Variation Propagation Control in Straight-Build Assemblies: 2D Case Study

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ABSTRACT

It is essential that rotating engines are assembled so that they are straight (straight-build) to satisfy the vibration and functional requirements of the engine. Geometric variations in different components can have a significant impact on the straight-build of an assembly. This paper considers the use of optimization methods to take account of these geometric variations. Five assembly-optimization methods for the straight build of an assembly are investigated with the aid of a connective assembly model that calculates the variation propagation in the assembly. The optimization methods used are; (i) minimizing the distances from component centres to table axis; (ii) minimizing the distance between actual and nominal component centres; (iii) minimizing the angular errors between actual and nominal planes; (iv) target-axis build assembly optimization; (v) changing-axis-build assembly optimization. Two 2D case-studies are investigated to analyse the suitability of the different optimization procedures. Simulation results show that optimization Procedures 1 and 3 have great potential to be applied to real assemblies.

Key Words: **Straight-Build Assembly, Tolerance Analysis, Variations Propagation.**

1. INTRODUCTION

Straight-build assembly is a way of joining parts together in order to have straight line between the component centres [1-2]. In assemblies like high speed rotating engines, the parts are assembled in order to meet the vibration requirements of the engine. In such cases, it is necessary to avoid internal bending of the rotor to meet the functional requirements of the engine. In mechanical assembly, the parts share their mating features with each other [3]. Due to the inherent nature of variability in manufacturing process, the mating features of parts are produced with imperfect shapes and in imperfect location and orientation [4]. These geometric variations play a significant role in error build-up during straight build of an assembly. For the straight build of an assembly, it is important to understand the impact of geometric variation of parts and predict and control assembly variation propagations [5]. The main aim of the straight build method is to assemble components compensating for the eccentricity error and angular orientation of mating surfaces to minimise internal bending of the central axis in the final assembly.

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Assembly optimization has not been given much attention in the past. Most of the research e.g. [6-8] has focussed on allocating assembly tolerances to minimize total assembly cost. Zhang, et. al. [9] developed an analytical model for optimal process sequence selection through planning the dimensions and tolerances for assembly tolerance allocation. Assembly optimization studies have also been carried out in the past to achieve product quality and robustness based on Genetic Algorithms to address the worst-case tolerance analysis [10]. The worst case tolerance analysis is not well suited, as it gives results that are overly passimistic. The current study suggests optimization methods to minimize the stage-by-stage variation propagation in the straight-build of a mechanical assembly.

In this paper, optimization methods are used in combination with a variation propagation algorithm to control error build-up in the assembly. The algorithm predicts the best possible combination of component parts to be assembled to minimise the error build-up during each assembly stage. A closed form error propagation algorithm also known as the connective assembly model is used in this paper to calculate part to part variation propagation in the assembly. A connective assembly model was also employed by Mantripragada, et. al. [3] and Whitney, et. al. [11] but they did not consider the effect of geometric feature variation. Five optimization methods are proposed in this paper to control variation propagations in the straight build of an assembly. These are discussed separately in this paper. The five optimization methods are considered for their applicability in practical assembly operations of axi-symmetric components. A comparison is made between the proposed optimisation methods and assembly of components without optimisation (simply stacking components without minimising the error at each stage) to investigate the potential to reduce the assembly error build-up in each optimization procedure.

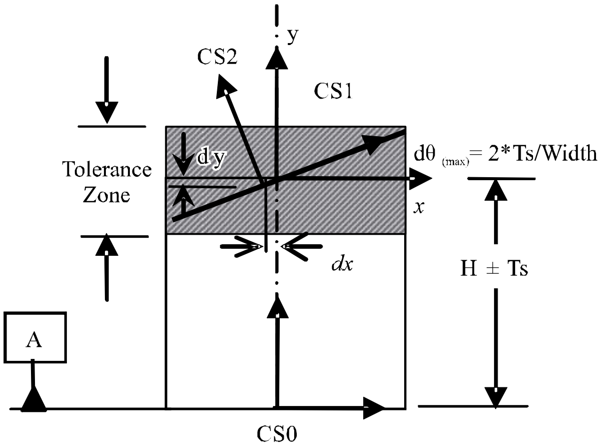
2. VARIATION PROPAGATION MODEL

To study and analyse assembly related problems it is essential to develop an appropriate mathematical assembly model which enables an assembly and its components to be represented and analysed using a suitable algorithm. In assembly modelling, it is assumed that reference frames are attached to the centre of each mating feature of a part by means of a matrix transform. A connective assembly model is used here to calculate variation propagation in the assembly. In the connective assembly model, accumulated assembly variations are described by the position and orientation of coordinate frames attached to mating features of the assembled component. The model also assumes that parts are assembled by joining mating features together [12]. The transform matrix represents the operation of rotation and translation on a coordinate frame originally aligned with a reference coordinate frame [13]. In the 2D case-study, the components are considered as axi-symmetric rectangular components. For a rectangle, the mating features are the straight lines that come into contact with the mating component (rectangle in 2D case study) during the assembly. In the 2D case study, component variation is represented as translation of the attached reference frame along the x and y-axes, whilst the rotation error is only considered about the z-axis perpendicular to the 2D plane (Fig. 1). From Fig. 1 it is clear that the angular variation ($d\theta$) in the orientation of the mating feature of the 2D rectangular component is about the z-axis. Hence, the variation propagation analysis in the current 2D case study will only consider the rotation matrix about the z-axis.

If the actual assembly contains variations from its nominal situation then the actual position of the i^{th} frame can be determined by the transform matrix (Fig. 2) [14]:

$$T_i = T_{Ni} + dT_i \tag{1}$$

where dT_i is the differential change in nominal transform, T_{Ni} is the nominal transform matrix (no errors are present) and T_i is the actual position of the i^{th} frame as shown in Fig. 2. Using this technique, we can model an assembly as a chain of frames. For '*n*' assembly components, the actual position of the n^{th} frame can be determined by multiplying the actual transforms of all stages as:



Tolerance Zone for 2D surface feature: The actual surface may lie anywhere within this zone at any angle, but may not extend beyond the zone.

FIG. 1. A TYPICAL EXAMPLE OF ASSEMBLY COMPONENT GEOMETRIC VARIATION

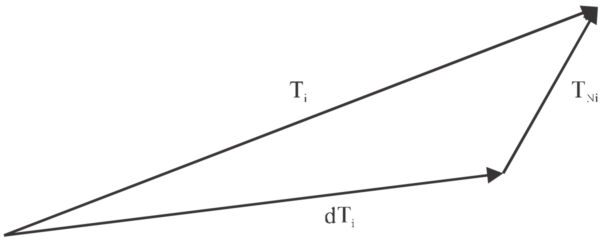


FIG. 2. EFFECT OF SMALL ERROR TRANSFORM ON NOMINAL TRANSFORM

$$T^F=T^F_N+dT^F = (T_{N1}+dT_1)\ x\ (T_{N2}+dT_2).....(T_{Nn}+dT_n) \tag{2}$$

or

$$dT^F = T^F - T^F_N \tag{3}$$

The details of the assembly variation propagation model can be found in [11]. This paper only considers the model for the prediction of stage-by-stage variation propagations in the assembly. To control error build-up in assembly, this paper proposes optimisation techniques to achieve straight build assembly.

3. ASSEMBLY OPTIMIZATION

To reduce the chance of assembly failure, it is essential to optimize assembly errors during each assembly stage. Optimization is used in many fields of engineering to minimize or maximize a function critical to the problem being solved [15-16]. Here, the goal is to minimize the error build-up in the assembly during each stage by altering a number of parameters in the assembly variation propagation model. The purpose of the proposed assembly optimization methods is to minimize variations propagated during each assembly stage by rotating/re-indexing a component about its central axis to achieve an optimum mating orientation. Regardless of the optimization technique used to solve the problem, the general procedure of optimization in combination with the variation propagation model can be illustrated as shown in Fig. 3.

Fig. 3 demonstrates that for given optimization techniques, errors from the previous stage and part variations at the current stage are provided as input variables to the variation propagation model. The variation propagation model calculates the accumulated error for the available number of orientations. The results of the accumulated error at each available orientation are evaluated to find out the orientation that gives the minimum value of accumulated error. The minimum output value of the accumulated error for the optimum orientation is given as input to the next assembly stage.

For 2D case study, there are two possible indexing orientations of each rectangular component to minimize error build-up during assembly. i.e. each component can be oriented in flipped or un-flipped positions about its central axis. In this paper error '*dx*' refers to eccentricity of the centre of an assembly feature to the table axis, while error '*e*' refers to eccentricity of the centre of an assembly feature to a target axis other than the table axis (Fig. 4). The details of each optimization are given below:

3.1 Straight-Build Assembly Optimization w.r.t. Table Axis (Optimization Method 1)

This assembly optimization method aims to reduce the stage-by-stage the eccentricity error (dp_x) from the centre of each component to the table/datum axis. Optimization Method 1 is taken from Hussain, et. al. [17]. Here the table axis is the axis perpendicular to the plane of the centre of base of the first component. For the 2D case study eccentricity error 'dp_x' is evaluated at each stage without flipping and after flipping the component. The position of the component having minimum error is then saved and used as the input to the next assembly stage. Fig. 4(a) illustrates that the components are assembled in such a way that they follow the datum axis.

3.2 Minimizing the Distance Between Actual and Nominal Centres of Component (Optimization Method 2)

Procedure 2 is introduced with the aim of analyzing the effect of vertical error in locating an assembly feature on variation propagation of assembly. The effect of vertical error is analysed by minimizing the vertical error along with the eccentricity error (dp_x) at each stage of assembly. Vertical error is referred to the position error in the location of an assembly feature from its nominal position in the y-direction. Vertical assembly error is represented by 'dp_y'. In Procedure 2, the combined effect of errors ('dp_x' and 'dp_y') is calculated by the vector norm as: $\|\text{CombError}\| = \sqrt{dp_x^2 + dp_y^2}$. In this case $\|\text{CombError}\|$ is calculated at each stage without flipping and after flipping the component. Component orientation with minimum accumulated error is then saved and used as the input to the next assembly stage.

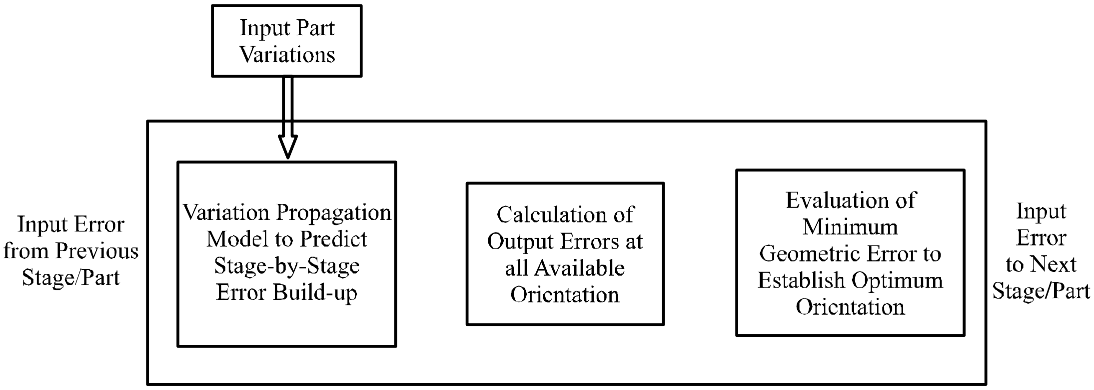


FIG. 3. THE OPTIMIZATION PROCESS.

3.3 Minimizing the Angular Errors Between Actual and Nominal Planes (Optimization Method 3)

Procedure 3 aims to minimize stage-by-stage the angular error between the nominal plane of the assembly feature and the actual plane of the assembly feature for each component (Fig. 4(b)). Angular error is represented as 'dr_θ' and is referred to as the parallelism error between the actual and the nominal plane. In Procedure 3, 'dr_θ' is calculated at each stage without flipping and after flipping the component. The position of the component with minimum error is then saved and used as an input to the next assembly stage.

3.4 Target-Axis Build Assembly Optimization (Optimization Method 4)

This optimization method considers a different way of assembling the components. For the straight build of an assembly, variation in the base component has greater influence on the assembly straight build than the other components. Due to this reason, Procedure 4 is introduced with the aim of targeting the axis generated by the two centers of the base component to minimise the error between the target axis and upcoming assembly components. In other words the assembly process follows the datum axis that passes through the centers of the two surfaces of the first component of the assembly. At each stage, the perpendicular distance from the assembly target axis to the centre of the top feature of the assembled component is calculated without flipping and after flipping the component (Fig. 4(c)). The position of the component with minimum perpendicular distance is then saved and used as the input to the next assembly stage.

3.5 Changing-Axis Build Assembly Optimization (Optimization Method 5)

Changing-axis-build assembly optimization process aims to align the axis of the assembly for stage '*k*' with the axis of assembly achieved during stage *k-1*. Assembly axis refers to the axis passing through the centre of the base of the first component to the centre of top of the 'k^{th}' component in the 'k^{th}' assembly station. At each stage of assembly, the perpendicular distance from the k^{th} assembly axis to the (*k-1*)st assembly axis is calculated without flipping and after flipping the component (Fig. 4(d)). The component orientation with minimum accumulated error is then saved and used as the input to the next assembly stage.

4. RESULTS AND DISCUSSION

Two case studies are considered for calculating assembly variation propagations. For the case studies considered following assumptions are made:

(i) Dimensional tolerances are known, and Normally distributed.

(ii) Assembly components are assumed to be 2D rectangles.

(iii) There is no error at the mating between the two components.

(iv) Assembly features are represented as being rectangular components described by straight lines.

(v) The base centre of the first component is the origin of the GCS (Global Coordinate System).

(vi) Each component is subjected to location variations in two directions, along the x- and y-axes (*dx* and *dy*), and an orientation error ($d\theta$) as shown in Fig. 1.

In assembly simulations the input part variations are considered as Normally distributed random variables $N(\mu,\sigma^2)$ based on the '±3σ principle'. The Tolerance Zone limit for 2D rectangle [11] is illustrated in Fig. 1.

4.1 Case Study-1: Assembly of Four Identical Components

Initially the 2D assembly consists of four rectangular components with the same nominal dimensions. Here the candidate assembly is analyzed to compare the six assembly procedures by checking their performance for a straight build assembly. Each rectangular component has nominal dimensions of 100mm width and 70mm height with tolerance limit of 0.1mm in both dimensions. In assembly, the rectangular components are assembled together without considering process and measurement errors. The simulation results of 10,000 runs for each optimisation method are produced to analyse the effectiveness of each optimization method.

4.2 Discussion on Results of Case Study-1

The results of average error 'dp_x' in Table 1 reveal that optimization methods 1and 2 result in the smallest stage-by-stage assembly variation and the two methods produce identical results. The average stage-by-stage reduction in the variation for Procedure 1 is 43.4% of that for Procedure 6. The histogram of error 'dp_x' (Fig. 5) also verifies that optimization methods 1and 2 produce the same results and have a smaller range of error distribution with higher level of confidence than the other methods. On other hand, results for eccentricity error 'e' reflect that optimisation method 5 produces the smallest stage-by-stage variation in the assembly.

4.3 Case Study-2: Assembly of Four Non-Identical Rectangular Components

In this case study, assembly of four non-identical components is considered for comparison of the six assembly procedures. The assembly has non-identical 2D rectangular components. Each component has allowable

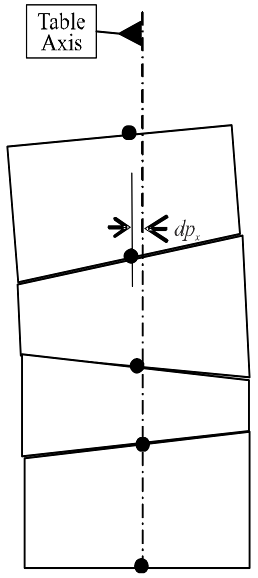


FIG. 4(a). OPTIMIZATION METHOD 1

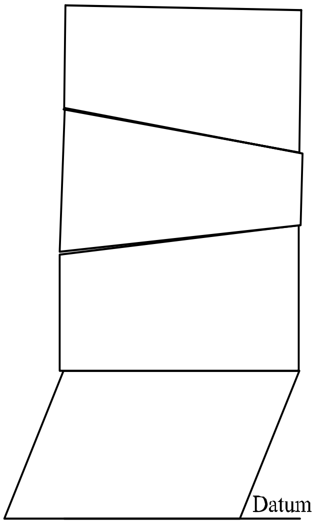


FIG. 4(b) OPTIMIZATION METHOD 3

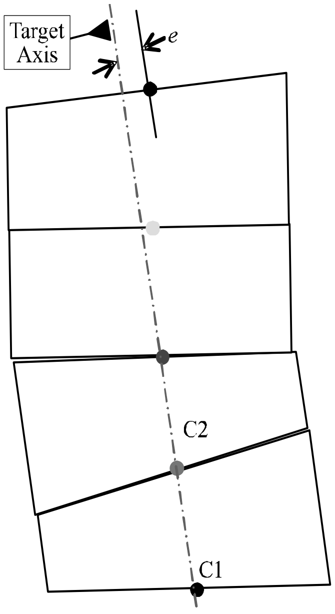


FIG. 4(c) OPTIMIZATION METHOD 4

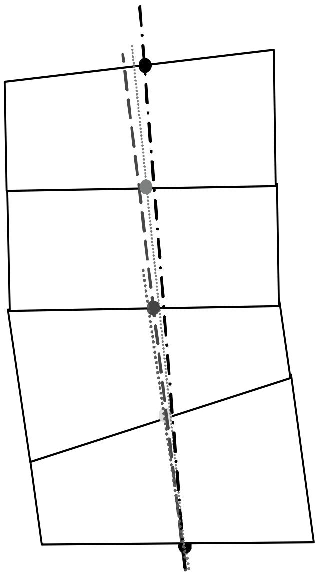


FIG. 4(d) OPTIMIZATION METHOD 5

variation limits of 0.1mm in both the *x* and *y* dimensions. The dimensions of all components are given in Table 2. Dimensions of the assembly components in case 2 are scaled dimensions of a practical assembly for a rotating engine. The principles of assembly for Case Study-2 are the same as Case Study-1. Results for the six assembly procedures, for the given assembly components are simulated 10,000 times to predict the effect of tolerance build-up during each assembly stage.

4.4 Discussion on Results of Case Study-2

The assembly results of non-identical components reveal that unlike Case Study-1, optimization methods 4 and 5 result in larger values of average error dp_x (Table 1) than assembly without optimization (Procedure 6). Similar to Case Study-1, optimization method 1 produces identical results as optimization method 2 for the accumulated error. Optimization methods 1 and 2 have the smallest value of average error dp_x for the six procedures considered. The average stage-by-stage reduction in the variation for Procedure 1 is 42% of that for Procedure 6. Optimisation method 3 also produces a higher value of average error. The overall results for Case Study-2 reveal that Procedures 1 and 2 produce smallest errors with a high level of confidence.

TABLE 1. MEAN OF ECCENTRICITY (dp_x) FROM TABLE AXIS, AND MEAN OF ECCENTRICITY (*e*) FROM TARGET AXES

Optimization Method	Mean of 'dp_x'				Mean of Eccentricity '*e*'			
	Stage-1 (mm)	Stage-2 (mm)	Stage-3 (mm)	Stage-4 (mm)	Stage-1 (mm)	Stage-2 (mm)	Stage-3 (mm)	Stage-4 (mm)
1	0.0267	0.0286	0.0425	0.0646	0.0267	0.0286	0.0425	0.0646
2	0.0267	0.0286	0.0425	0.0646	0.0267	0.0286	0.0425	0.0646
3	0.0267	0.0466	0.0645	0.0772	0.0267	0.0466	0.0645	0.0772
4	0.0267	0.0469	0.0681	0.0958	0.0000	0.0288	0.0519	0.0814
5	0.0267	0.0469	0.0701	0.0982	0.0000	0.0288	0.0301	0.0320
Procedure 6	0.0267	0.0470	0.0784	0.1180	0.0267	0.0470	0.0784	0.1180

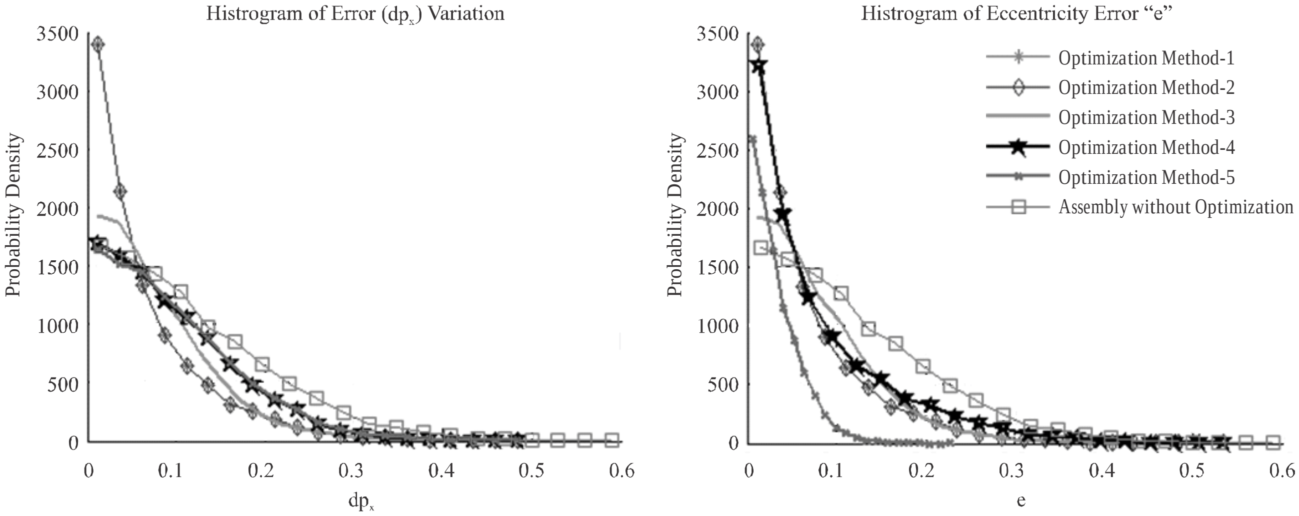


FIG. 5. HISTOGRAM OF FINAL ASSEMBLY VARIATION FOR ERROR 'dp_x' AND 'e'

TABLE 2. DIMENSIONS OF FOUR COMPONENTS OF THE ASSEMBLY

	Component-1 (mm)	Component-2 (mm)	Component-3 (mm)	Component-4 (mm)
Height (y)	9	202	32	390
Base Width (x_1)	206	538	572	572
Top Width (x_2)	206	572	572	310
Tolerance in $x(dx)$	±0.1	±0.1	±0.1	±0.1
Tolerance in $y(dy)$	±0.1	±0.1	±0.1	±0.1

TABLE 3. MEAN OF ECCENTRICITY (dp_x) FROM TABLE AXIS, AND MEAN OF ECCENTRICITY (e) FROM TARGET AXES

Optimization Method	Mean of 'dp_x'				Mean of Eccentricity 'e'			
	Stage-1 (mm)	Stage-2 (mm)	Stage-3 (mm)	Stage-4 (mm)	Stage-1 (mm)	Stage-2 (mm)	Stage-3 (mm)	Stage-4 (mm)
1	0.0263	0.0349	0.0308	0.0858	0.0263	0.0349	0.0308	0.0858
2	0.0263	0.0349	0.0308	0.0858	0.0263	0.0349	0.0308	0.0858
3	0.0263	0.0549	0.0639	0.1024	0.0263	0.0549	0.0639	0.1024
4	0.0263	0.0627	0.0850	0.1561	0.0000	0.5664	0.6352	1.7511
5	0.0263	0.0627	0.0850	0.1561	0.0000	0.5664	0.0206	0.1058
Procedure 6	0.0263	0.0541	0.0647	0.1392	0.0263	0.0541	0.0647	0.1392

5. CONCLUSIONS

In this paper, five assembly Optimization methods have been evaluated to reduce error build-up in straight build assemblies of identical and non-identical components. As expected, it has been noticed that the five optimisation procedures (except Optimization Method 4) produce lower levels of output error for both error 'dp_x' and error 'e' than the assembly without optimization (Procedure 6). The overall results of the assembly of identical components concludes that Optimization methods 1 and 2 produce best results for eccentricity error (dp_x) with respect to the table axis, whereas, Optimization method 5 produces best results for eccentricity (e) with respect to the target axis. For assembly of non-identical components, the simulation results reveal that Optimization methods 1 and 2 produce best results for both eccentricity error with respect to table axis and eccentricity error with respect to target axis. For the two case studies considered it is seen that Optimization method 2 produces identical results to Optimization method 1, which indicates that it is not necessary to use Optimization method 2 when Optimization Method 1 has already been used. The results of the two case studies also indicate that Optimization Methods 3 and 4 produce less satisfactory results than Optimization Method 1.

The overall conclusion of the study is that Optimization Procedure 1 has the most potential to minimise the error build-up in the straight build of an assembly and can be applied to practical assemblies.

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