Reduced Quantum General Relativity in Higher Dimensions

Lukasz Andrzej Glinka[1*] and Patrick Linker[2]

[1]Independent Science Editor & Author, Non-fiction & Philosophy Writer, Poland;
Member, American Association of International Researchers, USA
[2]M.Sc. Student, Fachgebiet Kontinuumsmechanik, Technische Universität Darmstadt, 64287 Darmstadt, Germany
Email: laglinka@gmail.com

Abstract Quantum General Relativity of a higher dimensional Riemannian manifold being an embedded space in a space-time being a Lorentzian manifold is investigated through a technique of differential topology. Consequently, in the reduced model of quantum geometrodynamics, the Wheeler-DeWitt equation is replaced through a first order functional quantum evolution and a supplementary eigenequation for a scalar curvature of an embedded space. The phenomenological approach, in the framework of objective quantum gravity and global one-dimensional conjecture, is applied in order to make the standard formalism of quantum mechanics applicable to quantum gravity, beyond a Feynman path integral and a Wilson loop techniques. It leads to the wave function which refers to quantum tunnelling through a manifestly exponential form and is determined through energy density of matter fields and a cosmological constant, and physical interpretation of quantum gravity through the Tomonaga-Schwinger equation and the Dirac interaction picture of relativistic quantum mechanics. The resulting model of quantum gravity creates the opportunity of potentially new theoretical and phenomenological applications for astrophysics, cosmology, and physics.

Keywords: quantum gravity, quantum geometrodynamics, Wheeler-DeWitt equation, phenomenological approach, objective quantum gravity, global one-dimensional conjecture, differential topology, quantum tunnelling, Tomonaga-Schwinger equation

1 Introduction

Quantum geometrodynamics, also known as quantum general relativity or canonical quantum gravity, is the pioneering formulation of the quantum theory of a gravitational field, for which the classical theory is the General Theory of Relativity, Cf. e.g. the Refs. [1,2], having straightforward applications in astrophysics and cosmology, Cf. e.g. the Refs. [3,4,5]. For briefness, let us recall only that the foundational standpoint of quantum geometrodynamics is the Dirac canonical primary quantization of constrained systems, applied to the Einstein-Hilbert action supplemented through the York-Gibbons-Hawking boundary action for a space-time metric splitted through the Arnowitt-Deser-Misner decomposition and solving the Einstein field equations of the General Theory of Relativity.

The crucial problem of quantum geometrodynamics is solving the Wheeler-DeWitt equation, a specific variant of the Schrödinger wave equation of quantum mechanics, which arises as the quantized Hamiltonian constraint and whose any wave function is believed to be a physical state of quantum gravity. The dynamical object of this quantum theory is a geometry of an embedded space, and all physically reasonable geometries are points in the Wheeler superspace, the configuration space of the General Theory of Relativity, which is equipped with the DeWitt supermetric. The most popular strategies of canonical quantum gravity include applications of a Feynman path integral and a Wilson loop, Cf. e.g. the Refs. [6,7,8,9] and references therein, and mostly give rise to various technical difficulties and interpretation obstacles which effectively block development of quantum gravity towards the status of an experimentally verifiable theory consistent with phenomenology of previously known effects, particularly astrophysics and physics at high and ultra-high energies.

Because, in the context of known experimental and observational results, both these standard theoretical frameworks are actually non-physical when applied in the context of the complicated structure of the Wheeler superspace, one can conclude that the physically relevant model of quantum gravity would arise from the correctly done phenomenological approach rather than a purely technical idea. Experienced practitioners of theoretical physics know well that a correct phenomenological approach is not an arbitrary choice, but is the most optimized compliance between a new argument and already known ones who received a multiple positive empirical verification through either experiment or observation. Unfortunately, even on the level of their origins both a Feynman path integral and a Wilson loop techniques remain at most mathematical ideas and are unable to produce empirically verifiable theoretical results. The prospective alternative to these basic mathematical approaches to quantum gravity is both clear and unambiguous physical context of a quantum wave function, actually this phenomenological standpoint has already produced the objective quantum gravity and the global one-dimensional conjecture, Cf. the Ref. [10,11,12,13,14,15,16], which formulate quantum gravity as a Cauchy problem and were successfully applied to receive the theoretical results having a really good compliance with the foundational approach of modern particle physics and observational results of modern cosmology.

In this brief article, we shall provide the route to quantum gravity which immediately gives a significant reduction of quantum geometrodynamics. As the result, the Wheeler-DeWitt equation is replaced through the equivalent system of two equations: a first order quantum evolution for a wave function and a supplementary eigenequation for a scalar curvature of an embedded space. Our approach makes use of a certain special argument of differential topology, which in our opinion modifies the concept of cohomology by the purposes of theoretical physics. Furthermore, in considerations of this article, we apply the previously formulated phenomenological approach in order to construct the wave function which is manifestly beyond either a Feynman path integral or a Wilson loop techniques, and through an exponential form has both clear and unambiguous physical interpretation related to the quantum tunneling. Through the well-defined phenomenology, the received model of Quantum General Relativity displays a prospective nature for the point of view of astrophysics, cosmology, and physics.

2 Wheeler-DeWitt Equation

Let us consider a $(1+D)$-dimensional space-time being a Lorentzian manifold characterized through $(1+D)\times(1+D)$ metric tensor $g_{\mu\nu}$ of signature $(1,D)$, the Ricci curvature tensor ${}^{(1+D)}R_{\mu\nu}$ and the Ricci scalar curvature ${}^{(1+D)}R = g^{\mu\nu}\ {}^{(1+D)}R_{\mu\nu}$, where a Greek index runs from 0 to D. Let a D-dimensional embedded space being a Riemannian manifold be characterized through $D\times D$ metric tensor h_{ij}, the Ricci curvature tensor ${}^{(D)}R_{ij}$ and the Ricci scalar curvature ${}^{(D)}R = h^{ij}\ {}^{(1+D)}R_{ij}$, where a Latin index runs from 1 to D. The foundational differential geometry formulas for 4-dimensional space-time curvatures remain unchanged in higher dimensions until application of the identity $g^{\mu\nu}g_{\mu\nu} = D+1$. Making use of the Einstein tensor ${}^{(1+D)}G_{\mu\nu} = {}^{(1+D)}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\ {}^{(1+D)}R$, in presence of a cosmological constant Λ_D and matter fields described through a stress-energy tensor $T_{\mu\nu}$, the field equations are ${}^{(1+D)}G_{\mu\nu} + \Lambda_D g_{\mu\nu} = \kappa_D T_{\mu\nu}$ with $\kappa_D = \kappa\ell_P^{D-3}$, where $\kappa = \dfrac{8\pi G}{c^4}$ is the Einstein constant and $\ell_P = \sqrt{\dfrac{\hbar G}{c^3}}$ is the Planck length.

2.1 Vacuum Gravitational Field

In quantum geometrodynamics, a wave function $|\Psi[h_{ij}]\rangle$ is considered as a physical state of quantum gravity. For a vacuum gravitational field, the Wheeler-DeWitt equation has the following form

$$\left(G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \ell_P^2\sqrt{h}\ {}^{(D)}R\right)|\Psi[h_{ij}]\rangle = 0, \tag{1}$$

where $h = \det h_{ij}$ and $G_{ijkl} = \dfrac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$ is the DeWitt supermetric on the Wheeler superspace. We shall consider the special situation

$$\ell_P^2\,{}^{(D)}R\,|\Psi[h_{ij}]\rangle = \delta^2\,|\Psi[h_{ij}]\rangle\,, \tag{2}$$

with the following definition of the first variation of a wave function

$$\delta\,|\Psi[h_{ij}]\rangle = \epsilon_{ij}\frac{\delta}{\delta h_{ij}}\,|\Psi[h_{ij}]\rangle\,, \tag{3}$$

where ϵ_{ij} are tensor coefficients, or equivalently

$$\delta\left|\Psi[h_{ij}]\right\rangle = 2i\kappa_D\ell_P\epsilon_{ij}\pi^{ij}\left|\Psi[h_{ij}]\right\rangle, \tag{4}$$

where $\pi^{ij} = -i\dfrac{1}{2\kappa_D\ell_P}\dfrac{\delta}{\delta h_{ij}} = -i\dfrac{\hbar c}{16\pi\ell_P^D}\dfrac{\delta}{\delta h_{ij}}$ is the canonically conjugate momentum operator in the Wheeler-Schrödinger representation. Certainly, the Eq. (2) can be discussed in the general framework of differential topology, but, by the purposes of this article, here we shall omit this aspect. Remarkably, the Eq. (2) is correct from the formal point of view, its both sides are of second order in differential quantities and have no physical dimensionality, and, moreover, this equation links geometry and topology of an embedded space.

Applying this technique, one can calculate the second variation

$$\delta^2\left|\Psi[h_{ij}]\right\rangle = \ell_P^2\,{}^{(D)}\!R\left|\Psi[h_{ij}]\right\rangle = \delta\epsilon_{ij}\frac{\delta}{\delta h_{ij}}\left|\Psi[h_{ij}]\right\rangle + \epsilon_{ij}\epsilon_{kl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}\left|\Psi[h_{ij}]\right\rangle, \tag{5}$$

or, equivalently,

$$\delta^2\left|\Psi[h_{ij}]\right\rangle = \ell_P^2\,{}^{(D)}\!R\left|\Psi[h_{ij}]\right\rangle = 2i\kappa_D\ell_P\epsilon_{ij}\pi^{ij}\left|\Psi[h_{ij}]\right\rangle - 4\kappa_D^2\ell_P^2\epsilon_{ij}\epsilon_{kl}\pi^{ij}\pi^{kl}\left|\Psi[h_{ij}]\right\rangle. \tag{6}$$

Whenever the relation (5) is rewritten in the form of a wave equation

$$\left(-\sqrt{h}\epsilon_{ij}\epsilon_{kl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \sqrt{h}\delta\epsilon_{ij}\frac{\delta}{\delta h_{ij}} + \ell_P^2\sqrt{h}\;{}^{(D)}\!R\right)\left|\Psi[h_{ij}]\right\rangle = 0, \tag{7}$$

then it coincides with the Wheeler-DeWitt equation if

$$\delta\epsilon_{ij}\frac{\delta}{\delta h_{ij}}\left|\Psi[h_{ij}]\right\rangle = 0, \tag{8}$$

and gives the orthogonality condition $\delta\epsilon_{ij}\pi^{ij}\left|\Psi[h_{ij}]\right\rangle = 0$ and leads to the conclusion ${}^{(D)}\!R\left|\Psi[h_{ij}]\right\rangle = 0$ suggesting that an embedded space is a Ricci-flat space, and is consistent with the standard cohomology. Moreover, this compatibility also requires $-\sqrt{h}\epsilon_{ij}\epsilon_{kl} = G_{ijkl}$ what gives the equation

$$\epsilon_{ij}\epsilon_{kl} = \frac{1}{2h}\left(h_{ij}h_{kl} - h_{ik}h_{jl} - h_{il}h_{jk}\right), \tag{9}$$

which, through contractions with space metric, gives the results

$$\epsilon_{ij} = \sqrt{\frac{D-2}{2D}}\frac{h_{ij}}{\sqrt{h}}, \tag{10}$$

$$G_{ijkl} = -\frac{D-2}{2D}\frac{h_{ij}h_{kl}}{\sqrt{h}} = -\frac{D-2}{2}\frac{h_{i(k}h_{jl)}}{\sqrt{h}}, \tag{11}$$

where the brackets refer to symmetrization. Calculating the first variation

$$\delta\epsilon_{ij} = \epsilon_{kl}\frac{\delta}{\delta h_{kl}}\epsilon_{ij}, \tag{12}$$

and making slightly tedious calculations with a help of the Jacobi formula $\delta h = hh^{ij}\delta h_{ij}$, one obtains

$$\delta\epsilon_{ij} = -\frac{(D-2)^2}{4D}\frac{h_{ij}}{h}, \tag{13}$$

$$\delta G_{ijkl} = 2\sqrt{\frac{2D}{D-2}}\frac{D-4}{(D-2)^2}h\delta\epsilon_{ij}\delta\epsilon_{kl}, \tag{14}$$

and, therefore, the aforementioned orthogonality condition is $h_{ij}\pi^{ij}\left|\Psi[h_{ij}]\right\rangle = 0$.

2.2 Non-Vacuum Gravitational Field

For a non-vacuum gravitational field, the Wheeler-DeWitt equation is

$$\left(G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}+\ell_P^2\sqrt{h}\left({}^{(D)}R-2\Lambda_D-2\kappa_D\rho\right)\right)|\Psi[h_{ij},\phi]\rangle=0, \quad (15)$$

where the symbol ϕ denotes presence of matter fields, and $\rho = T_{\mu\nu}n^\mu n^\nu$ is energy density of matter fields related to a vector field n^μ normal to an embedded space. Inclusion of the strategy of the previous subsection gives

$$\left(G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}+\sqrt{h}\left(\delta\epsilon_{ij}\frac{\delta}{\delta h_{ij}}+\epsilon_{ij}\epsilon_{kl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}-2\ell_P^2(\Lambda_D+\kappa_D\rho)\right)\right)|\Psi[h_{ij},\phi]\rangle=0, \quad (16)$$

and, through compatibility with a vacuum gravitational field, consistency holds for

$$h_{ij}\frac{\delta}{\delta h_{ij}}|\Psi[h_{ij},\phi]\rangle=-\frac{8D\ell_P^2h}{(D-2)^2}(\Lambda_D+\kappa_D\rho)|\Psi[h_{ij},\phi]\rangle, \quad (17)$$

or, equivalently,

$$ih_{ij}\pi^{ij}|\Psi[h_{ij},\phi]\rangle=-\frac{4D\ell_P h}{\kappa_D(D-2)^2}(\Lambda_D+\kappa_D\rho)|\Psi[h_{ij},\phi]\rangle, \quad (18)$$

and this is the reduced wave equation of quantum geometrodynamics. In this state of affairs, one can derive immediately

$$h_{ij}h_{kl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}|\Psi[h_{ij},\phi]\rangle= \\ =-\frac{8D\ell_P^2h}{(D-2)^2}\left[\kappa_D h_{ij}\frac{\delta\rho}{\delta h_{ij}}+(D-1)(\Lambda_D+\kappa_D\rho)-\frac{8D\ell_P^2h}{(D-2)^2}(\Lambda_D+\kappa_D\rho)^2\right]|\Psi[h_{ij},\phi]\rangle, \quad (19)$$

where the Jacobi formula has been used, and, for this reason, one can construct the equation

$${}^{(D)}R|\Psi[h_{ij},\phi]\rangle=-\frac{4}{D-2}\left[\kappa_D h_{ij}\frac{\delta\rho}{\delta h_{ij}}+\frac{D}{2}(\Lambda_D+\kappa_D\rho)-\frac{8D\ell_P^2h}{(D-2)^2}(\Lambda_D+\kappa_D\rho)^2\right]|\Psi[h_{ij},\phi]\rangle, \quad (20)$$

which in itself modifies the standard cohomology and together with the Eq. (17) forms the system of equations.

3 Phenomenological Quantum Gravity

Let us apply the phenomenological approach. Through the justification of a general nature, the quantum mechanics argues that a wave function of quantum geometrodynamics is a diffeoinvariant scalar-valued function, it particularly involves the DeWitt supposition on dependence on geometry of an embedded space. It leads to the conclusion that a wave function of quantum geometrodynamics, including the reduced quantum geometrodynamics as a particular model, is an objective function, i.e. is dependent on at most the matrix invariants c_n, $1\leq n\leq D$, of a space metric tensor h_{ij}. Recall that, in the light of the Cayley-Hamilton theorem, invariants c_n of a square matrix h_{lm} of dimension $D\times D$ are coefficients of its characteristic polynomial $(h_{lm})^D-\sum_{k=1}^{D-1}c_k(h_{lm})^{D-k}+c_DI_D=0$, where I_D is $D\times D$ identity matrix, which are linked each other through the Newton identities $kc_k=\mathrm{Tr}(h_{lm})^k-\sum_{i=1}^{k-1}c_i\mathrm{Tr}(h_{lm})^{k-i}$ for $k=1,\ldots,D-1$, while $c_D=(-1)^D\det h_{lm}$. Hence $|\Psi[h_{ij}]\rangle=|\Psi(c_1,\ldots,c_D)\rangle$, and, moreover, one can write $\frac{\delta}{\delta h_{ij}}=\sum_{n=1}^{D}\frac{\delta c_n}{\delta h_{ij}}\frac{\delta}{\delta c_n}$. Then, taking into account the global one-dimensional conjecture, one has $|\Psi(c_1,\ldots,c_D)\rangle = |\Psi[h]\rangle$, and, consequently, $\dfrac{\delta}{\delta c_n} = \dfrac{\delta h}{\delta c_n}\dfrac{\delta}{\delta h}$. Involving once again the Jacobi formula and the obvious identity $\displaystyle\sum_{n=1}^{D}\frac{\delta c_n}{\delta h_{ij}}\frac{\delta h_{ij}}{\delta c_n} = D$, this strategy gives $\dfrac{\delta}{\delta h_{ij}} = Dhh^{ij}\dfrac{\delta}{\delta h}$, $h_{ij}\dfrac{\delta}{\delta h_{ij}} = D^2h\dfrac{\delta}{\delta h}$, $h_{ij}h_{kl}\dfrac{\delta^2}{\delta h_{ij}\delta h_{kl}} = D^4h^2\dfrac{\delta^2}{\delta h^2}$, and so on. For matter fields the conjecture stands that $\rho = \rho[h]$.

In this state of affairs, the reduced wave equation derived in the previous section transforms into the objective quantum gravity

$$\frac{\delta}{\delta h}|\Psi[h]\rangle = -\frac{8\ell_P^2}{D(D-2)^2}\left(\Lambda_D + \kappa_D\rho[h]\right)|\Psi[h]\rangle\,, \tag{21}$$

which also applies to a vacuum gravitational field as $\dfrac{\delta}{\delta h}|\Psi[h]\rangle = 0$. Since in the received model of quantum gravity the dynamical evolution is formally a first-order ordinary differential equation, the wave function can be straightforwardly constructed as the solution to a Cauchy problem for an initial wave function $|\Psi[h_I]\rangle$, where h_I is an initial value of h, this is manifestly beyond either a Feynman path integral or a Wilson loop techniques. In this approach, the general wave function of the reduced quantum geometrodynamics is an objective global one-dimensional wave function

$$|\Psi[h]\rangle = N\exp\left\{-\frac{8\ell_P^2}{D(D-2)^2}\left(\Lambda_D(h-h_I) + \kappa_D\int_{h_I}^{h}\rho[h']\delta h'\right)\right\}|\Psi[h_I]\rangle\,, \tag{22}$$

where N stands for a normalization factor, and in itself undetermined initial state $|\Psi[h_I]\rangle$ is a solution to the vacuum theory. Furthermore, by the global one-dimensional nature of the model, one can adopt other rules of the standard quantum mechanics, including the Born normalization condition $\displaystyle\int_{h_I}^{h_F}\langle\Psi[h']|\Psi[h']\rangle\,\delta h' = 1$ with h_F being a final value of h and $\langle\Psi[h_I]|\Psi[h_I]\rangle = 1$, it leads to the conclusion

$$N = \left\{\int_{h_I}^{h_F}\exp\left[-\frac{16\ell_P^2}{D(D-2)^2}\left(\Lambda_D(h'-h_I) + \kappa_D\int_{h_I}^{h'}\rho[h'']\delta h''\right)\right]\delta h'\right\}^{-1/2}. \tag{23}$$

Furthermore, for the phenomenological theory one can easily establish the supplementary eigenequation

$${}^{(D)}R\,|\Psi[h]\rangle = -\frac{4D^2h}{D-2}\left[\kappa_D\frac{\delta\rho}{\delta h} + \frac{1}{2Dh}\left(\Lambda_D + \kappa_D\rho\right) - \frac{8\ell_P^2}{D(D-2)^2}\left(\Lambda_D + \kappa_D\rho\right)^2\right]|\Psi[h]\rangle\,. \tag{24}$$

For the aforementioned phenomenological model of quantum gravity one can derive two simplest situations. First of all, one can consider a presence of a cosmological constant and matter fields having a constant energy density, for which

$$|\Psi[h]\rangle = N\exp\left\{-\frac{8\ell_P^2}{D(D-2)^2}\left(\Lambda_D + \kappa_D\rho_0\right)(h-h_I)\right\}|\Psi[h_I]\rangle\,, \tag{25}$$

$$N = \frac{\dfrac{4}{D-2}\left[\dfrac{1}{D}\ell_P^2\left(\Lambda_D + \kappa_D\rho_0\right)\right]^{1/2}}{\left\{1 - \exp\left[-\dfrac{16\ell_P^2}{D(D-2)^2}\left(\Lambda_D + \kappa_D\rho_0\right)(h_F - h_I)\right]\right\}^{1/2}}, \tag{26}$$

$${}^{(D)}R\,|\Psi[h]\rangle = -\frac{2D}{D-2}\left[\Lambda_D + \kappa_D\rho_0 - \frac{16\ell_P^2h}{(D-2)^2}\left(\Lambda_D + \kappa_D\rho_0\right)^2\right]|\Psi[h]\rangle\,, \tag{27}$$

and two particular situations, the wave function for a lambda-vacuum field and vanishing cosmological constant, can be immediately derived from this case. A general case is a Ricci-flat embedded space, for which both the wave function and the normalization factor are in the aforementioned general forms, while the supplementary eigenequation is

$$\left[\kappa_D\frac{\delta\rho}{\delta h} + \frac{1}{2Dh}\left(\Lambda_D + \kappa_D\rho\right) - \frac{8\ell_P^2}{D(D-2)^2}\left(\Lambda_D + \kappa_D\rho\right)^2\right]|\Psi[h]\rangle = 0. \tag{28}$$

4 Discussion

Apparently a modified cohomology considered in the context of the quantum geometrodynamics is able to generate a model of quantum gravity wherein the Wheeler-DeWitt equation is replaced through the system of two equations for an embedded space: a first order differential equation describing quantum mechanics of a wave function understood as a physical quantum state, and a supplementary eigenequation for a scalar curvature which determines both geometry and topology through an energy density of matter fields and a cosmological constant. Looking through the prism of the Wheeler superspace, the derived orthogonality condition which appears for a vacuum gravitational field and is naturally related to the standard cohomology, has the physical interpretation as the analogue of the Lorentz/Lorenz gauge from the Maxwell electrodynamics. Meanwhile, for a non-vacuum gravitational field, the reduced model of quantum geometrodynamics appears as entirely defined through a modified cohomology, where modification is due to presence of matter fields and a cosmological constant, specifically both geometry and topology of an embedded space are affected.

Employed technique of differential topology has the most natural physical interpretation as the choice of a superspatial gauge, or, equivalently, as the choice of the specific coordinate system in the configuration space of the General Theory of Relativity. For a vacuum gravitational field, the choice refers to orthogonality between a metric space and its canonical conjugate momentum, whereas for a non-vacuum gravitational field this orthogonality is generalized through presence of a cosmological constant and matter fields. Under this choice of gauge, both the reduced Quantum General Relativity and its phenomenological variant are defined through the representation of quantum mechanics which non-trivially mixes both the Schrödinger picture and the Heisenberg picture. Moreover, in the phenomenological quantum gravity, which is defined as the global one-dimensional objective quantum gravity, the entire problem of quantum gravity is determined as a Cauchy problem. Remarkably, under this standard approach to differential equations, neglected through a Feynman path integral and a Wilson loop techniques, the crucial problems in application of formalism of quantum mechanics to quantum gravity are effectively absent, and, consequently, all well-known rules of quantum theory can be effectively transmitted onto the case of a quantized gravitational field in order to make its theory an empirically verifiable. In particular, as shown concisely in this article, in the phenomenological quantum gravity one can simply adopt the Born normalization condition, in contrary to the canonical quantum gravity considered through a Feynman path integral or a Wilson loop, wherein implementation of the standard quantum mechanics gives birth to both technical difficulties and interpretation obstacles and blocks development of quantum gravity towards the status of an empirically verifiable physical theory.

The phenomenological approach to the reduced Quantum General Relativity gives rise to quantum mechanics of an embedded space described through a first order quantum evolution, and generates a real exponential wave function whenever the natural assumptions of real cosmological constant, energy density of matter fields, and a space metric are applied. In this state of affairs, the context of the standard Schrödinger quantum mechanics gives the physical significance to the phenomenological quantum gravity throughout a straightforward relation to the quantum tunnelling. Strictly speaking, the phenomenological wave function appears like in the interior of a potential barrier, whereas direction of the barrier's penetration merely depends on the signs of a cosmological constant and energy density of matter fields. Remarkably, in the standardly physical interpretation, the Wheeler-DeWitt equation is considered as either the time-independent Schrödinger equation of non-relativistic quantum mechanics or the Klein-Gordon/de Broglie equation of relativistic quantum mechanics, or, in other words, conventionally quantum gravity is referred to the Bose-Einstein statistics. Meanwhile, through the first order quantum evolution, the reduced model of quantum geometrodynamics is more similar to the Dirac equation of relativistic quantum mechanics, which refers to the Fermi-Dirac statistics. There is a general compliance with the earlier discussion of quantum geometrodynamics on the level of quantum field theory, Cf. the Refs. [10]-[16], wherein the phenomenological quantum gravity refers to anyons, neither bosons nor fermions, which obey the one-dimensional Dirac equation for a two-component field. However, since in the reduced Quantum General Relativity one has to deal with a single-component wave function such like in the original Wheeler-DeWitt equation, both this model and its phenomenological variant should be interpreted in terms of either the time-dependent Schrödinger equation of non-relativistic quantum mechanics, or, if the relativistic nature of a gravitational field is fully included, the Tomonaga-Schwinger equation of relativistic quantum mechanics. For the latter variant, the reduced model and the phenomenological quantum gravity agree with the Dirac interaction picture, which is an intermediate representation between the Schrödinger picture and the Heisenberg picture. In this state of affairs, quantum gravity has a natural classification among the fundamental gauge theoretical considerations of particle physics, and would be considered along with quantum electrodynamics and the Yang-Mills theories of the Standard Model.

Remarkably, since in the reduced Quantum General Relativity all physical paths are compatible with the Wheeler-DeWitt equation, this model still can be approached through a Feynman path integral technique according to the idea of the Hartle-Hawking wave function, as well as a Wilson loop technique according to the idea of the Ashtekar new variables and the Rovelli-Smolin representation, but the change is a replacement of the quantized Hamiltonian constraint through a first order quantum evolution and a supplementary eigenequation. Naturally, the most intriguing consequences of the reduced Quantum General Relativity and the phenomenological quantum gravity would be particular applications in description of the quantum effects on different physical scales. At the first glance, the scope of describable phenomena affected through quantum gravity would apply to a wide spectrum of problems considered through astrophysics, cosmology, and physics, and, moreover, theoretical contributions from the phenomenological quantum gravity give the opportunity of immediate empirical verification. Moreover, pure theoretical applications of the proposed model of quantum gravity and its phenomenological variant in themselves would be the great challenges for further development.

Acknowledgements The authors thank each other for communication which led to this work, and to the Reviewers for valuable comments which significantly helped to improve the original manuscript.

References

1. A. Einstein, *The Meaning of Relativity: Including the Relativistic Theory of the Non-Symmetric Field: Fifth Edition, With a New Introduction by Brian Greene.* Princeton University Press, 2014.
2. P. A. M. Dirac, *General Theory of Relativity.* Princeton University Press, 1996.
3. S. Chandrasekhar, *The Mathematical Theory of Black Holes.* Oxford University Press, 2004.
4. S. Weinberg, *Gravitation and Cosmology. Principles and Applications of the General Theory of Relativity.* Wiley, 1972.
5. ——, *Cosmology.* Oxford University Press, 2008.
6. M. Bojowald, *Canonical Gravity and Applications: Cosmology, Black Holes, Quantum Gravity.* Cambridge University Press, 2011.
7. C. Kiefer, *Quantum Gravity.* Oxford University Press: Third Edition, 2012.
8. C. Rovelli, *Quantum Gravity.* Cambridge University Press, 2004.
9. T. Thiemann, *Modern Canonical Quantum General Relativity.* Cambridge University Press, 2007.
10. Ł. A. Glinka, *Aethereal Multiverse: A New Unifying Theoretical Approach to Cosmology, Particle Physics, and Quantum Gravity.* Cambridge International Science Publishing, 2012.
11. ——, "Objective quantum gravity, its possible relation to gauge theories and strings," *Applied Mathematics and Physics*, vol. 2, no. 3, pp. 82–93, 2014.
12. ——, "Novel solution of Wheeler-DeWitt theory," *Applied Mathematics and Physics*, vol. 2, no. 3, pp. 73–81, 2014.
13. ——, "Thermodynamical quantum gravity," *Applied Mathematics and Physics*, vol. 2, no. 3, pp. 66–72, 2014.
14. ——, "Bosonic quantum gravity according to the global one-dimensionality conjecture," *Applied Mathematics and Physics*, vol. 2, no. 3, pp. 59–65, 2014.
15. ——, "Do global virtual axionic gravitons exist?" *NeuroQuantology*, vol. 112, no. 1, pp. 35–39, 2014.
16. ——, "Global one-dimensionality conjecture within quantum general relativity," *Gravitation and Cosmology*, vol. 16, no. 1, pp. 7–15, 2010.

