Prediction of Pressure Difference and Velocity Profile in Steady Flow through Axi-Symmetric Plaque Deposited Arteries

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ABSTRACT

Numerical simulations of blood flow through plaque deposited arteries at different Reynolds numbers have been performed to investigate the impact of atherosclerosis on pressure drop and velocity profile at down stream. The predicated results are presented in terms of non-dimensional pressure isobars and velocity profiles at distinct Reynolds numbers and various levels of deposition at downstream of the artery segment. The scaled non-dimensional graph of pressure drop is also illustrated. The incompressible Navier-Stokes equation in the axi-symmetric frame of reference is solved numerically by employing FEM (Finite Element Method). Semi-implicit Taylor-Galerkin/pressure-correction scheme has been utilised to obtain steady state solutions. The effects of atherosclerosis on hemodynamic factors have been investigated. The results show that blockage disturbs the flow field in the wake of plaque deposited arteries and the trend of pressure and velocity is increasing as level of deposition or Reynolds number increases. The application of this research work can be utilised in the field of cardio vascular disease, design of device and further planning towards treatment.

Key Words: **Numerical Simulation, Flow of Blood, Arteries, Newtonian.**

1. INTRODUCTION

The deposition of plaque in the circulatory system of a human body has been reported by many researchers and considered as a major cause for various arterial diseases. The disease in which deposits of plaque damages inner layer of the artery segment, is referred as atherosclerosis, causing some hardness in the arteries wall and reduces the diameter of the artery, as a result flow of blood is not supplied as per requirement, which causes various diseases including heart attack. Therefore, hemodynamic factors have been under investigation for many years, which plays an important role in the formation of deposits of plaque in the arteries [1].

To study the flow of blood in arteries experimental, theoretical and numerical investigations have been carried out [2]. The complex geometry of arteries (viz. bending, bifurcation, stenosis, etc) is also an important factor, which obviously affects the local hemodynamics [3]. Atherosclerosis is characterised by localised arterial

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narrowing (deposition of plaque). Deposition of plaque may alter the local hemodynamics, at the same time the altered local hemodynamics can cause the re-narrowing of blood vessel [4]. Some studies have been made on the experimental work performed in rigid blockage, along with pulsatile flow in flexible blockage [5-7]. Numerical studies of pulsatile flow through axially symmetric smooth rigid deposition of plaque, with blood as an incompressible Newtonian fluid, have been reported [8]. Furthermore, non-Newtonian properties of blood have been considered by some researchers [9-12].Whilst, some studies have been made on the influence of blockage and the effects of elastic property of the vessel wall [13-15].

The present research can be used to predict steady flow of blood having Newtonian nature, through two-dimensional geometries. Such prediction may be quantitative and qualitative for viscous fluids, provided one can accurately characterises the viscous properties of the fluid. A technique known as Taylor-Galerkin/ Pressure-Correction method is presented to solve such problems in a transient sense and to overcome the problems associated with high values of elastic parameters and complex three-dimensional flows.

This paper presents a numerical method for simulating the blood flow in partially blocked arteries. This paper is well organised as: In Section 2, mathematical model is defined along with problem specification. Governing equations and numerical scheme are addressed in Section 3, whilst, results and discussions have been made in Section 4 to investigate effects of blood inertia at different Reynolds numbers on various levels of deposition on blood pressure and velocity.

2. MATHEMATICAL MODEL AND PROBLEM DEFINITION

A mathematical model of arteries having an axially symmetric deposition of plaque is established. To capture the Newtonian behaviour of blood flowing in large arteries through several blockage rigid models is investigated and numerical calculations are carried out using FEM. Since the development of atherosclerosis in arteries reduces the elastic property of the arterial wall, the assumption of a rigid wall may be reasonable. Considering the pressure differences and velocity profiles is an important diagnostic factor for examining the flow characteristics of blood through the arteries; here the obtained pressure differences and velocity profiles are presented.

Flow of blood in arteries is simulated through a circular rigid tube having an axially symmetric deposition of plaque, using the size of a typical artery. This is considered as a large vessel from a rheological point of view. A two dimensional axi-symmetric domain 70R in length has been used for this purpose, taken 15R for upstream and 55R for downstream of the artery segment. The deposition of plaque is considered in parabolic form and increasing in length with increasing in percentage of blockage. The numbers of elements in axial and radial direction are 560 and 8 respectively, where as mesh size is taken from 1.187-1.931 respectively.

The boundary conditions related with the problem are given by the assumption that the flow field is fully developed, no slip boundary conditions are imposed on the walls of the arteries, as walls of the arteries are rigid. Flow of blood is considered as axi-symmetric, therefore, only the upper half region of the plaque deposited artery segment is simulated. At the inlet of the artery; axial velocity is taken maximum defined by the function $v_z=v_{max}(1-r^2)$, while vanishing radial velocity is considered, as there is no cross flow of blood in radial direction, whereas pressure is considered zero at the out let of the artery segment.

3. GOVERNING SYSTEM OF EQUATIONS AND NUMERICAL SCHEME

The flow of blood in the plaque deposited arterial segment is considered to be axisymmetric, two dimensional and fully developed. The continuity and the Navier-Stokes equations, which govern the flow of blood subject to the boundary conditions, are written in the cylindrical coordinates in the absence of body force as:

$$\frac{\partial v_z}{\partial z}+\frac{1}{r}\frac{\partial (r v_r)}{\partial r}=0 \tag{1}$$

$$\frac{\partial v_r}{\partial t}+v_z\frac{\partial v_r}{\partial z}+v_r\frac{\partial v_r}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}\right) \tag{2}$$

$$\frac{\partial v_z}{\partial t}+v_z\frac{\partial v_z}{\partial z}+v_r\frac{\partial v_z}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right) \tag{3}$$

Where v_z and v_r are the axial and the radial velocity components, p is the isotropic pressure, ρ is the density of blood and μ represents the viscosity of blood.

For convenience, the Navier Stokes equations are written in the following non-dimensional form by introducing the following dimensionless variables:

$$r^*=\frac{r}{R},\ z^*=\frac{z}{R},\ v_r^*=\frac{v_r}{V},$$
$$v_z^*=\frac{v_z}{V},\ t^*=\frac{t*v}{R},\ p^*=\frac{p}{\rho V^2}$$

Where V and R are characteristic velocity and length respectively, V is taken as velocity of blood and R is taken as radius of artery. Substituting these non-dimensional values in Equations (1-3) and discarding asterisks for brevity and simplicity, the system of Equations (1-3) may be rewritten as:

$$\frac{\partial v_z}{\partial z}+\frac{1}{r}\frac{\partial (r v_r)}{\partial r}=0 \tag{4}$$

$$\frac{\partial v_r}{\partial t}+v_z\frac{\partial v_r}{\partial z}+v_r\frac{\partial v_r}{\partial r}=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_r}{\partial r^2}+\frac{1}{r}\frac{\partial v_r}{\partial r}+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}\right) \tag{5}$$

$$\frac{\partial v_z}{\partial t}+v_z\frac{\partial v_z}{\partial z}+v_r\frac{\partial v_z}{\partial r}=-\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right) \tag{6}$$

Where Re is a Reynolds number defined as:

$$\mathrm{Re}=\frac{\rho V R}{\mu} \tag{7}$$

The incompressible Navier-Stokes equation for a Newtonian behaviour of blood is simulated numerically by employing a semi-implicit Taylor-Galerkin/pressure-correction finite element scheme. This method is based on a time stepping procedure, that semi discretises temporal domain, employing Taylor series expansion in time and pressure-correction procedure to extract a time stepping scheme of second order accuracy [17-19].

4. RESULTS AND DISCUSSION

Numerical computations are computed for different levels of plaque deposition at various levels of inertia in an artery segment to realise the effects of inertia on blood pressure along with velocity profiles.

4.1 Influence of Inertia on Blood Pressure

In Tables 1(a-b), a non-dimensional pressure drop scaled by maximum pressure of blood in straight artery is listed against increasing Reynolds number for various levels of plaque deposition. Computations are carried out for different Reynolds numbers (Re=1, 50, 100, 150, 200, 250 and 300) respectively, along with various levels of plaque deposition (10, 20, 30, 40, 50, 60, 65 and 70%) respectively, for investigating the effects of blood pressure drop in the artery.

The validation of analytical solution of a scaled pressure drop in a straight artery that is constant and equal to one is demonstrated through Fig. 1(a-b). Further it is observed that scaled pressure drop is increases linearly and at 10% level of deposition and its slope is very small. The behaviour of scaled pressure drop increases as level of deposition or Reynolds number increases. In Fig. 2(a-b) a non-dimensional pressure drop scaled by maximum pressure of blood in artery segment is plotted against different plaque deposition in percentages for various Reynolds numbers. Fig. 2(a-b) clearly illustrates that the behaviour of scaled pressure drop remains linear up to 40% deposition of plaque and slope of line is very small. Beyond the 40% deposition of plaque, the behaviour of non-dimensional scaled pressure drop changes from linear to exponential form and is very high from 60% level of deposition.

Non-dimensional scaled pressure drop is presented in terms of empirical equations as:

p_s=m Re+C

Where m stands for the slope of the pressure gradient and C for its intercept, presented in Table 2.

TABLE 1 (A). SCALED PRESSURE DROP AGAINST REYNOLDS NUMBERS FOR DIFFERENT PERCENTAGES OF DEPOSITION

Re	Scaled Pressure Drop			
	10%	20%	30%	40%
1	1.009929	1.032393	1.078821	1.176107
50	1.010875	1.037911	1.098518	1.235821
100	1.011643	1.042571	1.116429	1.294571
150	1.012232	1.046357	1.131696	1.346196
200	1.012786	1.049714	1.145429	1.394
250	1.013304	1.052857	1.15875	1.44125
300	1.013143	1.055036	1.171071	1.486071

TABLE 2 (B). SCALED PRESSURE DROP AGAINST REYNOLDS NUMBERS FOR DIFFERENT PERCENTAGES OF DEPOSITION

Re	Scaled Pressure Drop			
	50%	60%	65%	70%
1	1.39582	1.96536	2.601	3.82929
50	1.5735	2.53625	3.69107	6.05768
100	1.75839	3.15032	4.87179	8.4725
150	1.92563	3.71721	5.97	10.733
200	2.08321	4.25871	7.02814	12.9364
250	2.23705	4.79027	8.08696	15.2063
300	2.391	5.33304	9.17732	17.5639

Empirical equations have been developed in terms of fourth order polynomial trend for non-dimensional scaled pressure drop against various levels of deposition on different Reynolds numbers displayed in Table 3.

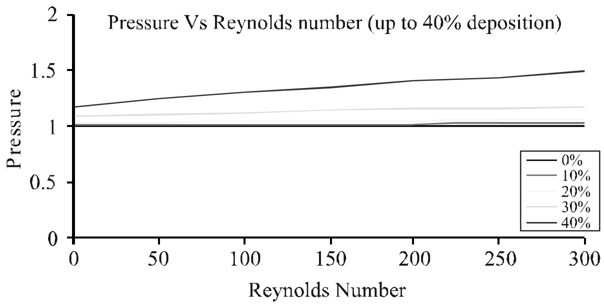

FIG. 1(a). SCALED PRESSURE DROP AT INLET AGAINST REYNOLDS NUMBERS (UP TO 40% DEPOSITION)

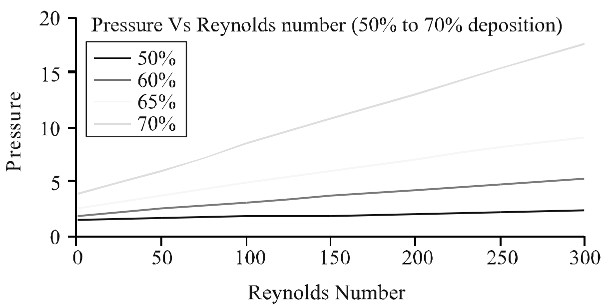

FIG. 1(b). SCALED PRESSURE DROP AT INLET AGAINST REYNOLDS NUMBERS (50-70% DEPOSITION)

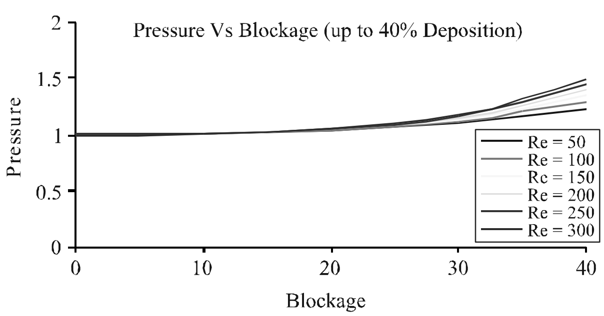

FIG. 2(a). SCALED PRESSURE DROP AT INLET AGAINST BLOCKAGE (UP TO 40% DEPOSITION) FOR VARIOUS REYNOLDS NUMBERS

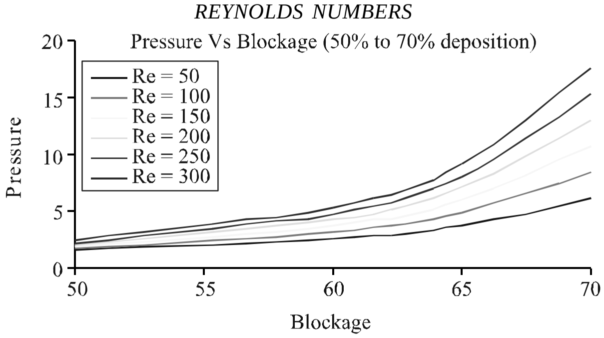

FIG. 2(b). SCALED PRESSURE DROP AT INLET AGAINST BLOCKAGE (50-70% DEPOSITION) FOR VARIOUS REYNOLDS NUMBERS

4.2 Pressure Isobars

In Figs. 3-5, pressure isobars are presented in terms of (Re=100, 200 and 300) respectively, along with 30, 50 and 70% respectively levels of deposition, with 15 contours at constant incremental values. It is observed that the trend of pressure drop is increases as level of deposition or Reynolds numbers increases in the downstream of the artery segment.

4.3 Velocity Profiles at Different Axial Locations

The velocity profiles of blood for tractions free boundary conditions are plotted in the set of Figs. 6-8 against increasing Reynolds number for different plaque depositions, in the downstream region at away from the deposition of plaque, inlet, mid plane and centre of the deposition, and in the upstream region, mid plane and exit of plaque deposition. It is observed that, at both low values of Reynolds number and percentage of blockage, the change in velocity remains very small and no trace of vortex is observed. At fixed value of Reynolds number (Re=100) with increasing percentage of deposition, the value of velocity profile increases and beyond 40% deposition the velocity profile at

TABLE 2. SCALED PRESSURE DROP AGAINST REYNOLDS NUMBERS FOR DIFFERENT PERCENTAGES OF DEPOSITION IN TERMS OF EQUATIONS

No.	Deposition	Equation
1.	10%	$Ps=1x10^{-5}$ Re+1.01
2.	20%	$Ps=8x10^{-5}$ Re+1.034
3.	30%	$Ps=3x10^{-4}$ Re+1.0829
4.	40%	$Ps=1x10^{-3}$ Re+1.1844
5.	50%	$Ps=3.3x10^{-3}$ Re+1.4109
6.	60%	$Ps=1.13x10^{-2}$ Re+1.9894
7.	65%	$Ps=2.2x10^{-2}$ Re+2.6211
8.	70%	$Ps=4.58x10^{-2}$ Re+3.811

upstream area is negative, this is may be due to formation of vortices behind the plaque deposition. Initially, the length and intensity of the vortex remain very small and as Reynolds number increases against increase in plaque deposition the length and intensity of vortex in increases. This phenomenon of vortex development is clearly demonstrated at Reynolds number (Re=200), where the vortex develops at even 30% of blockage. This development of vortex further strengthen at Reynolds number (Re=300).

4.4 Velocity Profiles at Axis of Symmetry

For traction boundary conditions at the axis of symmetry from Z=5 to Z=45, the velocity profiles of blood are plotted in the set of Fig. 9 against increasing Reynolds number at different plaque depositions. Velocity profile of blood given in Fig. 9 clearly illustrate that at low value of plaque deposition 10% with increasing Reynolds number the peak velocity remain low. As plaque deposition increases, means area of cross section decreases, the peak value of velocity profile over shoot at minimum radial direction i.e. the centre of the plaque deposition. It is also observed that velocity increases as Reynolds number or deposition level increases.

5. CONCLUSIONS

It is concluded that numerical results in a straight artery for a scaled pressure drop are constant and equal to one, at various Reynolds numbers as it should be in analytical solution. With the introduction of 10-30% level of deposition of plaque, the increase in scaled pressure drop is very small. However, from 40-60% of plaque deposition, the scaled pressure drop start to change its slope at some extent, subsequently, this change in slope over shoot from 60% and onwards deposition of plaque.

The velocity profiles of blood indicates that at different Reynolds numbers with increasing percentage of deposition, the value of velocity profile increases and beyond 40% deposition, the velocity profile at upstream area is negative, this is due to formation of vortices behind the plaque deposition. At the centre of deposition the flow of blood over shoots due to the development of stresses and the deformation of red blood cells takes place, hence the velocity profile is observed stretched with increasing Reynolds number or high level of deposition or both.

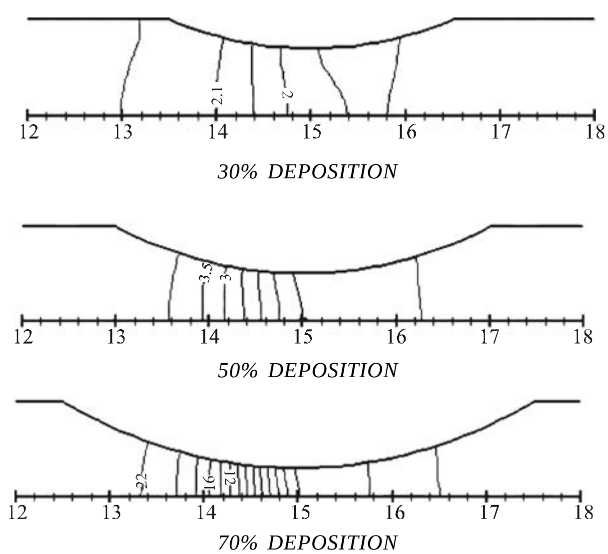

FIG. 3. PRESSURE ISOBARS AT REYNOLDS NUMBER Re=100 FOR DIFFERENT LEVELS OF DEPOSITION

TABLE 3. SCALED PRESSURE DROP AT INLET AGAINST BLOCKAGE (B_k) FOR VARIOUS REYNOLDS NUMBERS IN TERMS OF EQUATIONS

No.	Reynolds Number	Equation
1.	50	$Ps=2x10^{-6}\ (B_k)^4-2x10^{-4}\ (B_k)^3+6.2x10^{-3}\ (B_k)^2-6.16x10^{-2}\ (B_k)+1.0447$
2.	100	$Ps=2x10^{-6}\ (B_k)^4-3x10^{-4}\ (B_k)^3+9.6x10^{-3}\ (B_k)^2-9.66x10^{-2}\ (B_k)+1.07$
3.	150	$Ps=4x10^{-6}\ (B_k)^4-4x10^{-4}\ (B_k)^3+1.29x10^{-2}\ (B_k)^2-1.296x10^{-1}\ (B_k)+1.0939$
4.	200	$Ps=4x10^{-6}\ (B_k)^4-5x10^{-4}\ (B_k)^3+1.61x10^{-2}\ (B_k)^2-1.625x10^{-1}\ (B_k)+1.1169$
5.	250	$Ps=5x10^{-6}\ (B_k)^4-6\ x10^{-4}\ (B_k)^3+1.96x10^{-2}\ (B_k)^2-1.987x10^{-1}\ (B_k)+1.1437$
6.	300	$Ps=6x10^{-6}\ (B_k)^4-7x10^{-4}\ (B_k)^3+2.33x10^{-2}\ (B_k)^2-2.366x10^{-1}\ (B_k)+1.1706$

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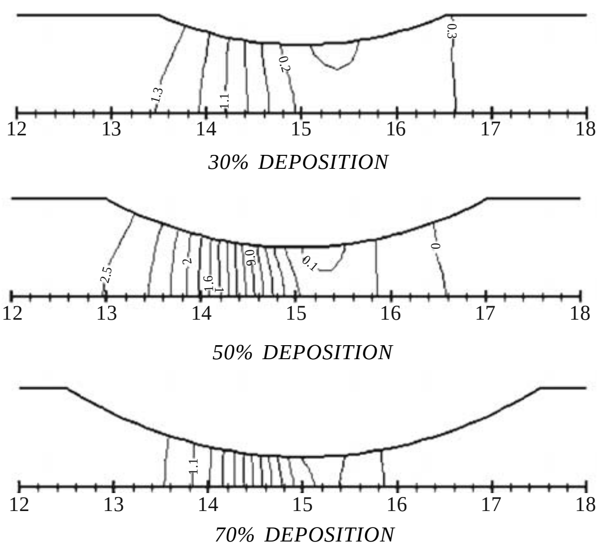

FIG. 4. PRESSURE ISOBARS AT REYNOLDS NUMBER Re=200 FOR DIFFERENT LEVELS OF DEPOSITION

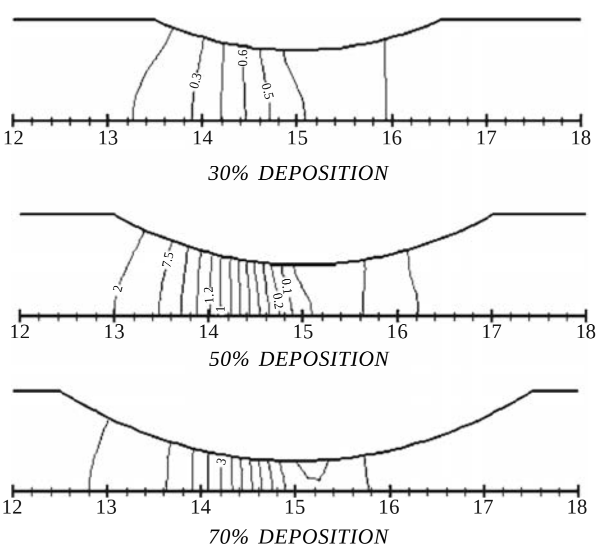

FIG. 5. PRESSURE ISOBARS AT REYNOLDS NUMBER Re=300 FOR DIFFERENT LEVELS OF DEPOSITION

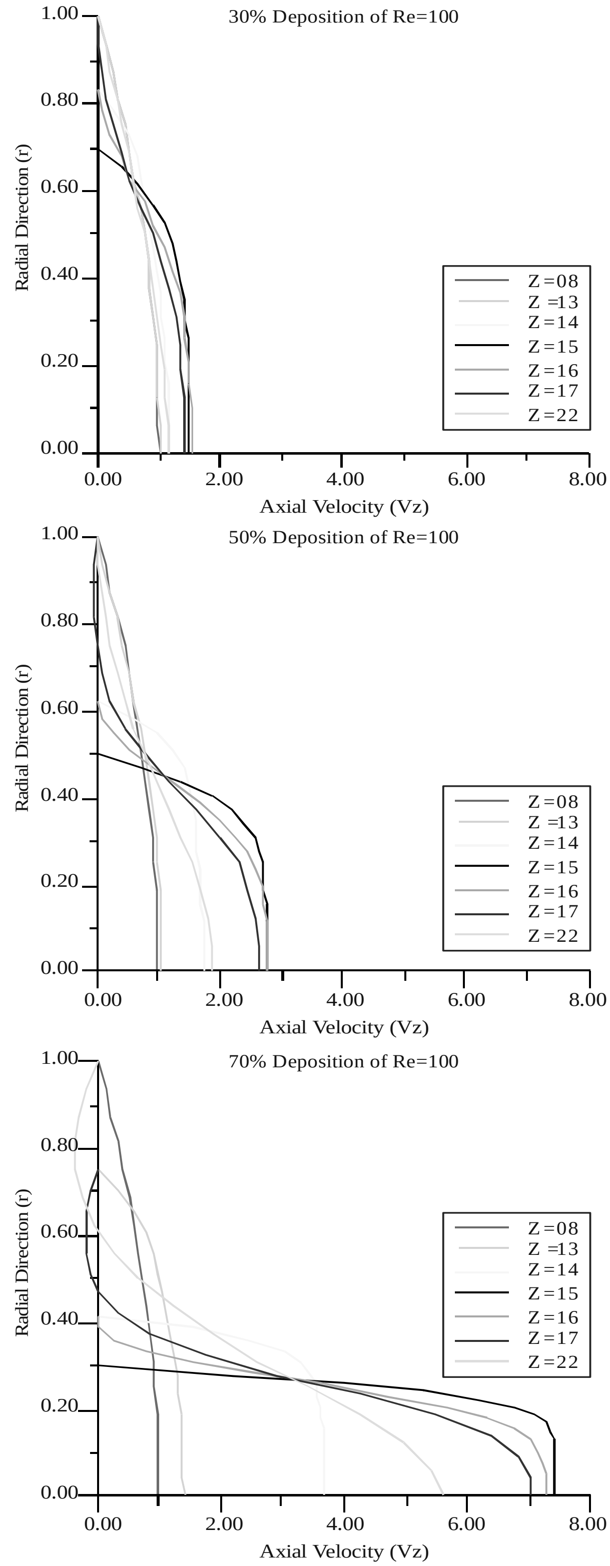

FIG. 6. VELOCITY PROFILES OF 30, 50, AND 70% DEPOSITION OF REYNOLDS NUMBER (Re=100)

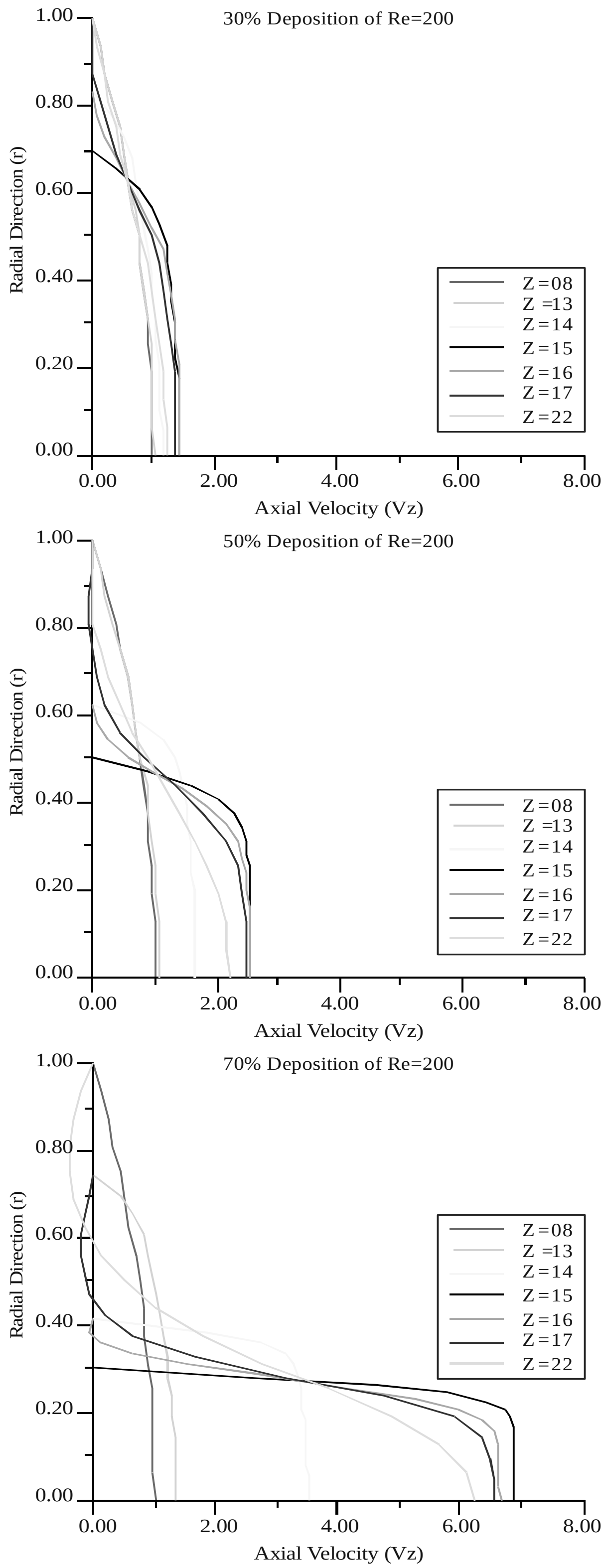

FIG. 7. VELOCITY PROFILES OF 30, 50, AND 70% DEPOSITION OF REYNOLDS NUMBER (Re=200)

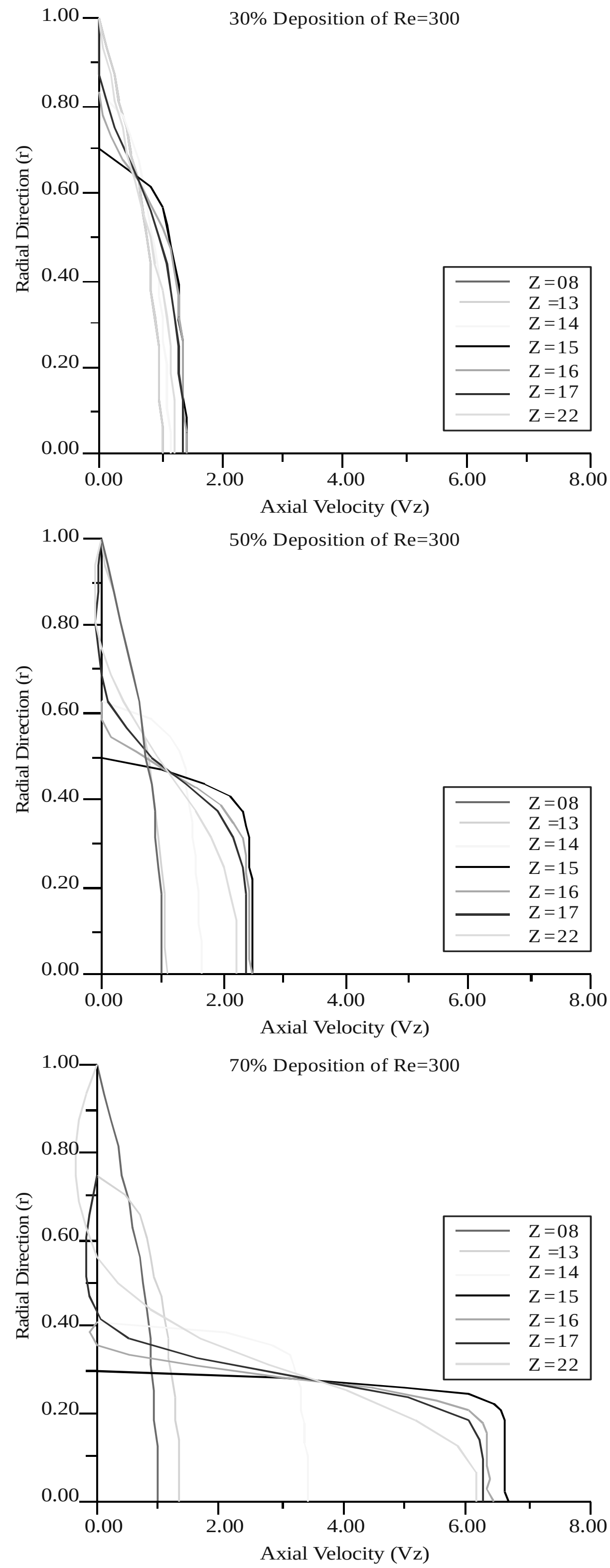

FIG. 8. VELOCITY PROFILES OF 30, 50, AND 70% DEPOSITION OF REYNOLDS NUMBER (Re=300)

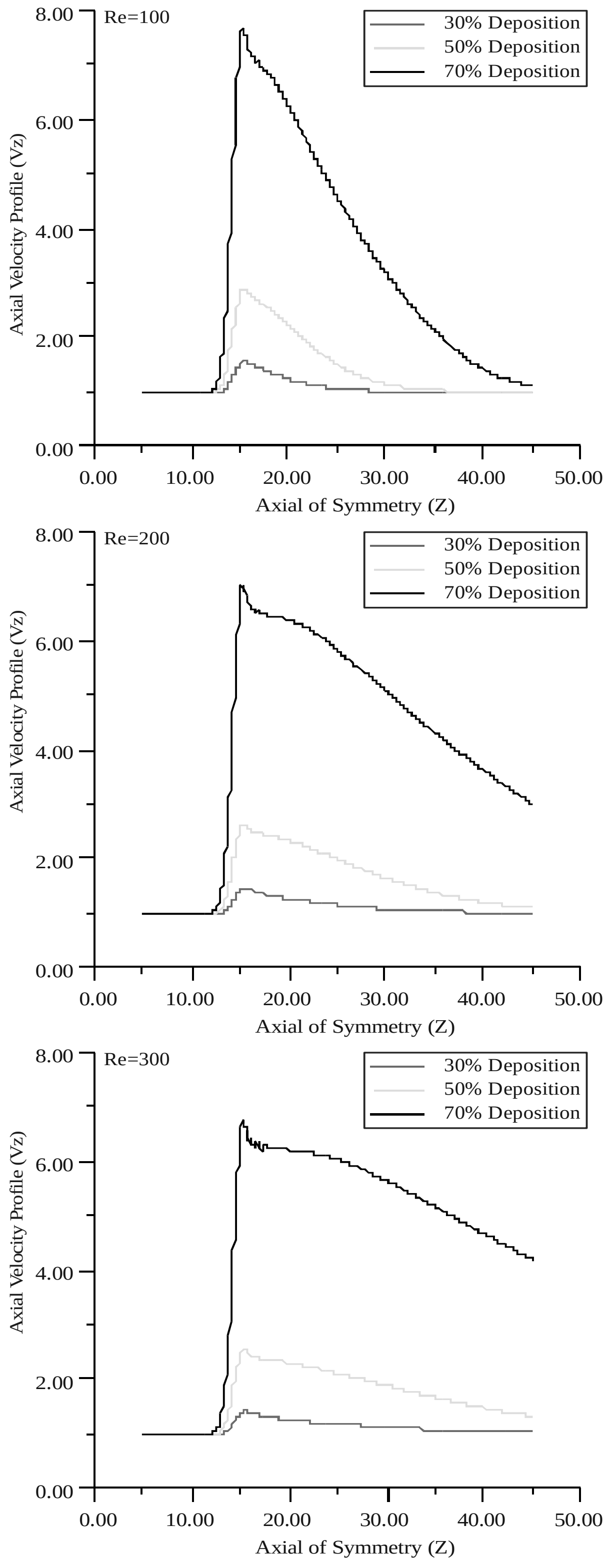

FIG. 9. VELOCITY PROFILES AT AXIS OF SYMMETRY FOR VARIOUS REYNOLDS NUMBERS

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