## Improvement in Machining Efficiency of Complex Profiles using 5-Axes Simultaneous Machining Approach

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#### **ABSTRACT**

This research paper focuses on the process of machining complex and varying contour aerofoil parts used in aerospace industry like wings, blades of turbine and diffusers, etc. The main theme is to enhance productivity and on the other hand achieving accurate machining results for complex profile parts. The research consists of three main parts viz. Accurate and optimized tool path generation of complex profile, understanding and conversion of a 5-axes machine kinematics into mathematical relationships and developing a post processor which converts cutter location file into machine readable file, and finally a time and accuracy study has been made and compared with techniques used previously for machining of such components. Tool path is generated on CAM software from where CL (Cutter Location) file is generated, then by use of mathematical calculations post processor is developed and used for the conversion of that CL file into machine readable file. Time and accuracy analysis has been made in the end on an impeller blade machining.

Tool paths generated in CAM software requires to be very closely analyzed in order to ensure collision free tool paths as movement in all 5-axes increases the chances of collision of head to work piece or with the table. For the study of 5-axes machine kinematics and calculations, a general 5-axes machine having 3 linear axes and two axes in rotary/tilting table has been used. By applying mathematical relationships using rotational matrix theorem and dot product of two vector quantities, equations were derived for rotary and tilting axes and then written in programming language (Visual  $C^{++}$ ) for line to line conversion of CL file into machine readable file.

For the generation of CL file three different tool paths were generated. First process was the roughing of the area between every two adjacent blades then the next step was to finish the surface of the blades and the leading/trailing edge and the last process was to finish the hub surface of the impeller. The cutting parameters like feed, speed, axial depth of cut, radial depth of cut are dependent on the spindle power of the machine and the type of the material of part to be machined and that of the cutter itself. The selection of the size of the cutting tool used for the machining of each process is also dependant on the minimum area between two adjacent surfaces of the blades of impeller and maximum curvature of the blade from shroud surface to hub surface in order to avoid collision of the cutting tool with the blade surfaces.

In the end, how the new approach has benefited is analyzed. The need for research on new approach was very demanding as to manufacture complex profile parts mostly used in aerospace industry with traditional machining approaches causes unpredictable delays due to manual calculations, inaccurate machining results like poor surface finish, tool breakage, etc, as well as requirement of various jigs/fixtures and skilled workers for accomplishing the task all the time. Now by adopting the approach as explored in this research gives a lot of advantages over traditional manufacturing approach. Some important features compared are presented in the tabular form.

Key Words: CL File, Impeller, 5-Axes Simultaneous Machining, Rotational Matrix.

#### 1. INTRODUCTION

Different types of impellers are used in jet engines, turbo engines, generators, turbines, compressors,

blowers and diffusers. The requirement in impellers is that it has to operate at high RPM, and at high pressure.

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The machining of such curved and twisted surfaces before the invention of state of the art 5-axes simultaneous machining technology was very difficult because of calculation of angles at every point was required. Then the tool was manually positioned at that point. In this way not only a lot of machining time was wasted but also use of long length tools and various jigs and fixtures results in poor surface finish because of tool marks on blade surfaces, inaccurate profiles due to manual positioning of the tool at every point and on the other hand end product was still expensive when compared to achieved quality. Also the major disadvantage in traditional machining is that the accuracy achieved on such profiles is inconsistently varying depending on the skill of the operator. The use of 5-axes simultaneous machining technology has made easier the machining of above mentioned complex profiles. This type of machining gives higher productivity and good surface finish. When using 5-axes simultaneous machining technology a big problem is to understand the machine kinematics and generation of collision free tool paths.

Many authors have proposed their suggestions in their research papers for accurate machining of such parts. Cho, et. al. [1] in his research suggested a method that tool path planning should consist of two steps: roughing of the surfaces between two blades, then point milling strategy for the finishing of blades and leading edge and hub surface. Lee, et. al. [2] proposed the method to machine the propeller in a single setup by a polyhedron model and choose the tool attitude by using the blade center-line vector.

Tsay, et. al. [3] in his research developed an algorithm to generate tool paths with global interference checking for 5-axes point milling of turbo machinery components. Based on projected distance between the surface data and cutter axes of ball nose, interference between the surface of a work piece and the cutter can be detected and from cutter contact points of the surface and the cutter size CL file can be produced.

Heo, et. al. [4] presented in his research an efficient rough cut plan for 5-axes machining by using characteristics curves of an impeller and their machine instead of simultaneous 5-axes control.

In this study, process of machining an impeller blade is presented. The method adopted in this research is different from the above mentioned researches on the technique used for calculating CL file as in this research three collision free tool paths are generated for blade roughing, blade finishing and hub finishing. Then technique used for developing mathematical relationships with CL file is also different from above researches as rotation matrix, Arfken, [5] and dot product of two vectors approach has been used. The technique developed is simple, powerful and helpful for machining any complex profile component with great ease and accuracy.

#### 2. IMPELLER SURFACES

Different surfaces of impeller are shown in the Fig. 1. Impellers can be divided in two types: Splitter Type and Non Splitter Type. In Splitter Type Impellers there is a small blade between every two larger blades. The function of these splitter type impellers is to generate more torque and power. The bottom surface of an impeller is called the hub surface of the impeller.

The leading edge is on the top side of the blade surface and the trailing edge is on the bottom side of the blade surface. The outer most surface of the blades of an impeller is called the shroud surface. The impellers are designed on data for the hub and shroud curves based on their compression ratio.

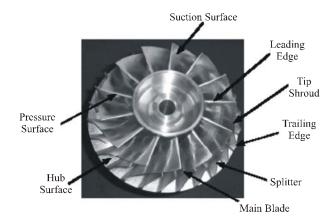


FIG. 1. DIFFERENT SURFACES OF AN IMPELLER, [1]

## 3. MACHINING OF COMPLEX PROFILES USING MANUAL POSITIONING MACHINING APPROACH

Before the invention of state of the art of 5-axes simultaneous machining technology, machining of curved and twisted profiles like blades of impeller was only possible by manual calculation of points and angles and also by the use of form tools and jigs and fixtures. By doing this a lot of set ups were required for machining of such profiles which ultimately increased the production time 10 times more than the machining using 5-axes simultaneous technology, other important factors are surface finish and accuracy of blade profile were also badly affected. Since the components are not machined in one setup, therefore, the finished component does not have good surface finish besides inconsistent geometry and accuracy.

MCV510 vertical machining center with 2-axes rotary and tilting table has been used for manual positioning of the tool to the required area. Cylindrical ball end mill of Dia 4 and 3mm with 4 degree taper angle was used. Experimental setup used for machining of an impeller is shown in Fig. 2. The material of the component was aluminum alloy for

test purposes. The machined component is also shown in Fig. 3. Different machining parameters [3] used are given in Table 1.



FIG. 2. EXPERIMENTAL SETUP FOR MACHINING OF IMPELLER BY MANUAL POSITIONING METHOD

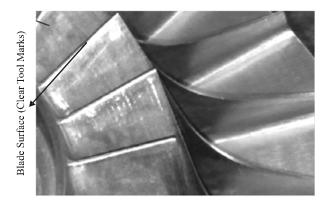


FIG. 3. MACHINED COMPONENT BY MANUAL POSITIONING
METHOD

TABLE 1. CALCULATIONS OF DIFFERENT MACHINING PARAMETERS FOR AN IMPELLER BY MANUAL POSITIONING MACHINING APPROACH

M 1' ' D	Machining Operations			
Machining Parameters	Blade Roughing	Blade Finishing	Hub Finishing	
Cutting Tool Used (Dia, mm)	4	3	3	
Cutting Speed Vc(m/min)	90	60	70	
Feed Per Tooth	0.03	0.012	0.04	
Plunging Depth ap(mm)	0.5	0.1	0.5	
Step Over ae(mm)	0.8	0.1	0.5	
Spindle RPM	7000	6000	7500	
Feed Rate Vf(mm/min)	650	225	900	
Setting Time (Hrs)	50	60	30	
Machining Time t(Hrs)	25	35	15	
Surface Finish (µm)		2.4		
Total Machining Time (Hrs)	75	95	45	
		215		

# 4. TOOL PATH GENERATION OF NON-SPLITTER TYPE IMPELLERS IN 5-AXES SIMULTANEOUS MACHINING

The tool path planning of non-splitter type impellers consists of three different processes. First process is the roughing of area between every two adjacent blades, next is the finishing of blades and leading/trailing edges, and the last process is the finishing of hub surface. In the following pages we describe the process in detail.

### 5.1 Designing/Machining of Work Piece (Blank) of Impeller

For the machining of impellers first of all a work piece as shown below in the Fig. 4 is machined. The outer most surface is most critical surface in the designing of impeller blank because this surface is generated from the shroud curve of the impeller which is designed in Pro-E CAD software. All other features other than blades are also machined while creating the work piece for the machining of impeller blades. For the machining of such work piece CNC turning machine is used.

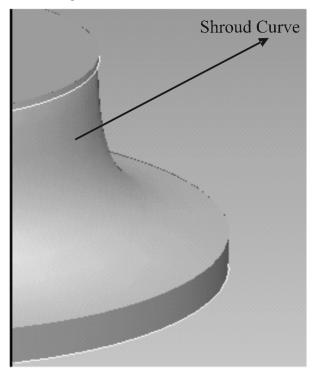


FIG. 4. WORK PIECE (BLANK) OF IMPELLER

#### **5.2** Roughing of Impeller Blades

After machining of the impeller work piece, next step is to machine the area between every two adjacent blades. In generation of tool path first step is to define the work piece zero point. This point is usually the center point and top most surface of impeller blank as shown in Fig. 4. Then the other important parameters are to check the tilt angle of the blade surfaces from shroud to hub surface, minimum distance between two adjacent blades and the radius on the root surface of impeller blades. Ball nose type cutting tools are used for the machining of impeller blades because they follow the contour of the surfaces of the impeller blades completely. Maximum size of the cutting tool which fulfills the above mentioned criteria i.e. generates collision free tool path is used in order to obtain maximum productivity and tool life. The other cutting parameters like feed, speed, depth of cut, step over are calculated from the cutting tool manufacturer's catalogue [6] and are given in Table 2. After defining all these parameters the NC sequence/tool path for roughing of impeller blades is completely defined. Now its CL (Cutter Location) file can be seen (Table 3). The viewing of simulation of the created tool path critically is the key factor as this will ensure error free working during the machining process. When the simulation is viewed and found satisfactory then the next step is to pattern this tool path into required number of blades so that in only single setup the roughing process of impeller blades could be completed (Fig. 5).

For this entire process Pro-E software has been used and Fanuc 11M controller MCV510 machine is used.

### 5.3 Finishing of Impeller Blades and Leading/Trailing Edges

When the process of rough machining of impeller blades is completed then the next step is to finish the blade surfaces of an impeller. Again the important parameters are the maximum tilt angle of the blade surfaces from shroud to hub surface, the minimum distance between the two adjacent surfaces of the blades, and the radius on the root surface of the impeller blades. The selection of the size of the ball nose is dependant on above mentioned three important parameters. The selection of machining

strategy is a key factor because of the requirement of surface finish. Point milling strategy [1] is the good choice for finishing of blades and leading/trailing edges for achieving good surface finish, tool life and productivity. Other machining parameters like feed, speed, step over are more important then the roughing because this is the finishing process, therefore calculations for above mentioned parameters were made after studying the behavior of the material to be machined, cutting tool used,

and on the machine tool maximum spindle power [6]. All these values are given in Table 2. After viewing the simulation the tool path can be patterned on the total number of the blades to be machined and then its CL file can be seen and checked for correction (if any) before execution/actual machining. Again ProE software has been used on MCV510 machine with fanuc 11M controller. The CL file as shown in Table 3 is generated automatically (Fig. 6).

TABLE 2. CALCULATIONS OF DIFFERENT MACHINING PARAMETERS FOR AN IMPELLER BY 5-AXES SIMULTANEOUS MACHINING APPROACH

N 11 1 D	Machining Operations				
Machining Parameters	Blade Roughing-1	Blade Roughing-2	Blade Finishing	Hub Finishing	
Cutting Tool Used (Dia, mm)	6	4	3	4	
Cutting Speed Vc (m/min)	120	100	80	100	
Feed per Tooth	0.05	0.04	0.02	0.04	
Plunging Depth, ap (mm)	0.6	0.5	0.1	0.5	
Step Over, ae (mm)	1.0	0.8	0.1	0.5	
Spindle RPM	6000	7500	8000	8000	
Feed Rate, Vf (mm/min)	950	900	500	1000	
Setting Time, (Hrs)	1	-	-	-	
Machining Time, t (Hrs)	10	4	8	2	
Surface Finish (?m)	0.8 on blade surfaces				
Total Machining Time, (Hrs)	11	4	8	2	
	25				

TABLE 3. CL FILE FOR ROUGHING OF IMPELLER BLADES

#### 5.4 Finishing of Hub Surface

The last process in the machining of impeller blades is to finish the hub surface of the impeller. For the machining of hub surface iso cut line method is used [1]. In this method the cutting tool approaches from bottom to the top surface of the hub and starts machining from the center of the two blades and expands its tool path towards both adjacent blades edges. The selection of the size of cutting tool is dependant on two important parameters of the blades; maximum tilt angle of the blade surface from shroud to hub surface and minimum distance between two adjacent surfaces of the blades. In the machining of hub surface

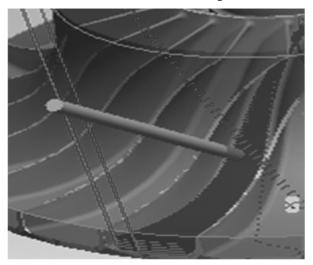


FIG. 5. ROUGHING OF IMPELLERS BLADE

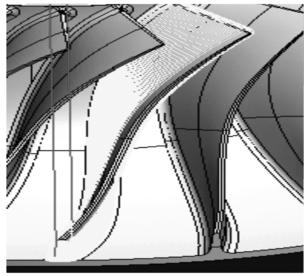


FIG. 6. FINISHING OF IMPELLER BLADES AND LEADING/ TRAILING EDGES

smooth continuous lines are required, therefore cutting parameters like feed, speed, and step over can be selected to a maximum value as calculated [6] in order to obtain highest productivity. Since amount of material to be removed is very small (only 0.5mm material axially) therefore, cutting tools works excellently. After viewing the simulation of the tool path, the tool path is patterned on the basis of total number of blades to be machined and then its CL file (Table 3) can be seen and checked for correction (if any) before execution/actual machining. Again ProE software has been used on MCV510 machine with fanuc 11m controller (Fig. 7). The important machining parameters are shown in Table 2.

#### 5. POST PROCESSING OF CL FILE

Now after generating all the required tool paths for machining of impeller blades the next step is to convert the respective CL files into machine readable files (NC File). For this purpose complete machine kinematics is mathematically modeled. A 5-Axes CNC machine having rotary table and tilting head with Nutator angle of 45° (A plane about which tilting axes moves) is selected for mathematical modeling. Rotational matrix method approach is used as the basis of calculations. Then the dot product of two vectors and their magnitude concept is utilized (Fig. 8).

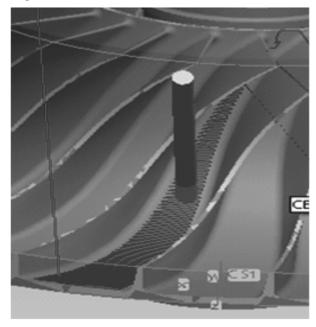


FIG. 7. FINISHING OF HUB SURFACE

The rotational matrix for a 3-axes co-ordinate system is given as [5,8]:

$$R_{\mathcal{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$
 (1)

$$R_{y} = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$
 (2)

$$R_z = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1 \end{bmatrix}$$
 (3)

Direction cosines are the projections of a vector on X, Y and Z axis (Fig. 9).

Mathematically direction cosines can be written as:

$$1 = \cos(\alpha) \tag{4}$$

$$m = \cos(\beta) \tag{5}$$

$$n = \cos(Y) \tag{6}$$

Where 'I' is the direction cosine about X-axes, 'm' is the direction cosine about Y-axes and 'n' is the direction cosine about Z-axes.

Since the movement of tilting axes of a machine follows a circular motion therefore we use parametric equation of circle as:

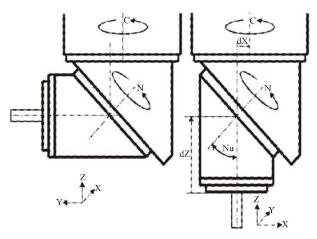


FIG. 8. MACHINE CONFIGURATION FOR FINDING KINEMATICS SOLUTION [7]

Let Q is a point on a circle of unit radius and B is the angle of Q vector along Z axes then equation of circle is:

$$Q = \begin{bmatrix} \sin B \\ 0 \\ \cos B \end{bmatrix}$$

Using 45° Nutator angle value (Fig. 10) in rotational matrix and dot product with parametric equation of Circle for obtaining value of Q:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(45) & -\sin(45) \\ 0 & \sin(45) & \cos(45) \end{bmatrix} \begin{bmatrix} \sin B \\ 0 \\ \cos B \end{bmatrix}$$

$$Q = \begin{bmatrix} \sin B \\ -\frac{1}{\sqrt{2}} \cos B \\ \frac{1}{\sqrt{2}} \cos B \end{bmatrix}$$
 (7)

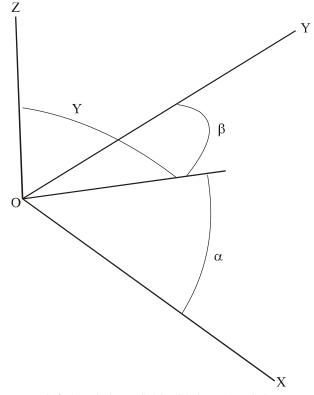


FIG. 9. A VECTOR IN SPACE CO-ORDINATE SYSTEM, MATHEMATICAL METHODS [5,8]

Now to finally find the tilting axes relation we present graphically the complete motion of tilting axes of a machine as shown in Fig. 11.

Q vector has already been calculated and to find the relation for tilting axes (B-axes) we have to find PQ vector. The vector P is the distance of tool from origin to tool tip as:

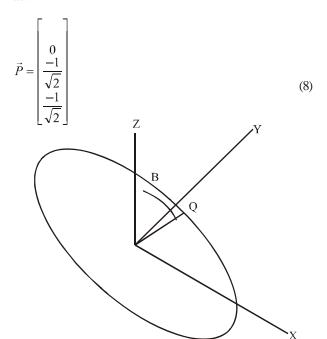


FIG. 10. A VECTOR Q ON A CIRCLE OF UNIT RADIUS, MATHEMATICAL METHODS [5,8]

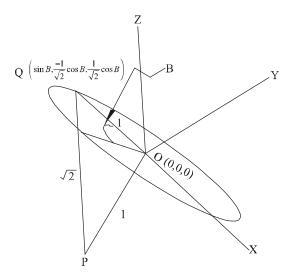


FIG. 11. GRAPHICAL REPRESENTATION OF TILTING AXES MOVEMENT OF MACHINE

Now to find vector between P and Q.

$$PQ = \vec{Q} - \vec{P} = \begin{bmatrix} \sin B \\ \frac{1}{\sqrt{2}} (1 - \cos B) \\ \frac{1}{\sqrt{2}} (1 + \cos B) \end{bmatrix}$$

Now by using equation for X-axes and other trigonometry relations B-axes relation is found and given as:

$$\Rightarrow B = \cos^{-1}(2n-1) \tag{9}$$

Where n is the direction cosine about Z axis and 6<sup>th</sup> value at every row in CL file and B is the required relation for tilting axes.

Now to find the relation for rotary axes we proceed as shown in Fig. 12.

Where C is angle which we have to find while as shown in Fig. 12 it is the angle which table has moved from "t' to "v" position, where the coordinates has also changed from I and m to new values as I' and m'.

Now to find relationship for C-axes we have to solve "t" and "v" vectors.

$$l' = \cos \alpha = \frac{\sin B}{\sqrt{2}}$$

$$m' = \cos \beta = \frac{1}{2} (1 - \cos B)$$

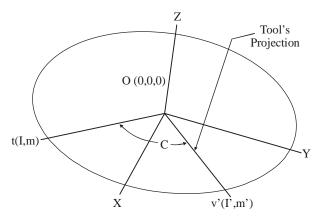


FIG. 12. GRAPHICAL REPRESENTATION OF ROTARY AXES
OF MACHINE

$$t = l\hat{i} + m\hat{j}$$

$$v' = l'\hat{i} + m'\hat{j}$$

Now by using dot product of two vectors we find C-axes relation as:

$$t' \bullet \vec{\mathbf{v}}' = |\vec{t}| \times |\vec{\mathbf{v}}'| \times \cos C$$

$$(l\hat{i} + m\hat{j}) \bullet (l'\hat{i} + m'\hat{j}) = \sqrt{l^2 + m^2} \times \sqrt{l'^2 m'^2} \times \cos C$$

Simpifing:

$$(ll' + mm') = \sqrt{l^2 + m^2} \times \sqrt{l'^2 m'^2} \times \cos C$$

$$\cos C = \frac{(ll' + mm')}{\sqrt{l^2 + m^2} \times \sqrt{l'^2 + m'^2}}$$

$$C = \cos^{-1} \left( \frac{(ll' + mm')}{\sqrt{l^2 + m^2} \times \sqrt{l'^2 + m'^2}} \right)$$

Now by putting the values of l' and m' as given above:

$$C = \tan^{-1} \left( \frac{m}{l} \right) - \tan^{-1} \frac{1}{\sqrt{2}} \frac{(1 - \cos B)}{\sin B}$$
 (10)

Where C is the required rotary axes relationship and l and m are the two direction cosines about X and Y axes respectively (which are 1st and 2nd value of CL file after three linear axes values).

Now for the conversion of CL file into machine readable file which is actually executed on CNC machine for impeller machine as NC program C-language is utilized. In which these relations (Equations 9-10) are used and CL as given in Table 3 is taken as input file and applies mathematical relations and machine readable file as shown in Table 4 is generated, which is then executed on CNC machine and component is finally machined. The component machined with this approach is shown in Fig. 13.

All the machining parameters used in 5-axes simultaneous machining approach and their results are given in Table 2.

Surface finish measured on impeller blades machined by two different approaches has been measured with "Surface Roughness Tester".

#### TABLE 4. NC FILE OF ROUGHING OF NON SPLITTER TYPE IMPELLER

%	N35 G01 X+43.239 Y+28.166 Z-11.011	N44 G01 X+43.024 Y+28.444 Z-10.522				
:0001	B150.677 C-36.616	B155.102 C-39.192				
N5G91 G28 X0 Y0 Z0	N36 G01 X+43.353 Y+28.016 Z-11.207	N45 G01 X+43.042 Y+28.404 Z-10.347				
N10G40 G17 G80 G49	B149.164 C-35.822	B156.981 C-40.514				
N15G0 G90 Z10		N46 G01 X+43.11 Y+28.287 Z-10.132				
N20T1 M6 S4000	N37 G01 X+43.505 Y+27.804 Z-11.393	B159.618 C-42.465				
N25G54 G90 T0	B147.825 C-35.229					
N26 G01 Z-9.479	N38 G01 X+43.628 Y+27.629 Z-11.513					
N27 G01 Z-9.383	B147.014 C-34.934					
N28 G01 X+52.948 Y+31.384	N39 G01 X+43.65 Y+27.614 Z-11.644	And so on				
N29 G01 X+44.345 Y+26.285 Z-9.368	B146.166 C-34.407					
N30 G01 X+43.973 Y+26.904 Z-9.494						
B172.566 C-53.291	N40 G01 X+43.358 Y+28.024 Z-11.331					
N31 G01 X+43.2 Y+28.154 Z-10.217	B148.264 C-35.224					
B158.523 C-41.894	N41 G01 X+43.181 Y+28.263 Z-11.082					
N32 G01 X+43.147 Y+28.248 Z-10.405	B150.113 C-36.116	N99994G0 Z10.				
B156.338 C-40.286	N42 G01 X+43.093 Y+28.376 Z-10.893	N99995 M9				
N33 G01 X+43.134 Y+28.285 Z-10.603	B151.651 C-36.98	N99996G91 G28 Z0				
B154.292 C-38.86		N99997 G49 H0				
N34 G01 X+43.165 Y+28.257 Z-10.805	N43 G01 X+43.042 Y+28.434 Z-10.704	N99998 G28 X0 Y0				
B152.413 C-37.644	B153.333 C-38.021	N99999 M30				

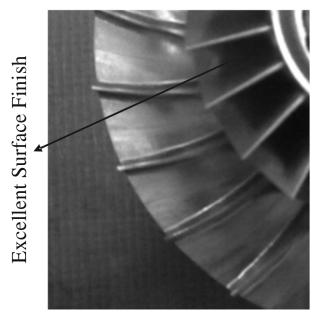


FIG. 13. MACHINED COMPONENT BY USING 5-AXES SIMULTANEOUS MACHINING APPROACH THROUGH CAM SOFTWARE

#### 6. CONCLUSION

This study focuses on the machining of complex profiles which involves 5-axes simultaneous machining technique. First part of study focuses on the generation of accurate tool path for the machining of blades of impeller. The next part discusses about the mathematical modeling of a machine kinematics by using Rotational Matrix theorem. The equations developed mathematically were used in a programming language and software is developed which performs the function of a post processor and converts the CL file into machine readable NC file. Then the machining results achieved from two different approaches have been shown in two tables. A lot of machining time (about 9 times) is saved and cost is minimized. The cost of post processor is also saved. The achievements made through this research include generation of tool path for any complex profile and mathematical modeling for a 5axes machine with different configuration to serve as post processor. By these efforts cost of machining of complex profiles can be minimized. The utmost accuracy can be achieved by machining the component in single setup besides saving the cost of post processor which is worth full in present time of economic crisis of the country.

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