Locating Center of Mass of Earth and Geostationary Satellites

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ABSTRACT

CoM (Center of Mass) of earth is a very important factor which can play a major role in satellite communication and related earth sciences. The CoM of earth is assumed to be around equator due to geometrical shape of earth. However, no technical method is available in the literature which can justify the presence of CoM of earth around equator. Therefore, in this research work the CoM of earth has been located theoretically with the help of mathematical relations. It also presents the mathematical justification against the assumption that equator is the CoM of earth. The effect of calculated CoM of earth on geostationary satellites has also been discussed. The CoM of earth has been found mathematically by using land to ocean ratios and the data is collected from the Google earth software. The final results are accurate with an approximate error of 1%.

Key words: **Center of Mass of Earth, Geo-Stationary Satellites, Crust Mass.**

1. INTRODUCTION

The point where the mass of object gets balanced and object remains in static equilibrium is known as the CoM of that object. This point plays an important role in the design of stable systems in which number of objects are involved. For example, in satellite communication for achieving stable orbits it is a required condition that the orbital path of satellite must form a great circle, which means that the satellite must pass through the CoM of earth. Therefore, with the help of accurate CoM of earth more stable orbits can be achieved that will eventually reduce the monitoring required for satellites to maintain their position in their orbits. Thus, the overall satellite communication could be affected by the calculated CoM of the earth. Besides this, CoM of earth has major impact on various fields related to earth. For example, the knowledge of CoM of earth can be used as a frame of reference through which the rate of change of continental drift can be estimated more accurately and this information can be very useful for the study of natural hazards such as earthquakes, global sea level changes etc. [1].

Many researchers and scientists have discussed the CoM of earth but it is not yet discovered [1]. Sometimes equator is assumed to be CoM of earth by merely considering the geometric shape of earth. In [2], the authors have given the concept of mass balancing and torque balancing latitude of earth for determining the CoM of earth. However, no mathematical proofs have been presented. Therefore, in this research the authors have determined mathematically the CoM of earth for the very first time to the best of their knowledge. This paper presents the methods which are used to calculate the CoM of earth and it also covers how these calculated results are advantageous for geostationary satellites.

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2. METHOD

The CoM of any object is a point where the mass of object get balanced. The net torque acting on the object at that point is zero and it remains in static equilibrium. Therefore, for calculating the CoM of earth the first step is to find out the point on the earth where the mass of earth is gets balanced.

Since the earth is divided in to two hemispheres (i.e. the northern and southern hemispheres) and the equator is lying in between the two (Fig. 1). Therefore, at first the mass of these two hemispheres has been calculated by using oceanic crust mass along with hydrosphere mass and the continental crust mass.

2.1 Calculating the Crust Mass of Northern and Southern Hemisphere

The Earth's interior is divided into three main layers i.e., core, mantle and crust. Crust is the outermost layer of the earth which is further classified into two parts i.e. the oceanic crust and the continental crust. In proposed method the center of mass of earth has been estimated by considering the outermost layer of the earth since it is the only layer accessible by the people. Mass of continental crust and the oceanic crust along with hydrosphere has been taken into account. The oceanic crust is very thin as compared to continental crust (Fig. 2); therefore hydrosphere is used along with oceanic crust so that a uniform level of earth's surface could be achieved.

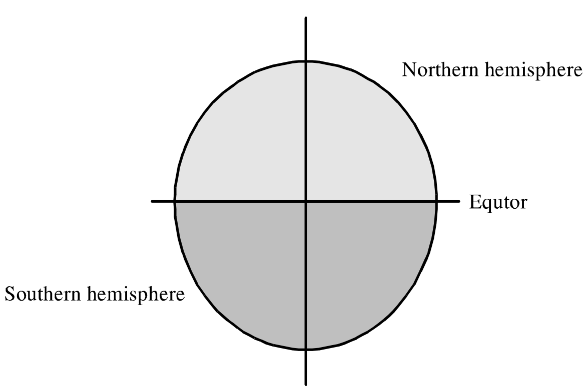

FIG. 1. BASIC EARTH MODEL

In [3-4], the O_{cm} (Oceanic Crust Mass) in total earths mass is given as 0.099% (where Earth's mass is, $5.9742 \mathrm{x} 10^{24}$ kg), and H_m (Hydrosphere Mass) is 0.023%. Therefore, the sum of O_{cm} and H_m can be given as:

$$TOH_{cm} = O_{cm} + H_m = 0.72885 \times 10^{22}\ kg$$

Similarly, the Continental crust mass makes up to 0.374% of the earth's mass [3-4],

$$TC_{cm} = 2.2344 \times 10^{22}\ kg$$

It is a well known fact that the earth's surface consists of 71% of ocean and 29% of land. For Northern hemisphere, the land to ocean ratio is 1:1.5 and for Southern hemisphere it is 1:4 as given in [5]. By using above ratios, the north to south land ratio is calculated to be 2:1 and ocean ratio is calculated to be 1:1.333. Thus, northern hemisphere consists of 19.34% of land and 30.433% of ocean (Fig. 3).

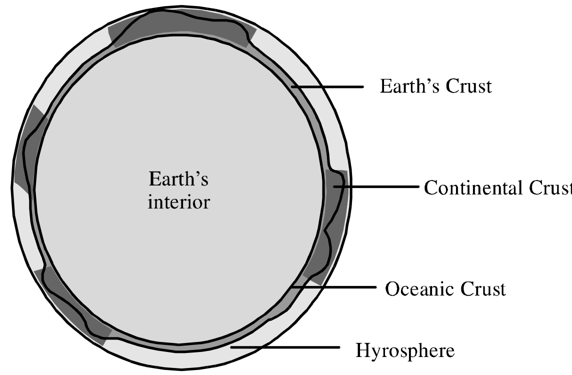

FIG. 2. STRUCTURE OF EARTH'S CRUST

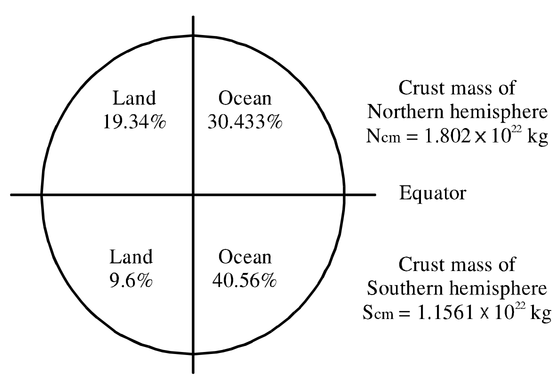

FIG. 3. CRUST MASS OF NORTHERN AND SOUTHERN HEMISPHERES

Therefore, the continental crust mass (NC_{cm}) of northern hemisphere is:

$$NC_{cm} = \frac{TC \times percent\ amount\ of\ land\ in\ Northern hemisphere}{total\ percent\ of\ land} = 1.49 \times 10^{22} kg$$

and the sum of oceanic crust mass and the hydrosphere mass (NOH_{cm}) is:

$$NOH_{cm} = \frac{TOH_{cm} \times percent\ amount of\ ocean\ in\ Northern\ hemisphere}{total\ percent\ of\ ocean} = 0.312 \times 10^{22} kg$$

Thus, the total crust mass of northern hemisphere (N_{cm}) is:

$$N_{cm} = NC_{cm} + NOH_{cm} = 1.802 \times 10^{22}\ kg \qquad (1)$$

Similarly, from given land to ocean ratio of 1:4 for southern hemisphere, the land is found to be 9.44% and the ocean to be 40.56% (Fig. 3). Rest of the followed procedure is same as mentioned earlier for northern hemisphere. The value of south continental crust mass (SC_{cm}) is found as:

$$SC_{cm} = 0.7297 \times 10^{22}\ kg$$

And the sum of Southern oceanic crust mass and hydrosphere mass denoted by (SOH_{cm}) is:

$$SOH_{cm} = 0.4164 \times 10^{22}\ kg$$

Thus, the total crust mass of southern hemisphere (S_{cm}) is:

$$S_{cm} = SC_{cm} + SOH_{cm} = 1.1561 \times 10^{22}\ kg \qquad (2)$$

By comparing Equations (1-2), it is clear that the two masses are not balanced along the equator; rather Northern hemisphere is heavier than Southern hemisphere because continental crust is heavier than the oceanic crust and northern hemisphere consists of more land as compared to southern hemisphere. Thus the assumption that "center of mass of earth is somewhere along the equator" is not correct. As clear from above results the two masses are not balanced along the equator. This could be the possible reason why the Earth's axis is tilted by 23.5°. As the northern hemisphere is heavier that is why it tries to shift the mass balancing axis from its original position and hence produces the tilt.

2.2 Calculating the Crust Mass of Earth above and below 21.5° Latitude

The next step is to determine that at which latitude of earth surface the mass of earth is getting balanced. After repeating the exercise for different latitudes we finally came up with 21.5°N as the mass balancing latitude of Earth and the procedure followed is explained in Fig. 4.

Since the crust mass of northern and southern hemispheres has already been found in section 2.1 therefore, the mass of crust in between equator and 21.5°N latitude (as shown by the shaded area in Fig. 4) has also been found. This calculated mass is added in crust mass of southern hemisphere to determine the mass of the earth's crust below 21.5°N latitude, and the same crust mass is subtracted from the crust mass of northern hemisphere to determine the mass of the earth's crust above 21.5°N latitude. Since the northern hemisphere consists of 90° latitudes each with 360° longitudes therefore, the total number of degree cells denoted as T_{dc} are 32,400 i.e. $T_{dc} = 90 \times 360$. Furthermore, the northern hemisphere consists of 40% land and 60% of ocean i.e. 1:1.5 therefore the number of degree cells occupied by land ($Nland_{cell}$) is:

$$Nland_{cell} = \frac{T_{dc} \times land\ \ prcentage\ in\ northern\ hemisphere}{100} = 12{,}960$$

Likewise the number of degree cells occupied by the Ocean ($Nocean_{cell}$) is:

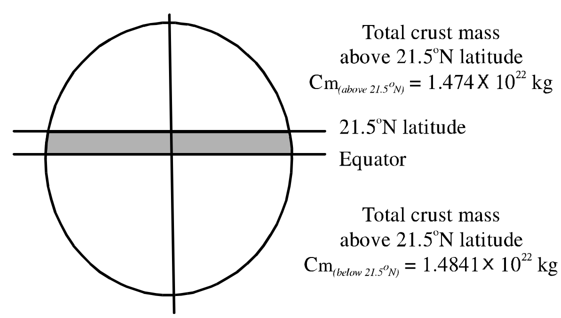

FIG. 4. MASS OF EARTH'S CRUST BELOW AND ABOVE 21.5° LATITUDE

$$Nocean_{cell} = \frac{T_{dc} \times ocean\ \ prcentage\ \ in\ \ northern\ \ hemisphere}{100} = 19{,}440$$

The degree cells in between equator and 21.5°N latitude are ($21.5° \times 360 = 7{,}740$) which constitute 23.89% of northern hemisphere. With the help of Google earth software it has been found that, the average amount of land per latitude from equator to 21.5°N is 96°, therefore, the number of land degree cells in this range of latitudes are:

$$land_{cell} = averageland\ \ per latitude \times rangeof\ \ latitudes = 2.064$$

The percent amount of land is:

$$land = \frac{land_{cel}}{Nland_{cell}} \times 100 = 15.926\%$$

Thus, the continental crust mass (BwC_{cm}) in between equator and 21.5°N is:

$$BwC_{cm} = \frac{land \times NC_{cm}}{100} = 0.237 \times 10^{22} kg$$

Similarly, it has been found that the average amount of ocean per latitude in between equator and 21.5°N latitude is 264°. Therefore the number of ocean degree cells in this range of latitudes are, $ocean_{cell} = 5.676$. The percent amount of ocean is:

$$ocean = \frac{ocean_{cell}}{Nland_{cell}} \times 100 = 29.197\%$$

So the sum of oceanic crust and hydrosphere mass ($BwOH_{cm}$) in between equator and 21.5°N is:

$$BwOH_{cm} = \frac{ocean \times NOH_{cm}}{100} = 0.091 \times 10^{22} kg$$

The total mass of earth's crust (Bw_{cm}) in between equator and 21.5°N is:

$$Bw_{cm} = BwC_{cm} + BwOH_{cm} = 0.328 \times 10^{22} kg \quad (3)$$

The total continental crust mass below 21.5°N latitude is:

$$BC_{cm} = 0.9767 \times 10^{22} kg \quad (4)$$

and the sum of oceanic crust and hydrosphere mass below 21.5°N latitude is:

$$BOH_{cm} = 0.5074 \times 10^{22} kg \quad (5)$$

Thus, the total mass of earth's crust below 21.5°N latitude is:

$$C_{m(below\ 21.5^o N)} = BC_{cm} + BOH_{cm} = 1.4841 \times 10^{22} kg \quad (6)$$

Following the same method as mentioned above, the continental crust mass of northern hemisphere above 21.5°N latitude (AC_{cm}) has been calculated and it results,

$$AC_{cm} = 1.253 \times 10^{22} kg \quad (7)$$

The sum of oceanic crust and hydrosphere mass (AOH_{cm}) is:

$$AOH_{cm} = 0.221 \times 10^{22} kg \quad (8)$$

Hence, the total mass of earth's crust above 21.5°N is:

$$C_{cm(above 21.5^o N)} = 1.474 \times 10^{22} kg \quad (9)$$

By comparing Equation (6 and 9) it is clear that the two masses are almost balanced along 21.5°N latitude with an approximate error of:

$$error = \left| 1 - \frac{C_{m(above 21.5^o N)}}{C_{m(below\ 21.5^o N)}} \right| \times 100 = 0.674\%$$

The approximate error is less than 1% which can be ignored. Thus 21.5°N latitude can be considered as more accurate mass balancing latitude of earth.

2.3 Calculating the Torque Balancing Latitude of Earth

In this section the other condition for CoM of earth has been verified, that is, the net effect of torque must be zero at 21.5°N latitude. For verifying this point 21.5°N has been considered as the reference latitude. The earth's part above this reference has been considered as object1 (O_1) and below this is considered as object2

(O_2) as shown in Fig. 5. By following the same method as mentioned in section 2.2, the mass balancing latitude for O_1 is found to be around 54°N and for O_2 it is around 31°S. Since the mass of object is concentrated at its center of mass, therefore, for determining the weight the average mass of reference latitude and average value of acceleration due to gravity for both objects has been calculated; their product results in the weight of two objects with respect to their reference latitudes. The distance between the reference latitude of earth and reference latitude of objects has been taken as the moment arm.

For O_1, on average there is 159° land/latitude and 201° ocean/latitude. It means on average the continental crust mass/latitude ($C_{cm/lat}$) for O_1 is:

$$C_{cm/lat} = \left[\frac{average\ degree\ of\ ocean\ per\ latitude}{total\ ocean\ in\ degree\ cells}\right]_{above\ 21.5^o N} \times AC_{cm} = 0.0183 \times 10^{22}\ kg$$

The average mass of oceanic crust and hydrosphere/latitude ($OH_{cm/lat}$) is:

$$OH_{cm/lat} = \left[\frac{average\ degree\ of\ ocean\ per\ latitude}{total\ ocean\ in\ degree\ cells}\right]_{above\ 21.5^o N} \times AOH_{cm} = 0.0032 \times 10^{22}\ kg$$

The average crust mass per latitude for O_1 ($TO_{1cm/lat}$) is:

$$TO_{1cm/lat} = C_{cm/lat} + OH_{cm/lat} = 0.0215 \times 10^{22}\ kg \quad (10)$$

Thus the average crust mass of 54°N latitude is 0.0215 × 10^{22} kg. The average value of acceleration due to gravity 'g' [6] for O_1 is:

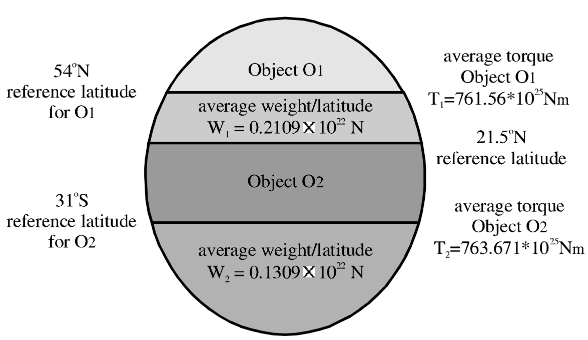

FIG. 5. TORQUE BALANCING LATITUDE

$$g_1 = \frac{g_{(21.5^o)} + g_{(90^o)}}{2} = 9.809\ m/s^2$$

Therefore, the average weight of 54°N latitude is:

$$W_1 = TO_{1cm/lat} \times g_1 = 0.2109 \times 10^{22}\ N \quad (11)$$

In [7], the approximated distance from reference latitude to 54°N is r_1=3,611 km. Therefore, the average torque for O_1 is found as:

$$T_1 = W_1 \times r_1 = 761.56 \times 10^{25}\ Nm \quad (12)$$

Similarly for O_2 the average land/latitude is 77° and average ocean/latitude is 283°. It means the average continental crust mass/latitude for O_2 is, $C_{cm/lat}$ = 0.0088 × 10^{22} kg. The average oceanic crust + hydrosphere mass/latitude is, $OH_{cm/lat}$ = 0.00455 × 10^{22} kg. The average crust mass per latitude for O_2 is:

$$TO_{2cm/lat} = 0.01335 \times 10^{22}\ kg \quad (13)$$

Thus average crust mass of 31°S latitude is 0.01335 × 10^{22} kg. The average value of acceleration due to gravity 'g' for O_2 is:

$$g_2' = \frac{g_{0^\circ} + g_{21.5^\circ}}{2} = 9.783\ m/s^2$$

Where

$$g_{0^\circ} = 9.78 m/s^2,\ g_{21.5^\circ} = 9.787\ m/s^2\ and\ g_{(90^\circ)} = 9.831 m/s^2$$

[6]:

$$g_2 = \frac{g_2' + g_{(90^\circ)}}{2} = 9.807\ m/s^2$$

The average weight of 31°S latitude is:

$$W_2 = 0.1309 \times 10^{22}\ N \quad (14)$$

In [7], the approximate distance from reference latitude to 31°S is r_2=5,834 km. Therefore, the average torque for O_2 is:

$$T_2 = W_2 \times r_2 = 763.671 \times 10^{25}\ Nm \quad (15)$$

By comparing the two Equations (12 and 15), it is clear that the net effect of torques of two objects is almost balanced at 21.5°N reference latitude with an approximate error of:

$$error = \left|1 - \frac{T_1}{T_2}\right| \times 100 = 0.28\%$$

Again the error is insignificant and can be ignored. Since 21.5°N latitude is satisfying all the conditions to be the CoM of earth. Therefore, it is clear that the CoM of earth is somewhere along 21.5°N latitude.

3. DISCUSSIONS

In satellite communication, the CoM of earth is generally assumed to be somewhere along the equator [8]. From above calculated results it is clear that equator is not satisfying any condition required for CoM of earth. Rather the conditions necessary for the CoM of earth are satisfied at 21.5° latitude as shown in Fig. 5. This newly calculated CoM of earth at 21.5°N latitude instead of traditional belief of Equator at 0° is certainly going to affect many fields such as satellite communications or related earth sciences. For example, in satellite communication, especially for geostationary satellites, more accurate CoM of earth has been found therefore it makes sense that the satellite in the orbit above 21.5°N latitude will remain static with respect to earth. In this orbit the forces and torques must be balanced therefore no external forces or torques will be required to maintain the position of geostationary satellites in the orbit, that are normally required for the geostationary satellites launched above equator. Besides this many other advantages can be achieved as discussed below:

(a) The earth bulge at 21.5°N latitude is very small as compared to equator. Earth bulge causes the satellite to drift slowly along the orbit, to one of two stable points at 75°E and 105°W, to counter this drift the jets are pulsed once every 2 or 3 weeks [8]. The geostationary satellite above 21.5°N latitude will suffer less from this problem since the earth bulge is very small.

(b) The radius of geostationary orbit is constant, that is, $a_{gso} = 42{,}164\,km$, this gives the height of orbit above earth's surface as $h_{gso} = 35{,}786\,km$ with equatorial radius of $r_e = 6378\ km$ [8]. Since from equator to pole the latitudinal radius decreases, therefore for keeping the radius of geostationary orbit constant the amount by which the latitudinal radius is decreased, by the same amount the height of orbit above earth's surface needs to be increased. Since the height of orbit above Earth's surface is increased therefore the coverage of geostationary satellite will be increased and it will also provide the increased limits of visibility. The two advantages altogether creates the possibility that two satellites will be enough to provide the full coverage of earth's populated areas.

(c) The geostationary satellite above equator cannot cover Polar Regions of the earth [8], but the geostationary satellite above 21.5°N latitude can easily cover the north polar region of the earth thus this is another very good point about this orbit.

4. CONCLUSION

The main focus of this paper is to locate the CoM of earth more accurately by providing some mathematical solutions. It also discusses the advantages of calculated results for satellite communication. These results can be proved very useful in various other fields such as in the study of global sea level change, earthquakes and volcanoes. The center of mass of earth has been found mathematically by using land to ocean ratios and the data is collected from Google earth software. The final results are accurate with an approximate error of 1%.

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