Application of Generalized Machine Theory in Analysis of Behavior of Universal Motor

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ABSTRACT

This paper teaches GME (Generalized Machine Theory) while investigating into Universal Motor's Dynamic Behavior for both transient and steady-state on various load applications. Formation of GME Equations of the Universal Motor is explained by formation of the transient equations of equivalent circuit of the universal motor, by applying Kirchhoff's Loop Law. They are numerically solved for transient behavior. After that the transient equations of the motor are converted into steady-state equivalent equations by replacing instantaneous currents and voltages into their rms values and substituting $\frac{d(Li)}{dt}$ by $L\frac{di}{dt}+\left(\frac{dL}{dt}\right)i$ and then $L\frac{di}{dt}$ by $X_L I$ etc. and solving for steady-state solution.

In each dynamic switching, transient and steady-state analysis, input power, current, power factor and speed of the motor are determined and plotted against common time and then analyzed. This research was essential to investigate the effect of applied load on input and output behavior of Universal Motor. Up to now, nobody has attempted to find effect of applied loads on performance of the Universal Motor.

The results obtained for the dynamic steady state and transient characteristics are presented in graphical form and discussed.

Key Words: **Transients, Dynamics, Universal Motor, Transient Behavior, Modeling of Universal Motor.**

1. INTRODUCTION

Universal Motors are very widely used in almost all types of applications [1-3] in home, commerce, office, factory and so on due to light weight, low initial and low running cost, self starting, high starting torque, easy to start, high speed, high efficiency, effectiveness, high output power to weight ratio, safe at relative high overload capacity, and operating good power factor. The

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universal motor is mostly built for fractional horse power sizes, which can operate satisfactorily from DC or AC supply mains. In such cases steady state and transient characteristics become important factors to be determined. It is a Universal fact that almost all transient work in Electrical Machines is done [4-6] in Differential Equations and particular in Generalized Machine Theory [4,6-8]. This paper emphasizes on teaching the systemic method of Generalized Machine Theory. Many authors have researched on various aspects of Universal Motor [9-14] but non of the researchers have studied effect of load variation on performance of the Universal motor. Research work in this paper is based upon ME Thesis by the authors of this paper [3]. In this research work, universal motor dynamic behavior is studied under switching transients with different loads applied.

2. TRANSIENT DYNAMIC EQUATIONS

Fig. 1 shows equivalent circuit of Universal Motor and it shows two circuits, (i) field circuit and (ii) armature circuit connected in series. Applying Kirchhoff's Loop Law, following equations are obtained.

$$V_a = R_a\, i_a + \frac{d}{dt}(L_a\, i_a) + \frac{d}{dt}\left[\left(M_m^{fa}\cos wt\right) i_f\right] \qquad (1)$$

$$V_f = R_f i_f + \frac{d}{dt}(L_f i_f) + \frac{d}{dt}\left[\left(M_m^{fa}\cos wt\right) i_a\right] \qquad (2)$$

Simplifying Equations (1-2) by partial differentiation $\frac{d(Li)}{dt}$ by $\left(L\frac{di}{dt} + \frac{dL}{dt}\right) i$, re-arranging the terms and substituting θ = –90° = –π/2, we get:

$$V_a = R_a\, i_a + L_a \frac{di_a}{dt} + M_m^{fa}\, i_f \omega \qquad (3)$$

$$V_f = R_f\, i_f + L_f \frac{di_f}{dt} + M_m^{fa}\, i_a \omega \qquad (4)$$

As magnetic flux of the armature do not cut the field windings and cannot induce a voltage in the field windings, substituting corresponding armature flux term $i_a=0$ for this purpose and hence the equations in matrix form became:

$$\begin{bmatrix} v_f \\ v_a \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_a \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix} + \begin{bmatrix} L_{Self}^f & 0 \\ 0 & L_{elf}^0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_f \\ i_a \end{bmatrix} + \omega \begin{bmatrix} 0 & 0 \\ M_m^{fa} & 0 \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix} \qquad (5)$$

And therefore induced Torque Equations (3-5) for $i_a = i_f = i$, becomes:

$$Te = \frac{pp}{2} \begin{bmatrix} i_f & i_a \end{bmatrix} \begin{bmatrix} 0 & 0 \\ M_m^{fa} & 0 \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix} = pp\ \ M_m^{fa}\ i^2 \qquad (6)$$

As applied voltage is sum of voltage across armature and voltage across field ($V=V_f+V_a$) and also the current is the same in both circuits $i_a=i_f=i$, the applied voltage and torque equations become:

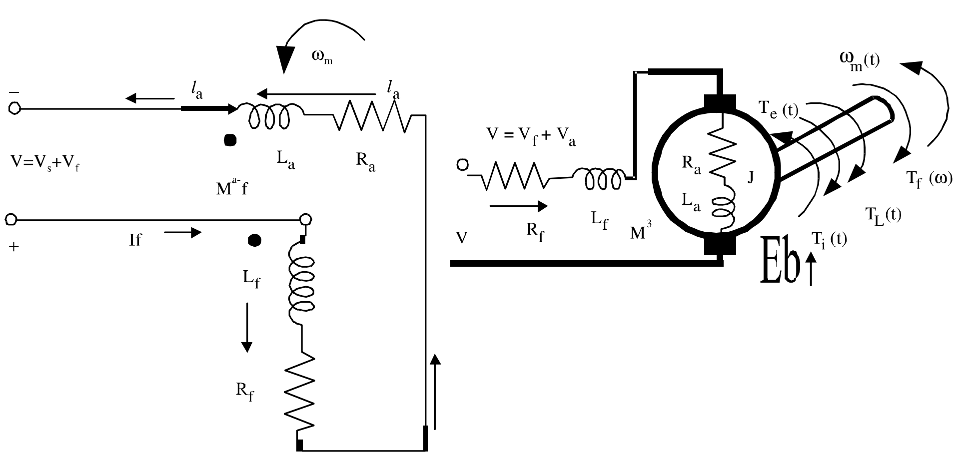

FIG. 1. GENERALIZED CIRCUIT OF UNIVERSAL MOTOR

$$V = Ri + L\frac{di}{dt} + M_m^{fa}\, i\ \omega \qquad (7)$$

$$T_e = pp.M_m^{fa}.i^2 \qquad (8)$$

Where as in Equation (7), $R = R_a + R_f$ and $L = L_a + L_f$.

Since the motor produces armature torque T_e to overcome load torque T_L, friction torque T_f and inertial torque $J_m \frac{d}{dt}\omega_m = \frac{J_m}{pp}\frac{d}{dt}\omega_e$, the dynamic torque balance equation and the angular speed balance equation become:

$$T_T = T_e - T_F - T_L = \frac{J_m}{pp}\frac{d}{dt}\omega_e \qquad (9)$$

$$0 = \frac{d\theta}{dt} - \omega \qquad (10)$$

Joining Equations (7,9–10) in matrix form and rearranging the matrices, a comprehensive equation for the numerical integration is given:

$$\frac{d}{dt}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} L_f + L_a & 0 & 0 \\ 0 & \frac{J}{pp} & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} V \\ T_T \\ 0 \end{bmatrix} - \begin{bmatrix} R_f + R_a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} - \omega \begin{bmatrix} M_m^{fa} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} \right\} \qquad (11)$$

Equation (12) is the final electro-dynamic equation, ready for numerical solution. Equation (11) is equivalent to:

$$\frac{d}{dt}[i] = [L]^{-1}\{[V] - [R]\,[i] - \omega[G]\,[i]\} \qquad (12)$$

When Equation (12) is numerically integrated, the solution is obtained in numbers. When these numbers are plotted, result is in graphical form. Interpretations of the graphical result, for transient is given in the section below.

3. STEADY-STATE DYNAMIC EQUATIONS

The steady-state computer program starts from first assumed value of armature torque T_e=16.5N-m in the first run and then an increase of 2.5N-m in each next run, till the maximum output of the motor is achieved. Armature current from Equation (2) at each armature torque is $I = \sqrt{\frac{T_e}{pp\ M_m^{fa}}}$. This current I is dc amperes for dc supply and rms amperes current for ac supply. Angular speed ω is calculated from Equation (1) by converting it into steady-state equivalent equation by $L\frac{di}{dt}$ by $X_L I$ and then rearranging its terms into Equation (13):

$$(V/I)^2 = \left(R_{eq} + \omega M_m^{fa}\right)^2 + \left(X_{eq}\right)^2 \text{ or}$$

$$\omega = \frac{1}{M_m^{fa}}\left| \sqrt{\left\{(V/I)^2 - (X_a + X_f)^2\right\}} - \left(R_a + R_f\right)\right| \qquad (13)$$

The speed ω can easily be converted into speed N_{rpm} by

$$N_{rps} = \omega / 2\pi \quad and \quad N_{rpm} = N_{rps}/60 \qquad (14)$$

At this stage, ω is inserted into friction torque $T_f = k_f\omega^2$. Considering the fact that an steady-state operation of an electrical machine gives only one value of current, speed, torque, efficiency etc. the computer program is made to calculate exactly those correct values at given magnitude of applied load & applied voltage V_{ac}, as under:

$$T_L = T_a - T_f,\ E_b = \omega M_m^{fa} I,\ \eta = \frac{P_{out}}{P_{in}} = \frac{\omega T_L}{VI_{pf}},$$

$$P_{out} = E_b I - \omega T_f - \omega T_L,\ P_{in} = V\,I\,pf,\ P_{arm} = E_b I,$$

$$T_j = T_e - T_f - T_L = \frac{J_m\, d\omega}{pp\, dt} = 0,$$

$$pf = \left(R_a + R_f + \omega M_m^{fa}\right) / \sqrt{\left\{\left(R_a + R_f + \omega M_m^{fa}\right)^2 + \left(X_a + X_f\right)^2\right\}} \qquad (15)$$

For the dc voltage supply, the above Equation (15), will become dc voltage equation by substituting Zero for frequency f=0 . The steady-state program is in fact execution of Equations (14-15) in the above given sequence.

4. MOTOR PARAMETERS

Transient behavior of the Universal Motor was obtained by applying either a dc voltage of 200V dc or an ac voltage of 200V rms, 50Hz frequency supply. The universal Motor has circuit parameters of R_f=0.82, R_a=1.43, R_{brush}=0. L_f= 0.0086, L_a=0.01656, and M_m^{fa} =0.7378, Poles=2, Jm=0.022, Five different loads (i) 50 N-m, (ii) 25 N-m, (iii) 0 N-m, (iv) sinosoidal 25 N-m, and (v) sinusoidal 50 N-m, were selected as shown in Fig. 2.

Effect of the different loads were investigated under steady state and transient dynamics. Both Steady-state and Transient Dynamic Behaviors are being discussed in this paper. The Transient currents of the Universal Motor (not shown here) are of transient nature dependent upon the magnitude and pattern of load and magnitude and frequency of applied voltage.

5. STEADY-STATE RESULTS FROM STEADY STATE ANALYSIS

5.1 AC Performance Characteristics

Fig. 3 shows results of conventional steady-state program when ac voltage in rms is supplied. It shows steady-state simulated efficiency, power factor pf, Input power Pin, feedback voltage E_b, armature power P_{arm}, Mechanical power output Pout, Input current I, armature torque T_{orq}, load torque T_{load} against speed ω rad/s. From the curve eff and pf of Fig. 3, with AC voltage supply to Universal Motor the highest efficiency and highest pf occurs between ω=20 radians per second to ω=30 radians per second. Power output in this range varies between 50*29=1450 watts to 60*29=1740 watts. This means the motor rating is approximately 2-2.5 hp.

5.2 DC Performance Characteristics

In Fig. 4, with dc supply to the motor, pf curve of the motor is unity. High dc efficiency remains in the speed range of 20-30 rad/sec. Ac and dc voltage steady-state results are almost same shape, smooth, average results of a series dc motor characteristics.

5.3 Comparison of AC/DC Outputs and Efficiencies vs Speed

Fig. 5 compares universal motor calculated from conventional method performed on ac and dc supply for efficiency, output torque and load torque against speed rad/s along x-axis. Feedback voltage and pf increase with speed and current decreases with increases speed.

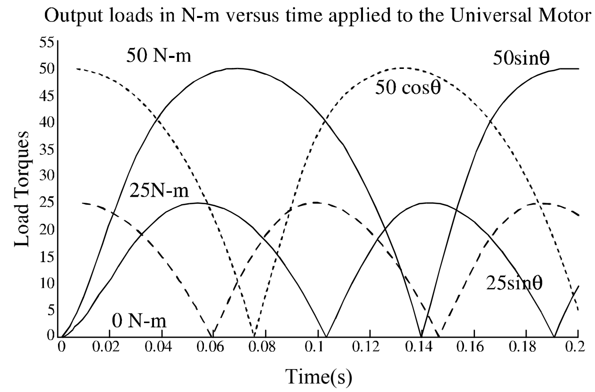

FIG. 2. FIVE TYPES OF VARIABLE & CONSTANT LOADS

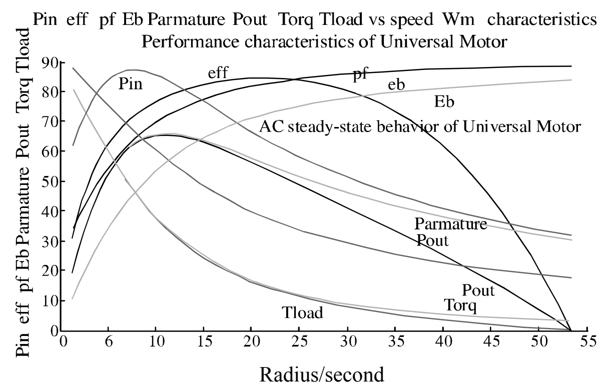

FIG. 3. AC VOLTAGE PIN VS SPEED, EFF VS SPEED, PF VS SPEED, EB VS SPEED, PRM VS SPEED, POUT VS SPEED GRAPHS

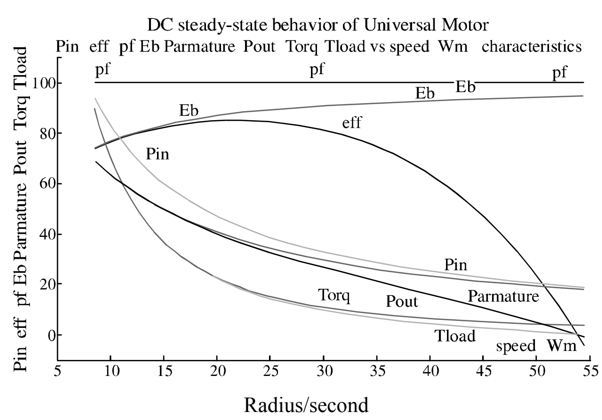

FIG. 4. DC VOLTAGE PIN/SPEED, EFF/SPEED, PF/SPEED, EB/SPEED, PRM/SPEED, POUT/SPEED, TLOAD/SPEED GRAPHS

5.4 Comparison of AC/DC Outputs and Efficiencies vs Current

Fig. 6 compares feedback voltages, pf, and currents against Current along x-axis for ac and dc voltages. DC values (solid lines) are either at the same magnitude or at a little higher value than the ac quantities (dotted lines) for the same current. These results show that the ac and dc voltage performance by conventional solution are almost same shape, nature and magnitude.

5.5 Comparison of AC/DC Torque and Efficiencies vs Speed

Fig. 7 compares universal motor performance on ac and dc supply for efficiency, output torque and load torque against speed along x-axis.

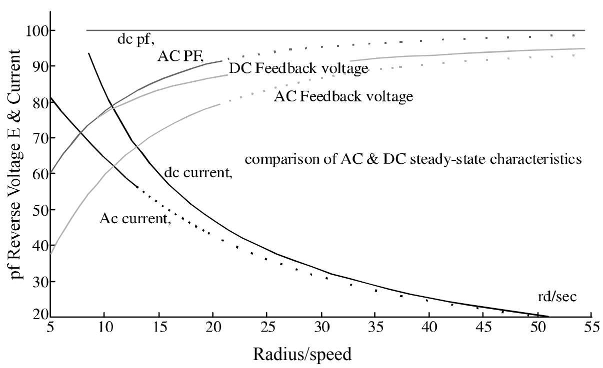

FIG. 5. GRAPH BETWEEN EFFICIENCY VS SPEED, TORQUE/SPEED AND TOUT/SPEED FOR DC (----) AND AC (.....)

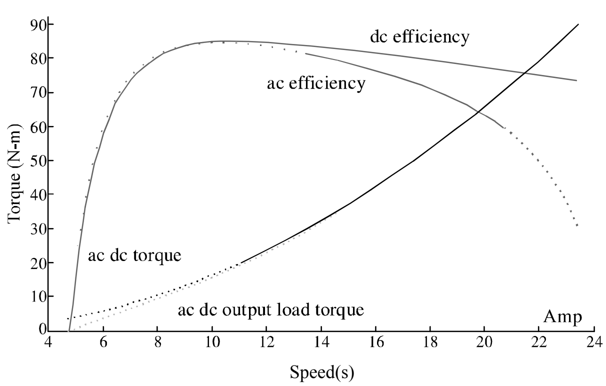

FIG. 6. GRAPH BETWEEN PF/SPEED, CURRENT/SPEED, AND REVERSE VOLTAGE/SPEED FOR DC (---) AND AC (......)

5.6 Comparison of AC/DC Outputs, pf and Efficiencies vs Current

Fig. 8 compares ac and dc feedback voltage, pf, and speed against current amperes along x-axis. These curves shows that efficiency and power factor of the Universal Motor are a little higher for dc supply than these values from ac supply. The curves in this graph also show that ac power supply efficiency is slightly lower than dc power supply efficiency of the motor. The curves show that the load torque is less than armature torque by the difference of friction torque. These curves show that for the same current, dc efficiency and torque output of the motor are higher than the same quantities with the ac supply. And the performance of the Universal motor under dc supply is a little superior than its performance with ac supply.

AC voltage Steady-state behavior obtained steady state ac voltage supply from transient program, is shown in Figs. 9-10.

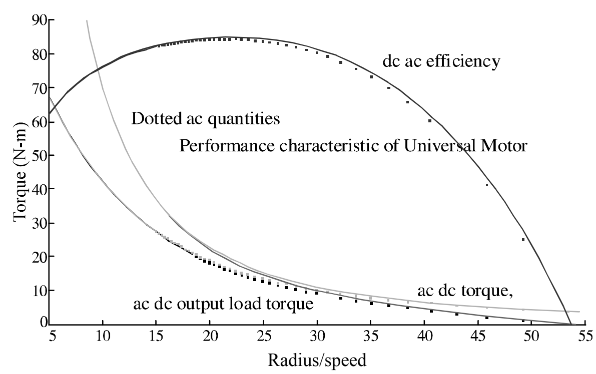

FIG. 7. GRAPH OF EFFICIENCY AND TORQUE VS SPEED FOR DC AND AC (.....)

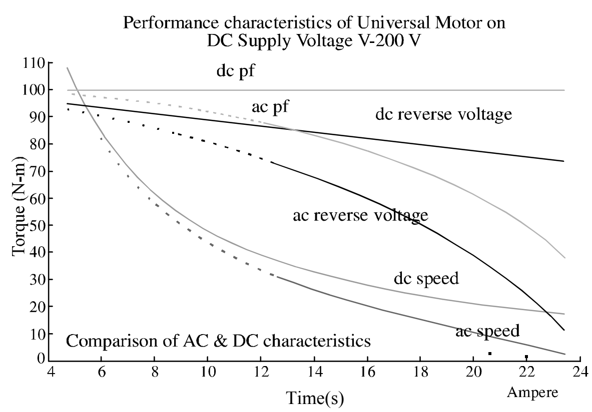

FIG. 8. GRAPH BETWEEN PF/CURRENT , EB/CURRENT & SPEED/CURRENT FOR DC (----) AND AC (.....)

6. STEADY-STATE RESULTS FROM TRANSIENTS

6.1 Comparison of Input Power vs Current for Different Loads

Fig. 9 compares ac and dc input power flow into the universal motor during steady state operations. The power drawn by the motor from ac source is positive and sinusoidal except a very small negative power dip at zero crossing of the current due to small out of phase voltage and current (or small out off phase fluxes on rotor and stator at higher load). It shows that reversal of the current is not exactly at the same time at the reversal of voltage. This small negative power dip appears only at high load outputs. Higher the output load, higher is the negative dip and also higher is the positive maximum peak load. With constant output load torque from the universal motor, the power input from the dc source supply is constant and always positive. Higher the constant output load, higher but constant is the power intake from the dc supply. Definitely for a constant output load torque and with ac supply, generated armature power is sinusoidal and with the dc supply, the armature torque and input power are constant. Higher the output load, higher is the power input to the universal motor and vice versa.

6.2 Comparison of Speed vs Time for Different Loads

Fig. 10 shows final steady-state speed of universal motor gained on 200V ac and 200V dc voltage supply at fixed output loads 50, 25 and 0 N-m. Speed of the universal motor at same constant load output and dc supply is higher than its speed at the same output load with ac supply. Also lower the speed, at higher load and vice versa. The graph shows that the universal

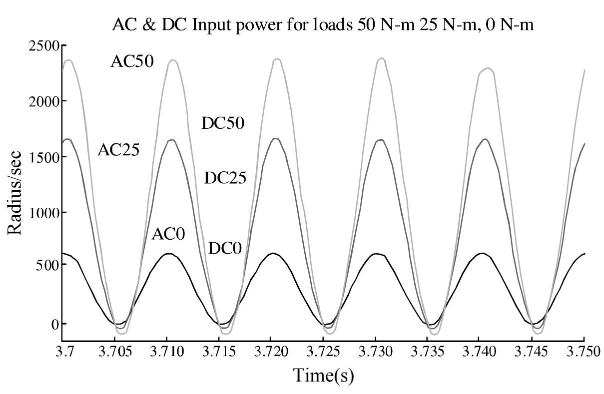

FIG. 9. GRAPH BETWEEN PIN/TIME AT DIFFERENT LOADS FOR DC (HORIZONTAL ST LINES) AND AC (SINUSOIDAL)

motor has highest speed at no load and low speed at high output load. For the constant output load and with the ac source, the motor has oscillatory periodic variable speed where as with constant output load and with dc source, the motor has constant speed. For the same supply voltage, higher is the mechanical power output, lower is the speed of the universal motor and vice versa.

6.3 Comparison of Armature Torque vs Time for Different Loads

Fig. 11 compares armature torque in N-m for constant values of output torques for ac and dc voltages. Although the output load torques are constant, with an ac supply, the armature torque is variable due to ac current. The armature torque is constant with constant output load torque and dc voltage source. Ac armature torque developed varies oscillatory sinusoidal from 0 N-m to some constant maximum peak. With constant output motor load torque, and with ac supply, the ac armature torque developed varies oscillatory periodically sinusoidal between 0 N-m and a constant maximum peak value. With dc source and constant

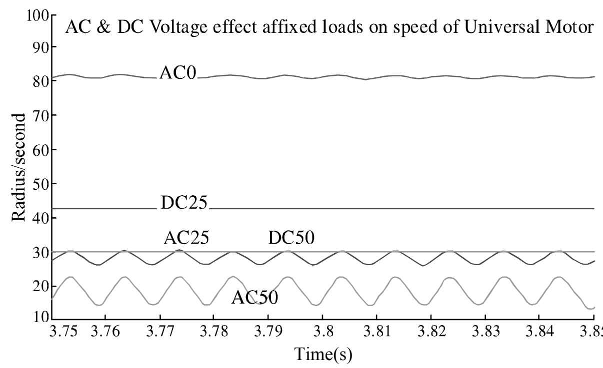

FIG. 10. GRAPH BETWEEN SPEED/TIME FOR VARIOUS LOADS FOR DC (STRAIGHT LINES) AND AC (SINUSOIDAL)

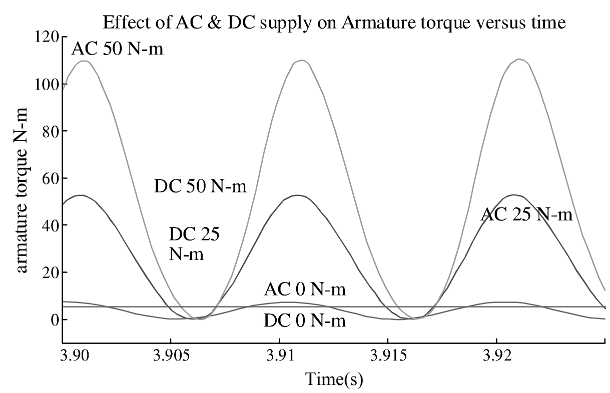

FIG. 11. GRAPH BETWEEN ARMATURE TORQUE/TIME AT VARIOUS LOADS FOR DC (CONSTANT HORIZONTAL) AND AC (SINUSOIDAL)

output load torque, dc armature torque developed is of a constant magnitude. During steady-state operation with ac supply, armature torque is oscillatory sinusoidal variable, irrespective of the shape of the motor output load torque, also the inertia torque is sinusoidal symmetrically across the zero axis along time axis. During steady-state operation of universal motor with dc supply voltage, armature torque is sinusoidal for sinusoidal output load torque and the armature torque is constant for constant output load torque with the inertia torque equal to zero. Armature torque is higher for higher output load torques and vice versa. During steady state operation with ac voltage supply, inertial torque is oscillatory sinusoidal symmetrical over zero torque axis in the direction of time increase. Steady state inertia torque is zero for dc supply voltage and constant output torque. Higher the output load torque, higher is the inertia torque with ac voltage supply.

6.4 Effect of Load Torque on Armature and Inertia Torques

Fig. 12 shows effect of load torque on generated and inertia torques. At all times, with the ac voltage supply to universal motor, steady-state generated torque is equal to friction torque plus load torque plus inertia torque. At any moment, with the dc voltage supply to the universal motor steady state generated torque is equal to friction torque plus load torque plus zero inertia torque. Inertia torque with dc supply during steady-state operation is zero. With ac supply during steady-state operation, generated torque is equal to sum of load, friction & inertia torques.

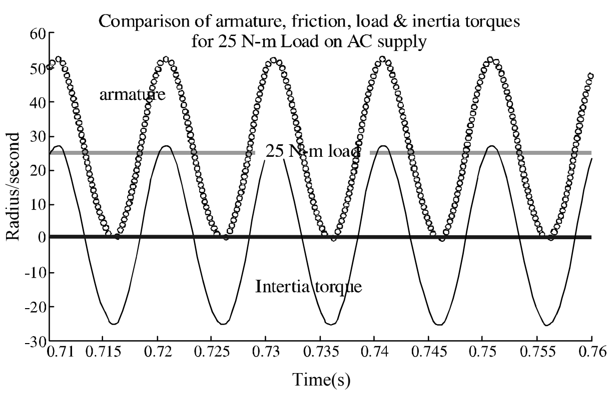

FIG 12. GRAPH BETWEEN INERTIA TORQUE/TIME AT VARIOUS LOADS FOR DC (CONSTANT HORIZONTAL AND AC (SINUSOIDAL)

7. TRANSIENT RESULTS FROM TRANSIENT PROGRAM

7.1 Transient Induced Torque

Fig. 13 shows that induced torque which is proportional to square of corresponding armature current, during ac voltage. For sinusoidal current, induced torque is sinusoidal. Current and load shape dictates the overall pattern of the induced armature torque of the Universal Motor. Some loads are sinusoidal and remaining loads are constant but induced torque in each case is sinusoidal because applied voltage is sinusoidal.

The dc voltage induced torque is shown in Fig. 14, which has also induced torque proportional to square of dc current. Fixed load and with ac supply, the induced torque responds to the frequency and less to the type of load. With dc voltage supply, the induced voltage has zero frequency response and the induced voltage shape responds to shape of applied load. It also shows that for higher applied load, higher is the generated torque & vice versa.

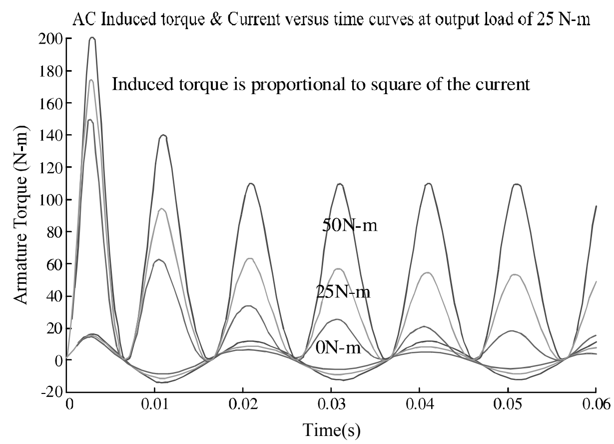

FIG 13. AC VOLTAGE TRANSIENT GENERATED TORQUE

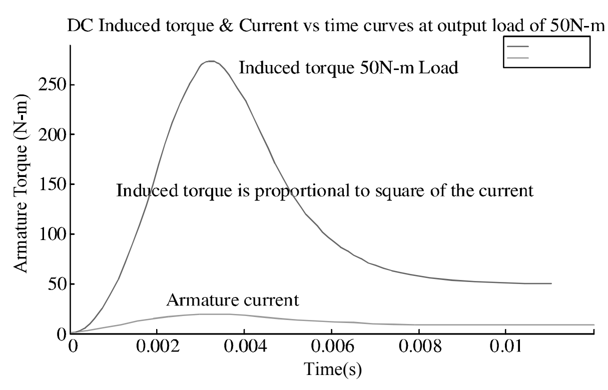

FIG. 14. DC VOLTAGE TRANSIENT TORQUE IN A UNIVERSAL MOTOR

7.2 Transient Effect of Supply Frequency on Induced Torque

Fig. 15 shows the effect of ac and dc voltage supply on induced torque with fix loads. Frequency of induced voltage is twice the frequency of applied voltage. In the case of DC voltage supply, frequency of the induced torque is zero. However, when the supply voltage is dc, shape of the induced voltage is similar to the load shape constant for constant and variable for variable. For ac supply of 50Hz, generated torque has 100Hz frequency with transient and steady-state nature.

7.3 Transient Behavior of Different Types of Torques

Fig. 16 shows generated torque, load torque and inertia torque for each constant applied load. The generated armature torque equals sum of load torque, friction torque and inertia torque. As generated torque is always positive, when the load torque is subtracted, it leaves negative inertial torque. Negative inertial torque affects the motor performance very badly. Negative inertial torque is deceleration or reduction in forward speed or roughness in speed.

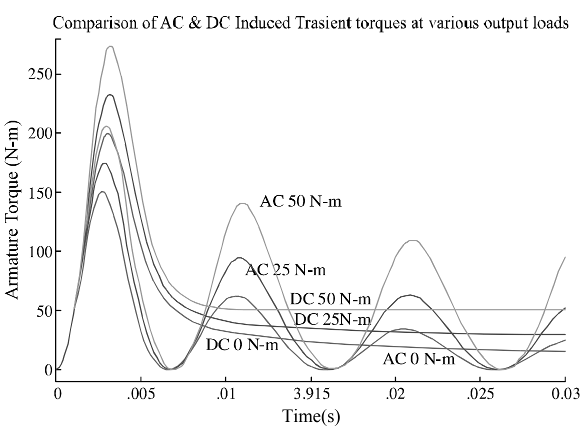

FIG 15. EFFECT OF AC/DC SUPPLY ON INDUCED ARMATURE TORQUE

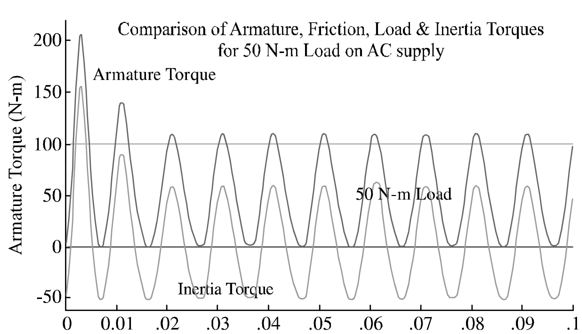

FIG. 16. EFFECT OF AC SUPPLY ON TORQUES

7.4 Transient Input and Output Power of the Motor

Fig. 17(a) shows transient input and output power of the Universal motor under ac voltage supply and Fig 17(b) shows transient input and output power of the Universal motor of dc voltage supply.

Fig. 17(a-b) show following facts; (i) Negative and positive transient peaks of input power are greater than output power peaks (ii) General shape of input and corresponding output power for same load is the same (iii) Output power wave form lags in phase the applies power waveform (iv) output wave shape crosses twice per phase in ac supply and finally (v) power input is bigger than power output. In the case of dc supply, load wave shape controls the internal and external wave form of the power.

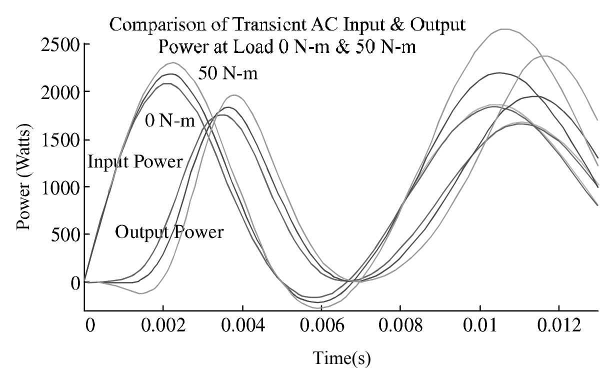

FIG. 17(a). PHASE SHIFT BETWEEN INPUT & OUTPUT POWERS FOR AC V

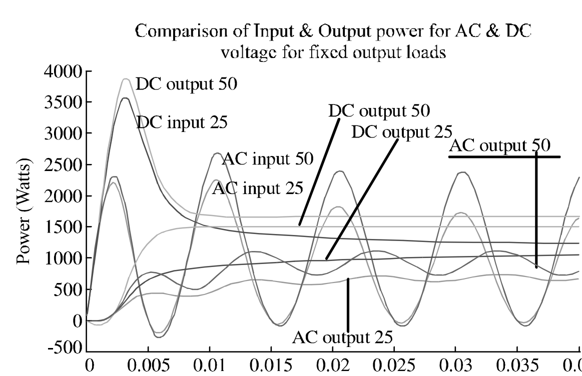

FIG 17(b). INPUT AND OUTPUT POWER FOR DC SUPPLY

7.5 Transient Efficiency of the Universal Motor

Fig. 18 shows transient efficiency of the Universal Motor. Dc voltage efficiency is constant at 90% whereas ac efficiency varies from 40-350% and sometimes goes into positive and negative peaks (not shown here). Positive and negative peaks are due to cross-overs and Lag of output power wave form on input power at two places per cycle.

7.6 Transient Speed of the Universal Motor

Fig. 19(a-b) show speed versus time characteristics of the Universal motor for applied ac voltage and applied dc voltage respectively. With the applied ac voltage, the speed is not smooth. The curves are caused by positive and negative parts of the Inertial torque.

For constant output loads 0, 25 and 50 N-m, the output speed is smooth and becomes constant after transient period. Whereas for sinusoidal output load torques, the speed becomes sinusoidal variable.

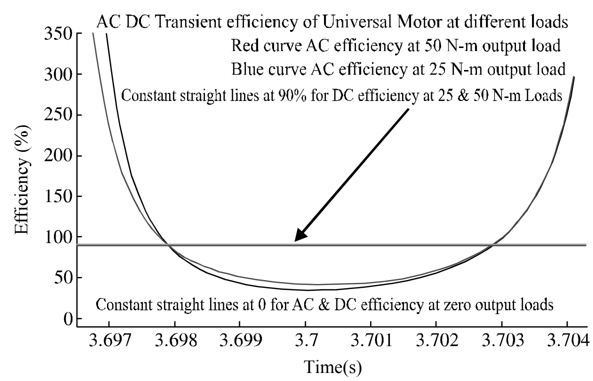

FIG. 18. AC AND DC VOLTAGE EFFICIENCY OF UNIVERSAL MOTOR

8. CONCLUSION

In this study the generalized circuit of the universal motor is developed and its circuit equations based upon the circuit are derived, described and applied for the dynamic performance of the Universal Motor. The electro-dynamic/electro-mechanical differential equations are programmed for dynamic steady state and transient solutions. The results show that the programming was carried out successfully and effectively. The program results are shown and analyzed. In this paper a detailed study was made for dynamic steady-state and transient performance with ac and dc voltage supplies using conventional steady-state and transient generalized electrical machine approaches for steady-state performance of the universal motor. With dc supply, results from generalized machine transient analysis are dc voltage fed like with starting transients in nature and shape. However steady-state and transient analysis with ac supply to universal motor, the torque, current, speed, power etc. from generalized machine transient approach are oscillatory periodic sinusoidal unlike the smooth average dc voltage like results from conventional approach method. The difference between these results are very clear and distinct and studied for the first time. The analysis of the universal motor confirms the work and highlights much more wonderful information to the end users and electrical machine industry not reported ever before. For example no author had ever before reported that the steady-state analysis can be done on the basis of transient differential equations and programming. Many authors determined their universal and series motor performance on the basis of conventional steady state equations & claimed them for steady-state and transient solution. Their ac and dc transient and steady-state solution characteristics were similar to the dc steady-state solutions. It was further observed that when the moment of inertias of armature and drive are

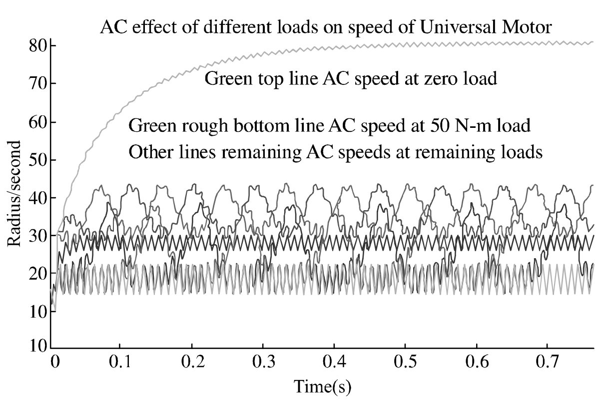

FIG. 19(a). AC VOLTAGE ANGULAR SPEED OF THE UNIVERSAL MOTOR

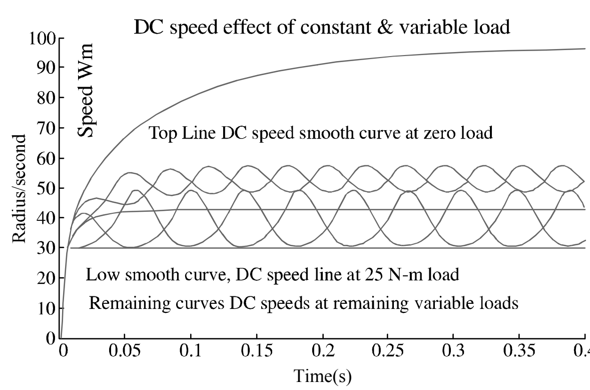

FIG. 19(b). DC VOLTAGE ANGULAR SPEED OF UNIVERSAL MOTOR large enough, the torque oscillations and variations in rotational speed were reduced.

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