Prediction of Elastic-Plastic Behaviour of Structures at Notches

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ABSTRACT

Under the condition of elastic-plastic deformation, aero engine casings experience local stress and strain concentrations along with associated variations in load paths and stiffness. The accurate prediction of such behaviour is clearly necessary for design optimisation, potentially leading to beneficial weight savings. The present research seeks to tackle the objective of accurate characterisation of elastic-plastic casing behaviour. The objective is to develop approximate techniques for predicting the elastic-plastic behaviour, for both generalised load-displacement responses (i.e. for global response) and notch stress-strain responses. Accurate prediction of the stress-strain distribution at a notch is difficult and existing notch prediction techniques can only be used for small strains. This paper seeks to develop novel techniques for predicting large elastic-plastic notch strain and associated stresses, with application to aero engine casing notches. The repeated local joints at the spoke-shell casing are of particular interest as they are the most likely sites for plastic deformation and possibly crack initiation. These local joints incorporate realistic notch-type features and the load cases cover a range of loading combinations, to develop insight and understanding of the elastic-plastic behaviour. This work analyse a double edge-notched flat bar with semicircular notches and a representative case of actual aero engine casing-type structures in a more simplified form. The investigation was carried out for structures for which stress and total strain are related by a power law. The equivalent stress at a notch can be estimated, given the value of n, by a linear interpolation between the stresses for a cases n=1 and n=0. The application of the notch stress-strain prediction method is illustrated through use of examples of notch components. The predictions are compared with results obtained using finite element analyses and approximate methods proposed by Nueber and Glinka.

Key Words: **Power Hardening, Creep, Notched Structures, Elastic-Plastic, Notch Stress and Strain.**

1. INTRODUCTION

In mechanical structures notches are important because these are the positions at which most failures originate [1-2]. Therefore, it is highly desired to predict stress-strain concentrations at notches during the design of structures. Without the use of nonlinear finite element analysis, the prediction of the stress-strain distribution at notches is difficult. Existing approximate notch strains prediction techniques are generally based on the use of elastic solutions, which are reasonably accurate for predicting small plastic strain [3-13]. Some methods [4-6] also use limit loads to assist in predicting the notch strains.

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Elastic-plastic notch stresses and strains are used in multiaxial fatigue life predictions, the equivalent stress (strain) approach is the most commonly used method for the evaluation of multiaxial fatigue life [14-16]. Therefore, accurate prediction of elastic-plastic notch stress-strain predictions, including large strains are clearly necessary for design optimisation, potentially leading to beneficial weight savings in structures such as those for aero engine casings.

Elastic-plastic FE (Finite Element) programs, with large strain facilities, exist and can be used to determine the stresses and strains in structures. However, in practice, the number of the elements required, in order to obtain accurate elastic-plastic results for complex structures, e.g. aeroengine casing structures, may be very large. Therefore, the current research seeks to develop of techniques for predicting the large elastic-plastic notch stress and strains without the need for excessive finite element analyses.

Simple structures, made of material for which the creep strain rate is a power law function of stress were studied by Calladine [17], which shows that the greatest stress in the structure varies linearly with the reciprocal of the power law exponent n'. The linear interpolation method is for structures made of materials where stress is related to total strain as power law. The notch stress for an appropriate value of n can be obtained by simple linear interpolation between elastic-stress ($n=1$) and perfectly plastic stress ($n'=0$).

Based on the linear interpolation method [18-19], local elastic-plastic stress and strain components, under monotonic load conditions, are predicted, for simple notched components, i.e. plane strain, plane stress, notched bars. Future publications include the behaviour of more complex casing-type structures. It was found that the predicted local elastic-plastic stress and strain components correlate well with Finite Element Analysis results.

2. PROPOSED NEW NOTCH STRAIN PREDICTION TECHNIQUE

Previous investigations on the performance of existing notch strain prediction techniques revealed some shortcomings of these approaches. Consequently, it was decided to explore the applicability of a technique proposed by Calladine [17] for predicting the maximum stress in a component undergoing power-law creep behaviour, i.e.

$$\varepsilon = B\sigma^{m} \quad (1)$$

where ε represents the secondary creep rate. B,m represents the material properties at the appropriate temperature. The Calladine approach is essentially an interpolation method based on observed near-linearity of the variation of maximum component stress with the inverse of stress exponent m for power-law creep [20]. The advantage of the Calladine method is that the interpolation can be achieved using (i) a linear elastic solution (m=1) and (ii) an elastic-perfectly plastic solution (m=∞), as shown in Fig. 1.

Based on the analogy between power-law creep (rate-dependent) and power-law hardening (rate-independent), it was anticipated that the same interpolation could be employed to give more accurate notch stress, and thus strain, predictions, than existing techniques, for the rate-independent case.

The new method developed in order to predict maximum notch stress and thus strains is based on a n-interpolation scheme. The prediction of maximum notch stress for appropriate values of n can be performed by linear interpolation between elastic-stress (n=1) and perfectly plastic stress (n =0). For the proposed method, the material's uniaxial stress-strain curve (Fig. 2) specified above yielding, i.e.

$$\sigma = K\varepsilon^{n} \quad (2)$$

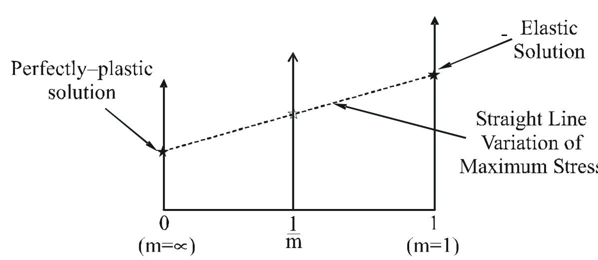

FIG. 1. DEMONSTRATION OF INTERPOLATION SCHEME FOR CREEP BEHAVIOUR

where $\sigma_{n,y}$ and E are the yield stress and Young's modulus of the power-hardening material. K,n are material constants.

Next step is determination of notch maximum equivalent stress σ_{eq} for given power-hardening material and applied load S. The elastic-plastic notch equivalent stress σ_{eq} can be estimated by simple linear interpolation between n=1, n=0 cases, as shown in Fig. 3.

The elastic-plastic notch equivalent stress, σ_{eq}, can be expressed as:

$$\sigma_{eq} = \sigma_{pp} + n(\sigma_{el}-\sigma_{pp}) \qquad (\sigma_{eq} \geq \sigma_{n,y}) \tag{3}$$

where σ_{el}, σ_{pp}, are elastic stress and perfectly plastic-stress respectively. The elastic stress σ_{el} for a applied load, S.

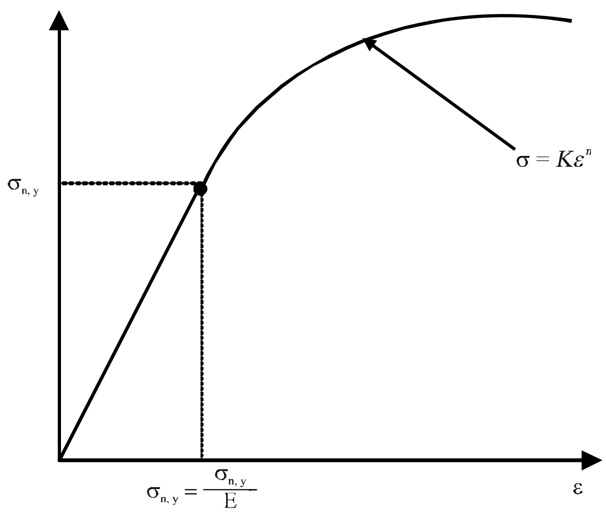

FIG. 2. GRAPHICAL REPRESENTATION OF MATERIAL CURVE

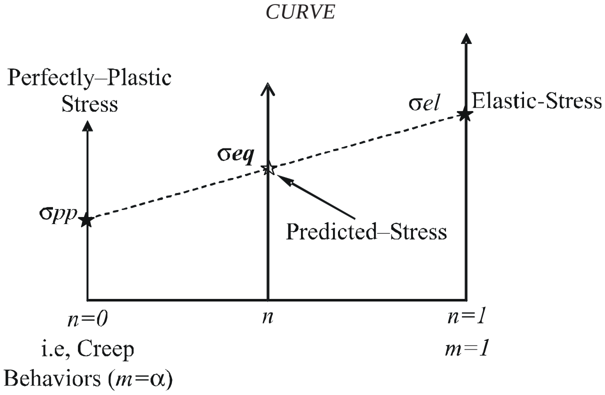

FIG. 3. SCHEMATIC DIAGRAM OF INTERPOLATION CURVE FOR GIVEN n AND APPLIED LOAD 'S'

$$\sigma_{el} = K_{tq}S \tag{4}$$

The equivalent stress concentration factor K_{tq} can be defined as the ratio of the maximum elastic-equivalent Mises stress in the structure to applied load. The K_{tq} can be obtained by considering equilibrium and compatibility equations or performing an elastic FE analysis. The perfectly-plastic stress σ_{pp}(n=0) is defined for given applied load, S.

$$\sigma_{pp} = C\frac{\sigma_{\gamma}S}{S_p} \tag{5}$$

where S_p is the plastic limit load level of the structure, for the elastic-perfectly-plastic material with the yield stress σ_y. The limit load of the structure can be estimated by equilibrium considerations, i.e, upper bound/lower bound theorems or performing single elastic-perfectly-plastic, FE analysis.

By rearranging Equation (3)

$$\sigma_{eq} = (1-n)\sigma_{pp} + n\sigma_{el}$$

$$\sigma_{eq} = \left[(1-n)\frac{\sigma_y}{S_p} + nK_{tq}\right]S \qquad \left(\sigma_{eq} \geq \sigma_{n,y}\right) \tag{6}$$

Subsequently, equivalent notch strain is calculated by using Equation and σ_{eq}. The notch equivalent strain can be written as:

$$\varepsilon_{eq} = \left[\frac{(1-n)\frac{\sigma_y}{S_p} + nK_{tq}}{K}\right]S^{\frac{1}{n}} \qquad \left(\sigma_{eq} \geq \sigma_{n,y}\right) \tag{7}$$

The elastic equivalent notch strains, below yielding, can be calculated by consideration of Hooke's law, i.e.

$$\varepsilon_{eq} = \frac{K_{tq}S}{E} \qquad \left(S < \frac{\sigma_{n,y}}{k_{tq}}\right) \tag{8}$$

3. NEUBER, GLINKA APPROXIMATION FORMULAS FOR THE MULTIAXIAL NOTCH STRESSES AND STRAINS, FOR THE POWER HARDENING MATERIAL

3.1 Neuber's Rule

Neuber's rule is probably the best-known approximate method for notch strain prediction and has been traditionally employed with cyclic stress-strain curves for low-cycle fatigue life prediction. Neuber established that:

$$K_t^2 = K_\varepsilon K_\sigma \tag{9}$$

for longitudinal notches in prismatic members under torsion, where K_t, K_ε and K_σ are the elastic stress concentration factor, the local strain concentration factor and the local stress concentration factor, respectively. This relationship permits a hyperbolic expression to be developed between the applied load, S, the equivalent stress concentration factor, the elastic modulus and the equivalent notch stress σ_{eq} and strain, ε_q

$$\sigma_{eq}\varepsilon_{eq} = \frac{K_t^2 S^2}{E} \tag{10}$$

Note that, Neuber's rule presented here doesn't taken into account behaviour of non-linear net section. For the material with power-hardening property after yielding a closed form solution for the elastic-plastic stresses and strains can be obtained. Using Equation (2) and Equation (10), the notch equivalent notch stress σ_{eq}, and notch strains ε_q can be derived as:

$$\sigma_{eq} = k\left[\frac{(K_{tq}S)^2}{EK}\right]^{\frac{n}{n+1}} \quad \left(S \geq \frac{\sigma_{n,y}}{k_{tq}}\right) \tag{11}$$

$$\varepsilon_{eq} = \left[\frac{(K_t S)^2}{EK}\right]^{\frac{1}{n+1}} \quad \left(S \geq \frac{\sigma_{n,y}}{k_{tq}}\right) \tag{12}$$

Below the yielding point, Hooke's law can be used to calculate notch strain as shown in Fig. 4.

3.2 Glinka Method

The Glinka method is based on the assumption that the strain energy density at a notch root does not change significantly if the localised plasticity is surrounded by predominantly elastic material. The Glinka method is therefore based on equivalence of the elastic energy (W_e) and elastic-plastic notch strain energy (W_p) densities and can be used to predict local stresses and strains at notch roots, again based on elastic results. The Glinka method can be written as:

$$W_p = W_e \tag{13}$$

$$W_e = \frac{(K_{tq}S)^2}{2E} \tag{14}$$

$$W_p = \int_0^\varepsilon \sigma \times d\varepsilon \tag{15}$$

For a power-law material, beyond the point of yielding, the following expression can be easily derived from Equations (2,13-15) to estimate the notch strain (Fig. 5).

$$\sigma_{eq} = k\left[\frac{(n+1)(k_t S)^2}{2Ek}\right]^{\frac{n}{n+1}} \tag{16}$$

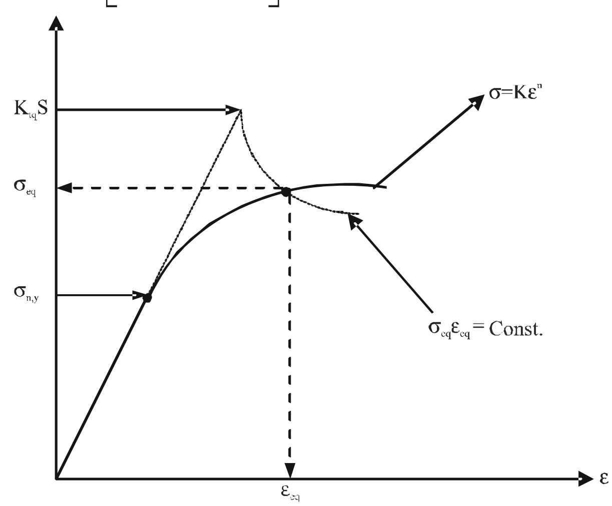

FIG. 4. GRAPHICAL REPRESENTATION OF NUEBER'S RULE FOR APPLIED LOAD 'S'

$$\varepsilon_{eq} = \left[\frac{(n+1)(k_t S)^2}{2Ek} \right]^{\frac{1}{n+1}} \qquad (17)$$

4. FE MODELLING OF SIMPLE NOTCH STRUCTURE

Application of the proposed method is illustrated by the example of the notched structures such as:

- Double edge-notched flat bar (3D models with 10mm thickness).
- Local joints of casing structure (multiaxial-condition).

Fig. 6 shows the geometry of the 10mm thick, double edge-notched flat bar with semicircular notches under applied axial load *S*. Only one quarter of the geometry is modelled due to the geometrical and loading symmetry and 20-node, reduced integration, brick elements that are employed.

The mesh of the local shell-to-outer casing connection model (local joint) is shown in Fig. 7. For the present work, the local joint incorporates realistic notch features, i.e. 9mm-notch radius. The loading modes considered is axial loading and reduced integration, twenty node solid elements are employed.

In order to calculate the values of K_{tq}, S_p of the structures, FE elastic-perfectly plastic analyses were carried out using ABAQUS (CAD tool for FE analysis) [21]. It was assumed to be made of an elastic-perfectly-plastic material with a Young's modulus of 71.2 GPa, a Poisson's ratio of 0.3 and a yield stress of 250 MPa. The results obtained from FE elastic-perfectly plastic analyses, presented in Table 1.

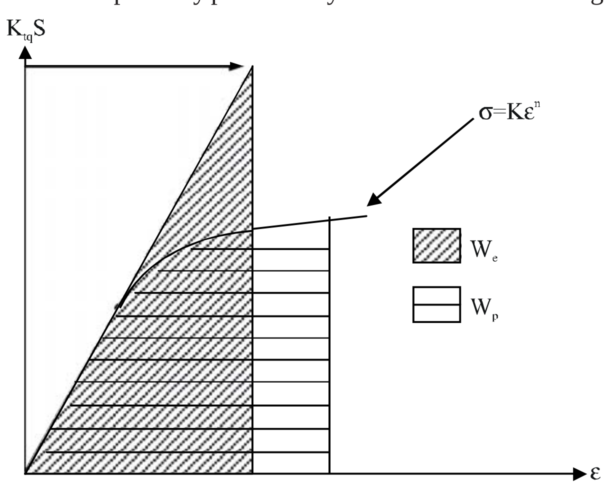

FIG. 5. GRAPHICAL REPRESENTATION OF GLINKA METHOD FOR APPLIED LOAD 'S'

5. THE APPLICATION OF PROPOSED METHOD

Application of the proposed procedures is illustrated by the example of the 3D model with 10mm thickness as shown in Fig. 6. The linear interpolation solution follows the scheme outlined.

The assumed material stress-strain law according the equation Equation (2).

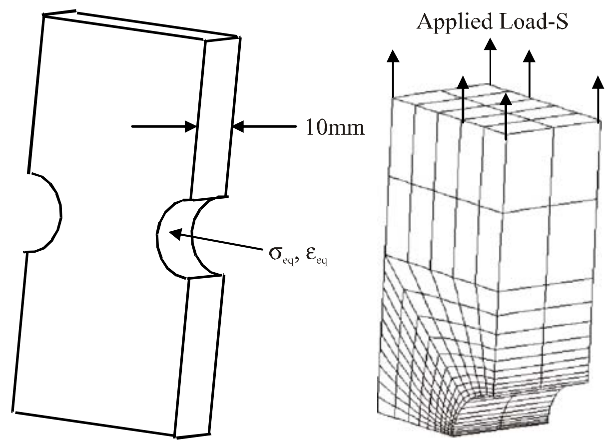

FIG. 6. GEOMETRY AND MESH OF 3D FE MODEL

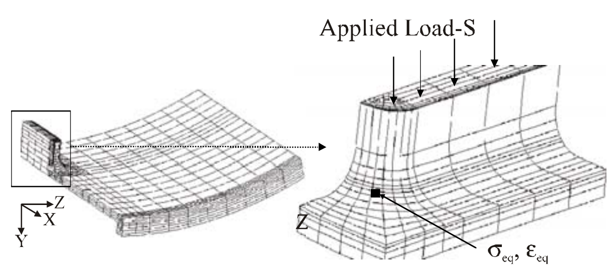

FIG. 7. FE MESH OF LOCAL JOINT

TABLE 1. THE VALUES OF KTQ, SP, FROM FE ANALYSES

Structure	Yield Stress σ_y(MPa)	SCF (K_{tq})	Limit Load S_p (Mpa)
3D Notched Flat Bar	250	2.978	196
Local Joint	250	25.45	30.5

$\sigma = 1800*\varepsilon^{0.6} \quad (\sigma \geq \sigma_{n,Y})$

FE solution for K_{tq} and S_p from elastic-perfectly plastic analysis with yield stress 250 MPa .

- Elatic-equivalent stress concentration factor K_{tq}=2.978.
- Limit-load S_p=196 MPa for yield stress σ_y=250 MPa.

The equivalent elastic-plastic, notch stresses calculated, according to the linear-interpolation scheme with Equation (6).

$\sigma_{eq} = 2.297S \qquad (S \geq 21.76MPa)$

The equivalent notch strains calculated, according to Equation (7).

$\varepsilon_{eq} = 1.5x10^{-05} S^{1.667} \qquad (S \geq 21.76MPa)$

The elastic equivalent notch strains, below the yielding, using Equation (8).

$\varepsilon_{eq} = 1.51938x10^{-04} S$

6. APPLICATION OF NUEBER'S RULE

Using Nueber's rule and the materials properties E, K_{tq}, K,n, the elastic-plastic notch equivalent stress-strains are calculated with Equations (11-12).

$\sigma_{eq} = 6.03xS^{0.75} \qquad (S \geq 16.789MPa)$

$\varepsilon_{eq} = 7.502x10^{-05} S^{1.25} \qquad (S \geq 16.789MPa)$

The elastic notch strains:

$\varepsilon_{eq} = 1.51938x10^{-04} x S \qquad (S < 16.78MPa)$

7. APPLICATION OF GLINKA METHOD

Using Glinka method and the materials properties E, K_{tq}, K,n, the elastic-plastic notch equivalent stress-strains are calculated with Equations (16-17).

$\sigma_{eq} = 5.5469xS^{0.75} \qquad (S \geq 16.789MPa)$

$\varepsilon_{eq} = 6.5257x10^{-05} S^{1.25} \qquad (S \geq 16.789MPa)$

The elastic notch strains:

$\varepsilon_{eq} = 1.51938x10^{-04} x S \qquad (S < 16.78MPa)$

8. RESULTS AND DISCUSSION

The elastic-plastic FE analysis was carried out for the assumed power-law material as shown in Fig. 8. For different values of applied load of S, the equivalent notch stresses-strains are calculated by the FE, linear interpolation method (proposed method), Nuber's rule and Glinka method. The applied load S versus equivalent notch stresses-strains (up to 2% total strains) curve is depicted in Fig. 9. The Neuber, Glinka solutions show a reasonably good agreement (up to 1% total strains) with FE results. However, discrepancies occur for the large notch stresses-strains and Nueber, Glinka methods tend to underestimate. For large notch stresses and strains, i.e. 10 % total strains, values obtained by the new method shows the good correlation with FE results, as shown in Figs. 10-11.

9. CONCLUSION

A new method is developed to predict large elastic-plastic notch stress and strains, for materials with power-hardening law. As a first step, the proposed method is implemented within simple structures with combination limit load S_p and elastic stress concentration factor K_{tq}. The predicted results using the new method show good correlation with FE results. Furthermore, Nueber's rule while considering general yielding (Neuber-2), also shows good correlation with FE results. Glinka, Neuber-1, based on elastic solution K_{tq}, works better for small notch stress/strains and shows significant under estimation with increasing applied stress 'S'.

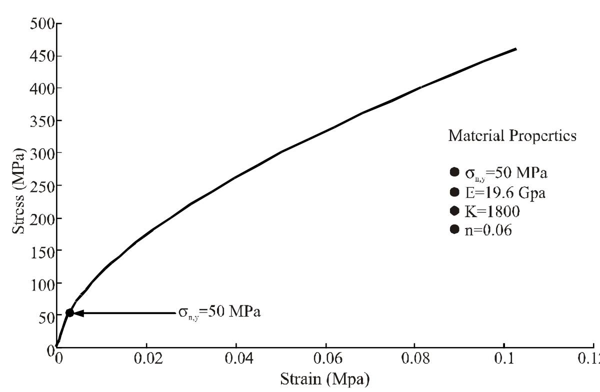

FIG. 8. THE STRESS-STRAIN CURVE OF ASSUMED POWER-LAW MATERIAL

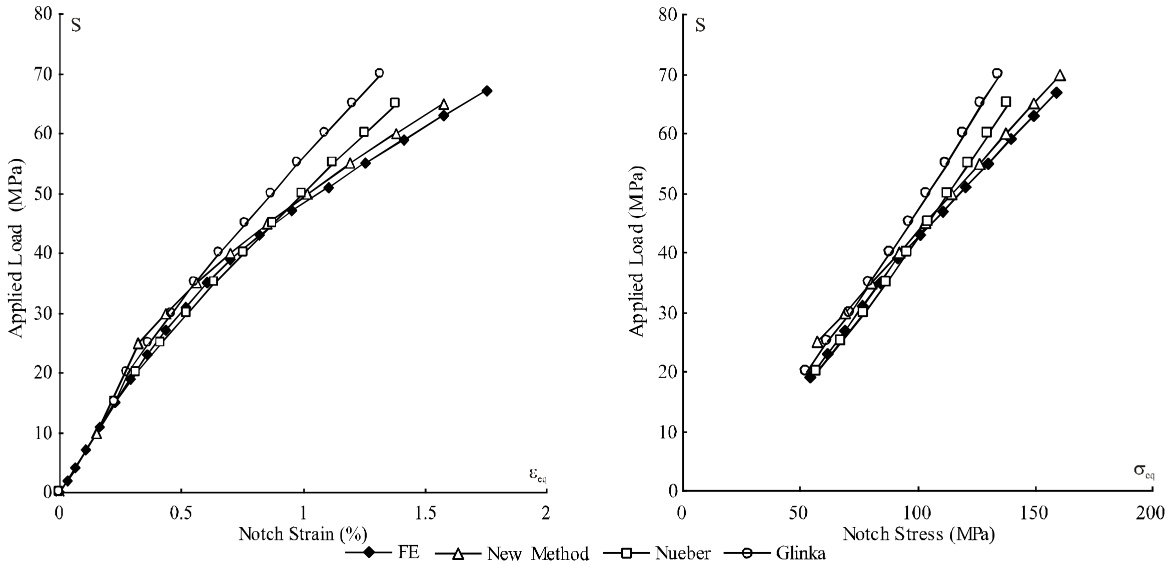

FIG. 9. EQUIVALENT NOTCH STRAINS AND STRESSES CALCULATED BY THE FE, LINEAR INTERPOLATION, NEUBER AND GLINKA METHOD, FOR SMALL STRAINS-STRESSES

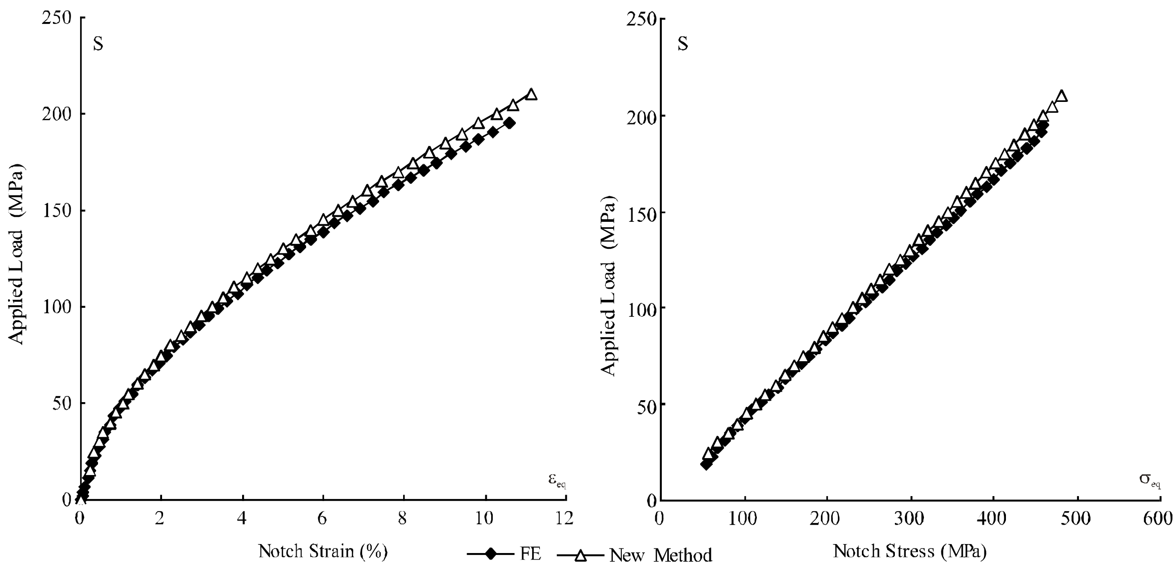

FIG. 10. FOR FLAT BAR, THE EQUIVALENT NOTCH STRAINS AND STRESSES CALCULATED BY THE FE, LINEAR INTERPOLATION, NEUBER AND GLINKA METHOD, FOR LARGE STRAINS-STRESSES

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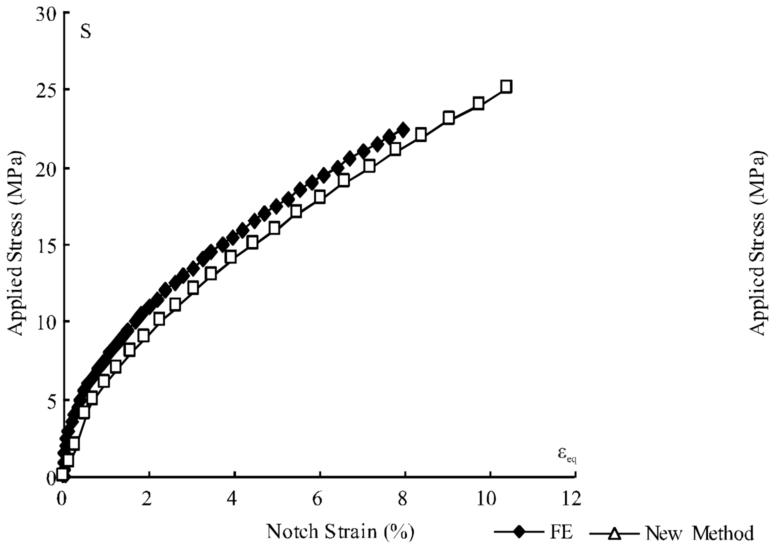

FIG. 11. FOR LOCAL JOINT, THE EQUIVALENT NOTCH STRAINS AND STRESSES CALCULATED BY THE FE, LINEAR INTERPOLATION, NEUBER AND GLINKA METHOD, FOR LARGE STRAINS -STRESSES

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