \frac{\partial \phi^{(1)}}{\partial X} + B \frac{\partial^3 \phi^{(1)}}{\partial X^3} = 0 \quad (19)$$

For steady state condition ($\partial \phi^{(1)} / \partial \tau \rightarrow 0$) Eq. (19) reduces to

$$-u_0 \phi^{(1)} + \frac{2}{3} A \phi^{(1)3/2} + B \frac{d^2 \phi^{(1)}}{dX^2} = C_1 \quad (20)$$

where C_1 is an integration constant. Now under appropriate boundary conditions, viz. $\phi^{(1)} \rightarrow 0$ and $d^2\phi^{(1)}/dX^2 \rightarrow 0$ at $X \rightarrow \pm\infty$, one can find C_1 as $C_1 = 0$. So, the above equation becomes

$$\frac{d^2\phi^{(1)}}{dX^2} = \frac{1}{B} \left(u_0\phi^{(1)} - \frac{2}{3}A\phi^{(1)3/2} \right) \tag{21}$$

Now, multiplying both sides of the above equation by $d\phi^{(1)}/dX$, one gets

$$\Rightarrow \left(\frac{d\phi^{(1)}}{dX} \right)^2 = \frac{u_0}{B} \phi^{(1)2} \left(1 - \frac{8A}{15u_0} \phi^{(1)1/2} \right) + C_2 \tag{22}$$

where C_2 is an another integration constant. Under the boundary conditions, $\phi^{(1)} \rightarrow 0$ and $d\phi^{(1)}/dX \rightarrow 0$ at $X \rightarrow \pm\infty$, one can express C_2 as $C_2 = 0$. So, the above equation becomes

$$\frac{d\phi^{(1)}}{dX} = \phi^{(1)} \sqrt{\frac{u_0}{B}} \sqrt{1 - \frac{8A}{15u_0} \phi^{(1)1/2}} \tag{23}$$

We now let

$$\begin{aligned} \sqrt{1 - \frac{8A}{15u_0} \phi^{(1)1/2}} &= Y \\ \Rightarrow \phi^{(1)} &= \left(\frac{15u_0}{8A} \right)^2 (1 - Y^2)^2 \end{aligned} \tag{24}$$

$$\Rightarrow \frac{d\phi^{(1)}}{dX} = -4 \left(\frac{15u_0}{8A} \right)^2 Y(1 - Y^2) \frac{dY}{dX} \tag{25}$$

Now, substituting Eqs. (24)-(25) into Eq. (23), one obtains

$$Y = \frac{(1 - e^{X\sqrt{\frac{u_0}{4B}}})}{1 + e^{X\sqrt{\frac{u_0}{4B}}}} \tag{26}$$

Now, substituting Eq. (26) into Eq. (24), one obtains

$$\phi^{(1)} = \phi_m \operatorname{sech}^4 \left[\frac{(\xi - u_0\tau)}{\Delta} \right] \tag{27}$$

Eq. (27) is the stationary SWs solution of the mK-dV equation, where, $\phi_m = \left(\frac{15u_0}{8A} \right)^2$ and $\Delta = \sqrt{\frac{16B}{u_0}}$ are the amplitude and the width of the SWs respectively.

5 Numerical Analysis

Figures 1 and 2 show the variation of the phase speed v_p of SWs with μ_d . From figure 1, it has been found that the phase speed of the SWs becomes slower if the density of positively charged dust particles is increased in a dusty plasma system. On the other hand the phase speed of this SWs becomes faster when the density of negatively charged dust particles is increased in that system which is shown in figure 2. Figures 3-7 show the variation of the amplitude of solitary wave potential ϕ_m with α , β_n , μ_d and μ_e , respectively. Figure 3 indicates that the SWs become flat shape when the thermal pressure of electrons is increased and the polarity of dust particles have little effect on the SW. But when increasing the thermal pressure of negative ions the solitary structure becomes more spike. Figure 4 shows that the amplitude of SWs increases for both positively as well as negatively charged dust particles when increasing the value of trapping parameter β_n but the amplitude is slightly greater for positively charged dust than for negatively charged dust. Figure 5 shows that, in the case of positively charged dust particles the potential of the SWs increases linearly when increasing the value of positive dust particles density. On the other hand,

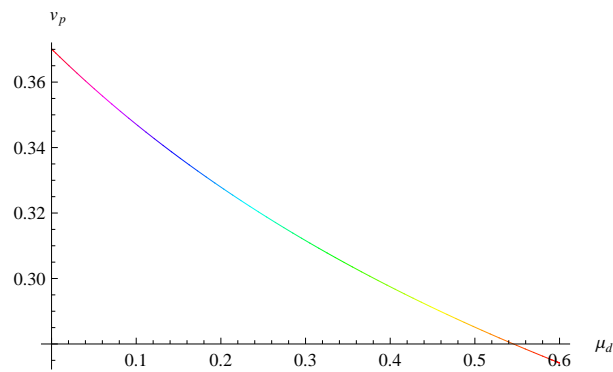


Figure 1. (Color online) The variation of the phase speed (v_p) of SWs with μ_d for $j = +1$, $\mu_e = 0.3$ and $\alpha = 10$.

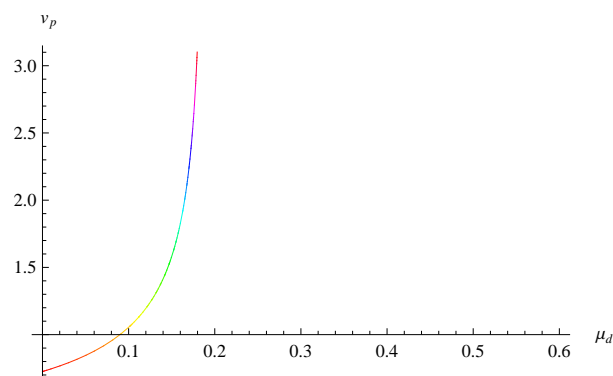


Figure 2. (Color online) The variation of the phase speed (v_p) of SWs with μ_d for $j = -1$, $\mu_e = 0.3$ and $\alpha = 10$.

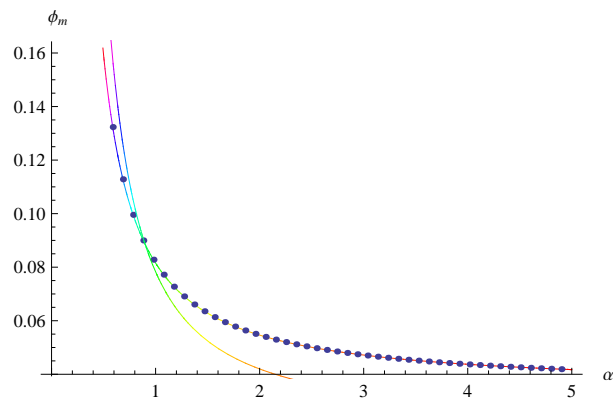


Figure 3. (Color online) The variation of the amplitude (ϕ_m) of SWs with α for $j = 1$ ($j = -1$) dotted curve (solid curve), $u_0 = 0.1$, $\mu_e = 0.3$, $\mu_d = 0.2$, and $\beta_n = -0.7$.

the SWs potential decreases linearly with dust particle density μ_d when dust particles are negatively charged which is shown in figure 6. From figure 7, it has been seen that the amplitude decreases with the increase of the value of μ_e no matter dust particle is positive or negative but the amplitude is slightly larger for positively charged dust than for negatively charged dust. Figure 8 indicates that the width increases with the increase of the value of the density of electrons no matter dust particles are positive or negative but the width is slightly lower when the dust particles are positively charged. That means increasing the electron number density makes the solitary structure more flat shape. Figures 9 and 10 show the variation of the width Δ of solitary wave with μ_d for $j = 1(-1)$ figure 9 (figure 10). From this

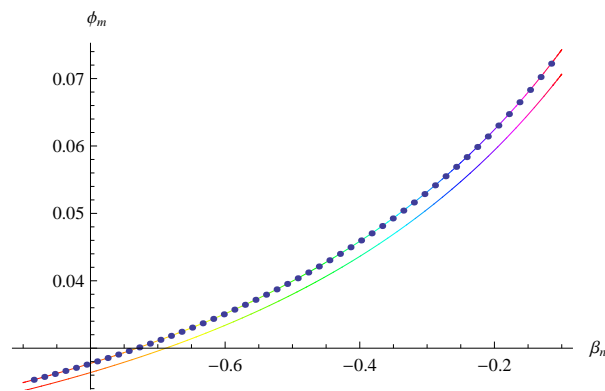


Figure 4. (Color online) The variation of the amplitude (ϕ_m) of SWs with β_n for $j = 1$ ($j = -1$) dotted curve (solid curve), $u_0 = 0.1$, $\mu_e = 0.3$, $\mu_d = 0.02$, and $\alpha = 10$.

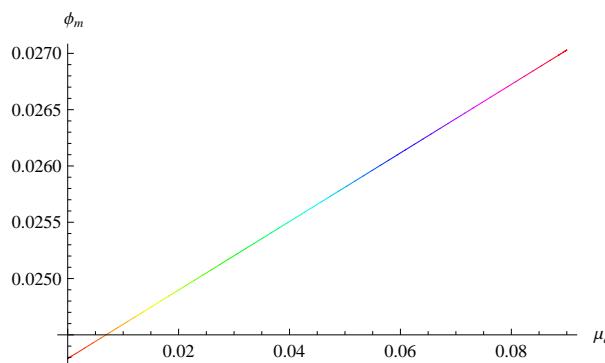


Figure 5. (Color online) The variation of the amplitude (ϕ_m) of SWs with μ_d for $j = +1$, $u_0 = 0.1$, $\mu_e = 0.3$, $\alpha = 10$, and $\beta_n = -0.9$.

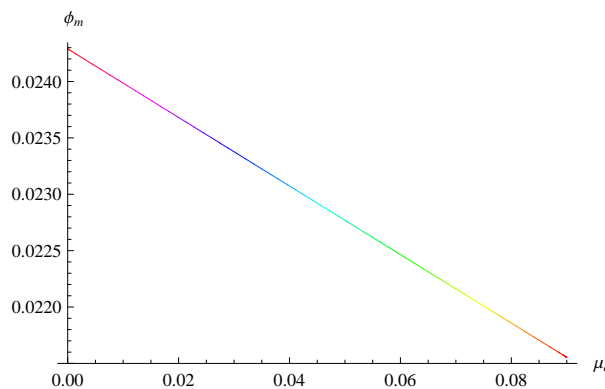


Figure 6. (Color online) The variation of the amplitude (ϕ_m) of SWs with μ_d for $j = -1$, $u_0 = 0.1$, $\mu_e = 0.3$, $\alpha = 10$, and $\beta_n = -0.9$.

figure it is clear that the width decreases with the increase of the number density of positively charged dust but it increases for the number density of negatively charged dust.

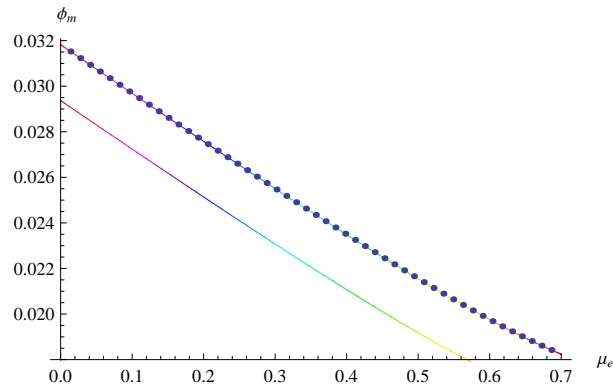


Figure 7. (Color online) The variation of the amplitude (ϕ_m) of SWs with μ_e for $j = 1$ ($j = -1$) dotted curve (solid curve), $u_0 = 0.1$, $\mu_d = 0.04$, $\alpha = 10$, and $\beta_n = -0.9$.

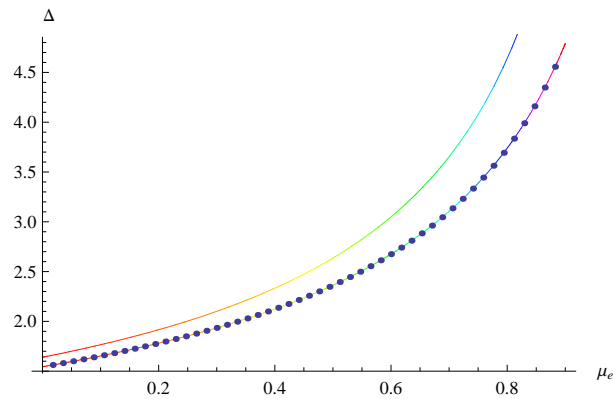


Figure 8. (Color online) The variation of the width (Δ) of SWs with μ_e for $u_0 = 0.1$, $j = 1$ ($j = -1$) dotted curve (solid curve), $\alpha = 10$, and $\mu_d = 0.04$.

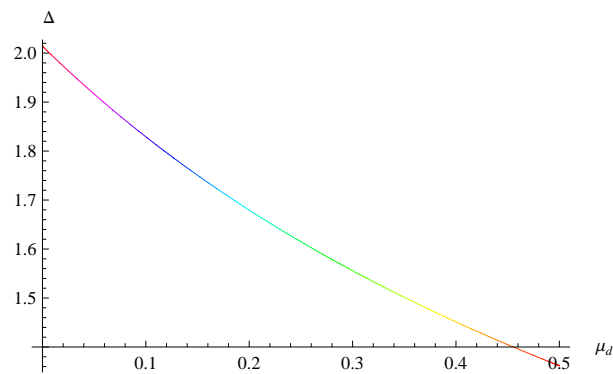


Figure 9. (Color online) The variation of the width (Δ) of SWs with μ_d for $u_0 = 0.1$, $j = 1$, $\alpha = 10$, and $\mu_e = 0.3$.

6 Discussion

We have considered an unmagnetized four components DENP system consisting of electrons which obey Maxwellian distribution, negative ions satisfying vortex-like distribution, cold mobile positive ions, and arbitrarily charged stationary dust, and have studied the DIASWs by deriving mK-dV equation. It has been observed that the effect of dissipation in this case is negligible in comparison with those of the nonlinearity and dispersion. The basic features (phase speed, amplitude, and width) of such DIASWs are

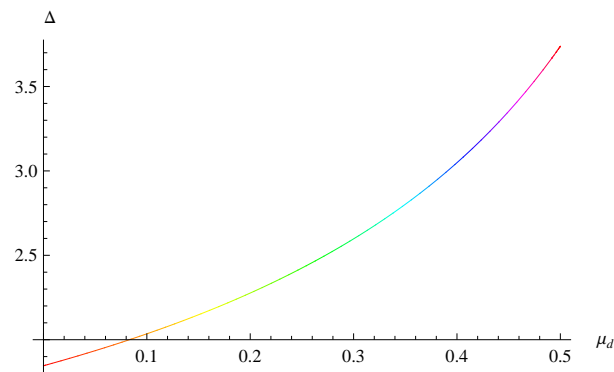


Figure 10. (Color online) The variation of the width (Δ) of SWs with μ_d for $u_0 = 0.1$, $j = -1$, $\alpha = 10$, and $\mu_e = 0.2$.

significantly modified by the presence of trapped negative ions, and arbitrarily charged stationary dust particles. The results which have been obtained from this investigation can be summarized as follows:

1. Dusty plasma system, whose constituents are cold mobile positive ions, Maxwellian electrons, trapped negative ions, and arbitrarily charged stationary dust of different constant temperatures, is found to support SWs associated with the non-linear DIA waves.
2. In the presence of the trapped negative ions distribution, the dynamics of weakly dispersive non-linear DIA waves are governed by the mK-dV equation instead of K-dV equation. Thus, the potential polarity of the DIA solitary waves in our dusty plasma is different from the usual IA solitary waves in an electron-ion plasma.
3. The profile of this wave can be represented in the form of $\text{sech}^4(z/\Delta)$, instead of $\text{sech}^2(z/\Delta)$ which is the stationary solution of the standard K-dV equation for isothermal ions.
4. The dusty plasma system under consideration supports the DIA solitary waves that are associated with positive potential only. The fixed polarity of the potential structures is due to the effect of trapped negative ion distribution.
5. It has been found that the effect of trapped negative ion gives rise to a stronger nonlinearity (i.e., we have solitary structures with larger amplitude, smaller width, and higher propagation velocity).
6. From these above figures 1-2, it has been also found that the phase speed of the solitary wave increases with the increase of the values of μ_d if dust particles are negative, which completely agree with the laboratory experiments [63], [69] on the other hand phase speed decreases with μ_d if dust particles are positive. To verify this prediction, we propose to perform an experiment on the variation of the phase speed with the positive dust number density by using an experimental setup where positive dust can be produced [70].
7. The potential of the solitary wave increases with the increase of the values of α , β_n , and μ_d (if dust particles are positive) but it decreases with the increase of the values of μ_e and μ_d (if dust particles are negative). On the other hand, the width of the solitary waves increases with the increase of the values of μ_e and μ_d (if dust particles are positive) but it decreases with the values of μ_d (if dust particles are positive).

In the present work we have studied nonlinear wave with mK-dV equation. To observe the higher order nonlinearity one can derive modified Zakharov Kuznetsov (mZK) equation [71], Gardner equation [72], [73] which is combined into KdV-mKdV equations, modified Gardner equation [74], Kadomtsov Petviashvilli (KP) equation [75], Gardner KP (GKP) equation [76] and nonlinear Schrödinger equation [77]. We have intuit the model based on the study of different astrophysical plasma systems like localized electrostatic disturbances in space (viz. auroral plasma, Saturn's E- and F-rings, etc.) [27], [78], [79], [80]. However, our present results may be useful for understanding the space [27], [78], [79], [80] and laboratory [27], [69], [81] dusty plasmas.

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