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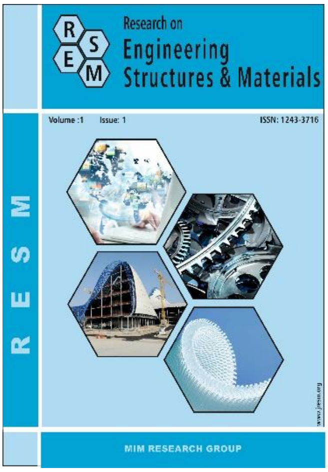

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Online Publication Date: 7 Oct 2015
URL: http://www.jresm.org/archive/resm2015.12me0814.html
DOI: http://dx.doi.org/10.17515/resm2015.12me0814

Journal Abbreviation: *Res. Eng. Struct. Mat.*

To cite this article
Demirdag O, Yildirim B. Comparing transfer matrix method and ANFIS in free vibration analysis of Timoshenko columns with attachments. *Res. Eng. Struct. Mat.*, 2016; 2: 1-18.

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Research Article

Comparing transfer matrix method and ANFIS in free vibration analysis of Timoshenko columns with attachments

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Article Info

Article history:
Received 14 Aug 2015
Revised 01 Oct 2015
Accepted 02 Oct 2015

Keywords:
Transfer Matrix Method, Adaptive Network Based Fuzzy Inference System, Natural frequency, Timoshenko column

Abstract

In this study, two approaches having different characteristics, one being Transfer Matrix Method (TMM) that reduces computational effort and time by reducing the dimension of the considered matrix to four for all problems and the other being The Adaptive Network based Fuzzy Inference System (ANFIS) used in The Fuzzy Logic Toolbox of Matlab software that again needs less computational effort and time are compared in the free vibration analysis of Timoshenko columns with attached masses having rotary inertia. The governing equation of the column elements is solved by applying the separation of variables method in the TMM algorithm. The same problems are solved, also, by fuzzy-neural approach in which ANFIS model is used by establishing Neuro Fuzzy Frequency Estimation (NFFE) models. Natural frequencies for the first three modes of an elastically supported Timoshenko column with 1, 5 and 10 attached masses are computed using NFFE models, and the results are compared with the ones of TMM. The comparison graphs are presented in numerical analysis to show the effectiveness of the considered methods, and it is resulted that neuro-fuzzy approach may give encouraging results for these kinds of models having great number of attached masses.

1. Introduction

Many researchers investigate the vibration of beam-columns with attached masses using conventional methods, most of them requiring much computing effort and time. For instance, Bapat and Bapat [1] investigated the natural frequencies of an Euler beam with attached masses using TMM, and modeled all supports by elastic springs against rotation and translation. Karami et all. [2] proposed a differential quadrature element method for free vibration analysis of nonuniform Timoshenko beams with elastic support and attachments. Lin and Chang [3] studied fee vibration analysis of multi-span Timoshenko beam with an arbitrary number of flexible constraints by TMM. Posiadala [4] considered the transverse free vibration of Timoshenko beams having rotation and translation springs, attached mass with moment of inertia, linear undamped oscillators and additional supports, and obtained the frequency equation by Lagrange multiplier formalism. TMM is used with Holzer method for torsional vibration of systems with attached masses [5], and with Myklestad-Thomson method for flexural vibrations of discrete systems with attached masses [6]. Esmailzadeh and Ohadi [7] made vibration and stability analysis of non-uniform Timoshenko beams under axial and distributed tangential loads. Gokdag and

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DOI: http://dx.doi.org/10.17515/resm2015.12me0814
Res. Eng. Struct. Mat. Vol. 2 Iss. 1 (2016) 1-18

Kopmaz [8] studied the coupled flexural-torsional free and forced vibrations of a Timoshenko beam with tip and/or in-span attachments. Ozkaya [9] obtained the non-linear equations of motion for transverse vibrations of a simply supported beam carrying attached masses. Demirdag [10] compared the transfer matrix and finite element methods in obtaining frequencies of elastically supported columns with attached masses. Demirdag and Catal [11] obtained the response spectra of semi-rigid supported single-storey frames modeled as a Timoshenko column with a tip mass.

In this study, two approaches having different characteristics, one being TMM that reduces computational effort and time by reducing the dimension of the considered matrix to four for all problems and the other being neuro-fuzzy approach that again needs less computational effort and time; however, since neuro-fuzzy approach is an estimation method it cannot be used to obtain exact results in any discipline and this is the main disadvantage of the estimation methods.

2. Problem Definition

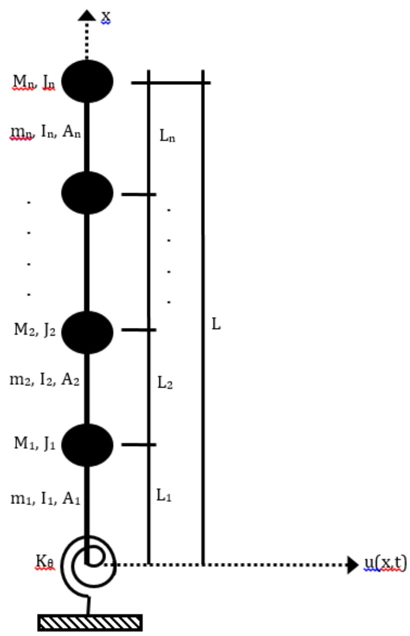


Fig. 1 Mathematical model of *n* uniform Timoshenko columns with *n* attached masses

The mathematical model of n uniform Timoshenko columns with n attached masses given in Fig. 1 is used in this study for multistory frames. Elastic support is modeled by rotation spring. In order to reflect the relative stiffness of the column and the rotational spring an end fixity factor is defined. Thus, the fixity factor is defined in Eq (1) from the rotational stiffness so that it takes as limits: null (0) value for a theoretically pinned joint and unity (1) for a theoretically rigid one [12].

$$f = \frac{1}{1 + \frac{3EI}{K_\theta L}} \qquad (0 \leq f \leq 1) \qquad (1)$$

where EI and L are flexural rigidity and length of the Timoshenko column, K_θ is the rotational spring constant. The governing equation of the free vibration is derived by including bending and shear deformation with rotary inertia of the columns. The rotary inertia of the attached masses is also included in the analysis. In order to study with nondimensionalized values the multiplication factors are defined for attached mass and its rotary inertia, respectively, as in the following

$$\bar{M}_i = \frac{M_i}{m_i L_i} \qquad \bar{J}_i = \frac{J_i}{m_i L_i^3} \tag{2}$$

where M_i and J_i are ith attached mass and its rotary inertia, L_i is the length of ith column. In this study, the natural frequencies of the model having different number of attached masses are obtained by three algorithms considering the variation of fixity factor, nondimensionalized attached mass and its rotary inertia values. Firstly, a TMM approach considering the continuity relations of displacement, slope, moment and shear at the interface of adjacent columns is performed to determine eigenfrequencies of the model. Considering the compatibility conditions at the interface of adjacent columns the relations between two adjacent spans is obtained; thus, exact values of eigenfrequencies of the entire system are determined for different number of masses by using TMM algorithm. Neuro-fuzzy algorithm is the second method used to obtain the frequencies of the model for the same conditions by establishing Neuro Fuzzy Frequency Estimation (NFFE) models. Nondimensionalized attached mass and its rotary inertia values, and fixity factors are the three inputs and the natural frequencies are the outputs necessary for the neuro-fuzzy algorithm.

3. Analysis by TMM

3.1. Determination of Eigenfunction

Differential equation of motion for the i^{th} Timoshenko column is

$$\frac{\partial^4 u_i}{\partial x_i^4} - \left(\frac{m_i k_i}{AG_i} + \frac{m_i r_i^2}{EI_i} \right) \frac{\partial^4 u_i}{\partial x_i^2 \partial t^2} + \frac{m_i^2 r_i^2 k_i}{EI_i AG_i} \frac{\partial^4 u_i}{\partial t^4} + \frac{m_i}{EI_i} \frac{\partial^2 u_i}{\partial t^2} = 0 \tag{3}$$

where $u_i(x_i,t)$, m_i, r_i, k_i, EI_i and AG_i, are displacement at x_i ($0 \le x_i \le L_i$), distributed mass, radius of gyration, effective shear area factor due to cross-section geometry, flexural and shear rigidities, respectively, of the ith column [10]. Applying the separation of variables method to Eq (3) in the form of Eq (4) for $T(t) \neq 0$ and rearranging with the dimensionless parameters gives the eigenfunction $X(x_i)$ of the i^{th} storey column as in Eq (5).

$$u_i(x_i,t) = X_i(x_i)T(t) = X_i(x_i)\left[A\sin(\omega t) + B\cos(\omega t) \right] \tag{4}$$

$$X_i(x_i) = C_{1i}\sinh(\lambda_{1i} x_i) + C_{2i}\cosh(\lambda_{1i} x_i) + C_{3i}\sin(\lambda_{2i} x_i) + C_{4i}\cos(\lambda_{2i} x_i) \tag{5}$$

where

$$\alpha_{1i} = m_i k_i \omega^2 / AG_i$$

$$\alpha_{2i} = m_i \omega^2 / EI_i$$

$$\alpha_{3i} = \alpha_{1i} + \alpha_{2i} r_i^2$$

$$\Delta_i = \left(\alpha_{1i} - \alpha_{2i} r_i^2\right)^2 + 4\alpha_{2i}$$

$$n_{1i} = \left(-\alpha_{3i} + \sqrt{\Delta_i}\right)/2$$

$$n_{2i} = \left(-\alpha_{3i} - \sqrt{\Delta_i}\right)/2$$

$$\lambda_{1i} = \sqrt{n_{1i}}\ ;\ \lambda_{2i} = \sqrt{|n_{2i}|}$$

C_{1i}...C_{4i} are integration constants. Moment, shear, slope functions of the i^{th} Timoshenko column are [13]

$$M_i(x_i,t) = -EI_i u_i''(x_i,t) - EI_i \alpha_1 u_i(x_i,t) \tag{6.1}$$

$$V_i(x_i,t) = \left[-EI_i / \left(1 - \alpha_{1i} r_i^2\right)\right]\left[u_i'''(x_i,t) + \alpha_{3i} u_i'(x_i,t)\right] \tag{6.2}$$

$$\theta_i(x_i,t) = u_i'(x_i,t) - V_i(x_i,t) k_i / AG_i \tag{6.3}$$

3.2. Boundary Conditions

Boundary conditions at the interface of the adjacent (i-1)th and i^{th} columns (Fig. 2) are written as in Eq (7) using the continuity of displacement and slope and the equilibrium of moment and shear [14].

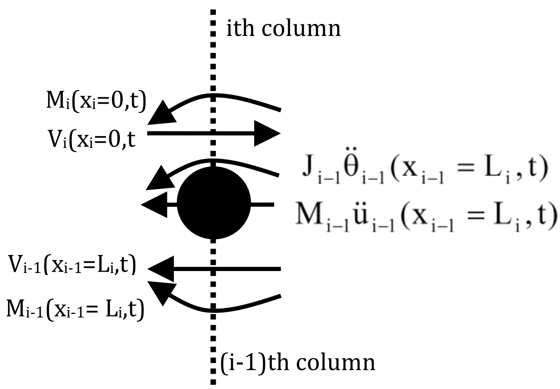


Fig. 2 Free body diagram for the interface of (i-1)th and ith columns

$$u_{i-1}(x_{i-1} = L_{i-1}, t) = u_i(x_i = 0, t) \tag{7.1}$$

$$\theta_{i-1}(x_{i-1} = L_{i-1}, t) = \theta_i(x_i = 0, t) \tag{7.2}$$

$$M_{i-1}(x_{i-1} = L_{i-1}, t) - J_{i-1}\ddot{\theta}_{i-1}(x_{i-1} = L_{i-1}, t) = M_i(x_i = 0, t) \tag{7.3}$$

$$V_{i-1}(x_{i-1} = L_{i-1}, t) + M_{i-1}\ddot{u}_{i-1}(x_{i-1} = L_{i-1}, t) = V_i(x_i = 0, t) \tag{7.4}$$

Since continuity of displacement and slope is not valid for the support and the n^{th} attached mass, one gets 4(n-1) relations from Eq (7). However, four more relations are needed for the entire system, two given in Eq (8) from the elastic support in Fig. 3 and two given in Eq (9) from the n^{th} attached mass in Fig. 4 where K_θ is rotational spring constant [14].

$$\left.\begin{array}{l} u_1(x_1=0,t)=0 \\ M_1(x_1=0,t)=-K_\theta\theta_1(x_1=0,t) \end{array}\right\} \quad (8)$$

$$\left.\begin{array}{l} M_n(x_n=L_n,t)=J_n\ddot{\theta}_n(x_n=L_n,t) \\ V_n(x_n=L_n,t)=-M_n\ddot{u}_n(x_n=L_n,t) \end{array}\right\} \quad (9)$$

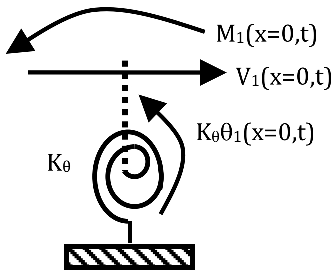


Fig. 3 Free body diagram of elastic support

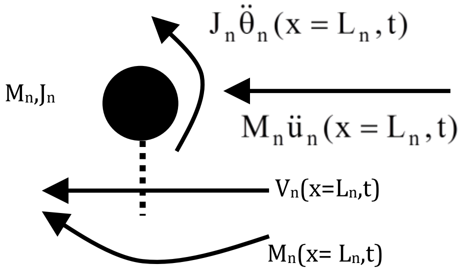


Fig. 4 Free body diagram of n^{th} mass

3.2. Obtaining Transfer Matrix

The relation between $C_{1i}...C_{4i}$ and $C_{1i-1}...C_{4i-1}$ is written from Eq (7) in matrix form as

$$\begin{Bmatrix} C_{1i} \\ C_{2i} \\ C_{3i} \\ C_{4i} \end{Bmatrix} = [T_i] \begin{Bmatrix} C_{1i-1} \\ C_{2i-1} \\ C_{3i-1} \\ C_{4i-1} \end{Bmatrix} = \begin{bmatrix} T_{i11} & T_{i12} & T_{i13} & T_{i14} \\ T_{i21} & T_{i22} & T_{i23} & T_{i24} \\ T_{i31} & T_{i32} & T_{i33} & T_{i34} \\ T_{i41} & T_{i42} & T_{i43} & T_{i44} \end{bmatrix} \begin{Bmatrix} C_{1i-1} \\ C_{2i-1} \\ C_{3i-1} \\ C_{4i-1} \end{Bmatrix} \quad (i=2,3,...,n) \quad (10)$$

where

$$T_{i11}=\alpha_{26i}\alpha_{9i-1}ch_{i-1}+\alpha_{27i}\alpha_{14i-1} \qquad T_{i12}=\alpha_{26i}\alpha_{9i-1}sh_{i-1}+\alpha_{27i}\alpha_{15i-1}$$

$$T_{i13} = \alpha_{26i}\alpha_{10i-1}c_{i-1} + \alpha_{27i}\alpha_{16i-1} \qquad T_{i14} = -\alpha_{26i}\alpha_{10i-1}s_{i-1} + \alpha_{27i}\alpha_{17i-1}$$

$$T_{i21} = \alpha_{31i}sh_{i-1} - \alpha_{30i}\alpha_{21i-1} \qquad T_{i22} = \alpha_{31i}ch_{i-1} - \alpha_{30i}\alpha_{22i-1}$$

$$T_{i23} = \alpha_{31i}s_{i-1} - \alpha_{30i}\alpha_{23i-1} \qquad T_{i24} = \alpha_{31i}c_{i-1} - \alpha_{30i}\alpha_{24i-1}$$

$$T_{i31} = \alpha_{28i}\alpha_{9i-1}ch_{i-1} - \alpha_{29i}\alpha_{14i-1} \qquad T_{i32} = \alpha_{28i}\alpha_{9i-1}sh_{i-1} - \alpha_{29i}\alpha_{15i-1}$$

$$T_{i33} = \alpha_{28i}\alpha_{10i-1}c_{i-1} - \alpha_{29i}\alpha_{16i-1} \qquad T_{i34} = -\alpha_{28i}\alpha_{10i-1}s_{i-1} - \alpha_{29i}\alpha_{17i-1}$$

$$T_{i41} = \alpha_{32i}sh_{i-1} + \alpha_{30i}\alpha_{21i-1} \qquad T_{i43} = \alpha_{32i}s_{i-1} + \alpha_{30i}\alpha_{23i-1}$$

$$T_{i42} = \alpha_{32i}ch_{i-1} + \alpha_{30i}\alpha_{22i-1} \qquad T_{i44} = \alpha_{32i}c_{i-1} + \alpha_{30i}\alpha_{24i-1}$$

$$\alpha_{32i} = \alpha_{4i}\alpha_{30i} \qquad \alpha_{31i} = \alpha_{5i}\alpha_{30i}$$

$$\alpha_{30i} = 1/\left(\alpha_{4i} + \alpha_{5i}\right) \qquad \alpha_{29i} = \alpha_{9i}/\alpha_{25i}$$

$$\alpha_{28i} = \alpha_{12i}/\alpha_{25i} \qquad \alpha_{27i} = \alpha_{10i}/\alpha_{25i}$$

$$\alpha_{26i} = \alpha_{13i}/\alpha_{25i} \qquad \alpha_{25i} = \alpha_{9i}\alpha_{13i} + \alpha_{12i}\alpha_{10i}$$

$$\alpha_{24i} = \alpha_{5i}c_i - \alpha_{20i}s_i \qquad \alpha_{23i} = \alpha_{5i}s_i + \alpha_{20i}c_i$$

$$\alpha_{22i} = \alpha_{19i}sh_i - \alpha_{4i}ch_i \qquad \alpha_{21i} = \alpha_{19i}ch_i - \alpha_{4i}sh_i$$

$$\alpha_{20i} = \alpha_{18i}\alpha_{10i} \qquad \alpha_{19i} = \alpha_{18i}\alpha_{9i}$$

$$\alpha_{18i} = J_i\omega^2 \qquad \alpha_{17i} = \alpha_{13i}s_i - \alpha_{11i}c_i$$

$$\alpha_{16i} = -\alpha_{13i}c_i - \alpha_{11i}s_i \qquad \alpha_{15i} = \alpha_{12i}sh_i - \alpha_{11i}ch_i$$

$$\alpha_{14i} = \alpha_{12i}ch_i - \alpha_{11i}sh_i \qquad \alpha_{13i} = \alpha_{6i}\alpha_{8i}$$

$$\alpha_{12i} = \alpha_{6i}\alpha_{8i} \qquad \alpha_{11i} = M_i\omega^2$$

$$\alpha_{10i} = \lambda_{2i} + \alpha_{6i}\alpha_{8i}k_i/AG_i \qquad \alpha_{9i} = \lambda_{1i} - \alpha_{6i}\alpha_{7i}k_i/AG_i$$

$$\alpha_{8i} = \lambda_{2i}\left(\lambda_{2i}^2 - \alpha_{3i}\right) \qquad \alpha_{7i} = \lambda_{1i}\left(\lambda_{1i}^2 + \alpha_{3i}\right)$$

$$\alpha_{6i} = -EI_i/\left(1 - \alpha_{1i}r_i^2\right) \qquad \alpha_{5i} = EI_i\left(\lambda_{2i}^2 - \alpha_{1i}\right)$$

$$\alpha_{4i} = EI_i\left(\lambda_{1i}^2 + \alpha_{1i}\right) \qquad sh_i = \sinh\left(\lambda_{1i}L_i\right)$$

$$ch_i = \cosh\left(\lambda_{1i}L_i\right) \qquad s_i = \sin\left(\lambda_{2i}L_i\right)$$

$$c_i = \cos\left(\lambda_{2i}L_i\right)$$

Applying Eq (10) consecutively for *n* storey gives

$$\begin{Bmatrix} C_{1n} \\ C_{2n} \\ C_{3n} \\ C_{4n} \end{Bmatrix} = \left[T_t\right] \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \end{Bmatrix} = \left[T_n\right]\left[T_{n-1}\right].....\left[T_3\right]\left[T_2\right] \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \end{Bmatrix} \tag{11}$$

where $[T_t]$ is the transfer matrix of the entire system. Substituting Eq (11) into Eq (9) gives two more equation related to $C_{11}...C_{41}$, therefore, there exists 4 homogeneous equations together with Eq (8) that characterize free vibration of the entire system as

$$\left[F\right] \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \\ C_{41} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{12}$$

where [F] is coefficient matrix. Equating the determinant of [F] to zero gives frequency equation of the entire system, and every root of this frequency equation is the eigenfrequency of the model. These frequencies are computed by a program written by the authors considering the secant method [15].

4. Neuro-Fuzzy Modeling

In fuzzy modeling, the membership functions and rule base are generally determined by trial-and-error approaches. Although this approach is straightforward, the determination of best fitting boundaries of membership functions and number of rules are very difficult. In order to calibrate the membership functions and rule base in fuzzy modeling, the neural networks have been employed by researchers [16-21]. This system has been called fuzzy neural, neuro-fuzzy or adaptive network based system. The key properties of neuro-fuzzy systems are the accurate learning and adaptive capabilities of the neural networks, together with the generalization and fast-learning capabilities of fuzzy logic systems. The Adaptive Network based Fuzzy Inference System (ANFIS) was developed by Jang [16] and is used in The Fuzzy Logic Toolbox of Matlab software.

To explain the ANFIS architecture, the first order Sugeno model with the following rules is taken into account:

Rule 1: If (x is A_1) and (y is B_1) then ($f_1 = p_1 x + q_1 y + r_1$)

Rule 2: If (x is A_2) and (y is B_2) then ($f_2 = p_2 x + q_2 y + r_2$)

where *x* and *y* are the inputs, A_i and B_i are the fuzzy sets, f_i are the outputs within the fuzzy region specified by the fuzzy rule, p_i; q_i and r_i are the design variables that are ascertained during training process. The ANFIS architecture to implement these two rules is shown in Fig. 5, in which a circle indicates a fixed node, whereas a square indicates an adaptive node.

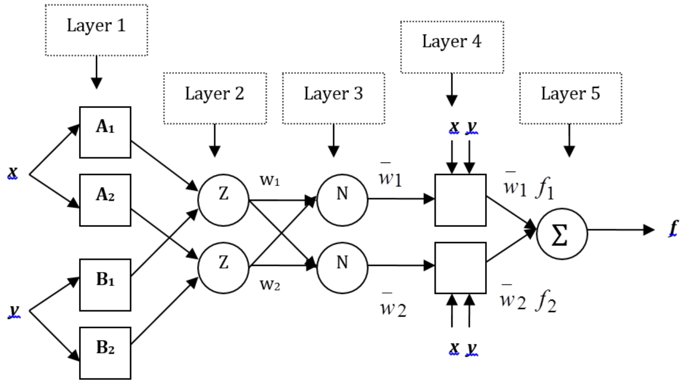


Fig. 5 ANFIS architecture

In the first layer, all the nodes are adaptive nodes. The outputs of Layer 1 are the fuzzy membership grade of the inputs, which are given by:

$$O_i^1 = \mu_{A_i}(x) \qquad i=1,2 \tag{13}$$

$$O_i^1 = \mu_{B_{i-2}}(y) \qquad i=3,4 \tag{14}$$

where $\mu_{A_i}(x)$, $\mu_{B_{i-2}}(y)$ can adopt any fuzzy membership function. For instance, if the Gaussian function is employed, $\mu_{A_i}(x)$ is given by:

$$\mu_{A_i}(x) = \exp\left[-\left(\frac{x-c_i}{a_i}\right)^2\right] \tag{15}$$

where a_i and c_i are the variables of the membership function. As the values of these variables change, the Gaussian function varies accordingly, thus exhibiting various forms of membership functions on linguistic label A_i. Variables in this layer are referred to as premise variables.

In the second layer, the nodes are fixed nodes. They are labeled with *Z* which multiplies the incoming signals and sends the product out. The outputs of this layer can be represented as:

$$O_i^2 = w_i = \mu_{A_i}(x)\mu_{B_i}(y) \qquad i=1,2 \tag{16}$$

which are the firing strengths of a rule.

In the third layer, the nodes are also fixed nodes. They are labeled with *N*; indicating that they play a normalization role to the firing strengths from the previous layer. The outputs of this layer can be represented as:

$$O_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2} \qquad i=1,2 \tag{17}$$

which are the so-called normalized firing strengths.

In the fourth layer, the nodes are adaptive nodes. The output of each node in this layer is simply the product of the normalized firing strength and a first order polynomial (for a first order Sugeno model). Thus, the outputs of this layer are given by:

$$O_i^4 = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \tag{18}$$

In the fifth layer, there is only one single fixed node labeled with Σ. This node performs the summation of all incoming signals. Hence, the overall output of the model is given by:

$$O_i^5 = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{19}$$

In order to tune premise (a_i, c_i) and design variables (p_i, q_i, r_i) the hybrid learning algorithm was proposed by Jang et al. [22]. The hybrid learning algorithm combines gradient descent and least square methods and it is faster than a back propagation algorithm. The least squares method (forward pass) is used to optimize the consequent variables with the premise variables fixed. Once the optimal consequent variables are found, the backward pass starts immediately. The gradient descent method (backward pass) is used to adjust optimally the premise variables corresponding to the fuzzy sets in the input domain. By this passing process, optimum variables are determined. The details of this algorithm can be obtained from Jang et al. [22]. This approach is also used in different engineering disciplines by many researchers [23-24].

5. Numerical Analysis

Natural frequencies for the first three modes of an elastically supported Timoshenko column with 1, 5 and 10 attached masses are computed by both TMM and Neuro-Fuzzy approaches for parameters of f=0.1, 0.25, 0.5, 0.75, 0.99, 0.999; $\bar{M}_i$ =0.1, 0.5, 1, 2.5, 5, 7.5, 10; $\bar{J}_i$ =0.1, 0.5, 1, 5, 10. m_i=0.32 kNs^2/m^2, L_i=1 m, EI=1353870 kNm^2, AG=3240000 kN, k=2.426, S_x=0.00743 m^3, A=0.04 m^2, I=0.006447 m^4 are the characteristics of the IPB profile column used for the numerical analysis.

According to Eq (1), the relationship between the connection stiffness ($K_\theta L/EI$) and the fixity factor (f) is approximately linear when the fixity factor values are between 0.0 and 0.5 and nonlinear from 0.5 to unity as shown in Fig. 6. It can be seen from the graph that as the fixity factor approaches unity the curve increases asymptotically to infinity since the fixity factor of unity is used for theoretically ideal fixed support.

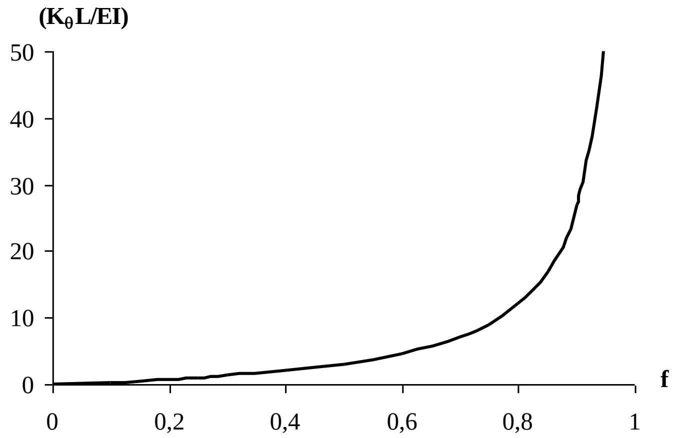


Fig. 6 Relationship between the connection stiffness ($K_\theta L/EI$) and the fixity factor (f)

5.1. The NFFE Models

The NFFE models were developed for estimating vibration frequencies of an elastically supported Timoshenko column with attached masses for different conditions. NFFE1, NFFE2, NFFE3; NFFE4, NFFE5, NFEE6 and NFFE7, NFFE8, NFEE9 are the models for the first, second, third modes of one, five and ten attached masses system, respectively. The effective variables of the vibration phenomenon are determined considering previous studies and models. The Attached Mass (AM), Rotary Inertia (RI) and Fixity Factor (FF) are ascertained as the fuzzy logic vibration estimation model variables.

The natural frequencies are affected by support condition. Rotational spring is used for the elastic support to model the general support conditions. Increases in spring coefficient values cause increases in frequencies. Fixity factor concept is used in the study to formulate elastic support behavior. Theoretically, zero for fixity factor value denotes a pinned support whereas infinity denotes a fixed support. Therefore, variation of fixity factor is considered one of the effective variables on frequency values.

The attached mass on the column is determined as the second variable. The number and the value of the attached masses are directly related to frequency values of the column. Increasing the number and the value of the attached masses decreases the frequency values. The third variable determined for the input parameter is the rotary inertia of the attached mass. An increase in the value of rotary inertia of the attached mass causes, also, a decrease in the frequency values. Generally in vibration problems, however, the models like in this study are the mathematical models that are formed to model more complex real systems. Therefore, determination of the value of the attached mass and of its rotary inertia is very hard and includes uncertainties. As a result, the nondimensional parameters for the attached mass and its rotary inertia are selected as the effective variables on vibration frequency. Membership functions of the variable are determined using the data obtained by ANFIS approach.

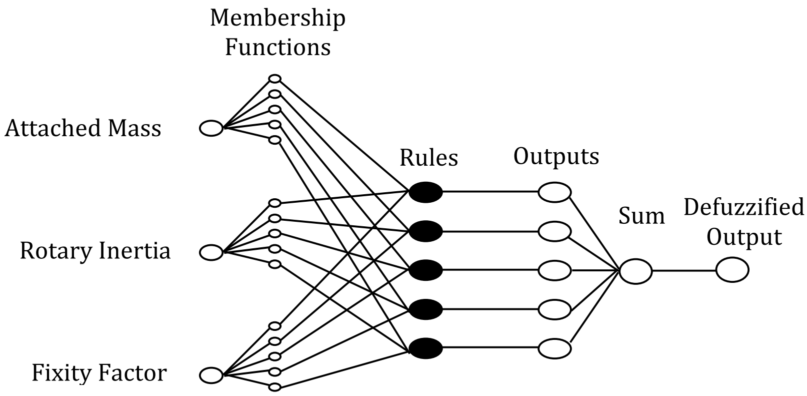


Fig. 7 NFFE model structure

The NFFE model is developed using Fuzzy Logic Toolbox of the software Matlab 7.0. The NFFE model structure is indicated in Fig. 7. The membership functions are determined by using ANFIS approach. The Adaptive Neuro Fuzzy Inference System (ANFIS) has three input variables and one output variable. As a process used by ANFIS systems, the initial values of the antecedents' variables can be defined in a way that the centers of the membership functions are equally spaced along the range of each input variable. Then, the variables of the fuzzy rules are optimized to get the final membership range. The Gaussian membership functions are used in definition of NFFE model variables. The tuned membership functions of the input variables are shown in Fig. 8. The Sugeno fuzzy inference system is used in the NFFE model. In Sugeno fuzzy inference models, the crisp output function (or value) is described using the input fuzzy variables. General form of the output function (linear) used in NFFE model is given in Eq (20). The coefficients and intercepts (design variables for Layer 4) of output membership functions are determined after training process in NFFE model. These values are given in Table 1.

$$f_i = p_i AM + q_i RI + s_i FF + r_i \tag{20}$$

The hybrid learning rule was applied for identifying the output variables in the neuro-fuzzy optimization process. The learning hybrid rule combines steepest descent and least squares estimator for identifying the variables of the consequent part of the inferential rules [19-22].

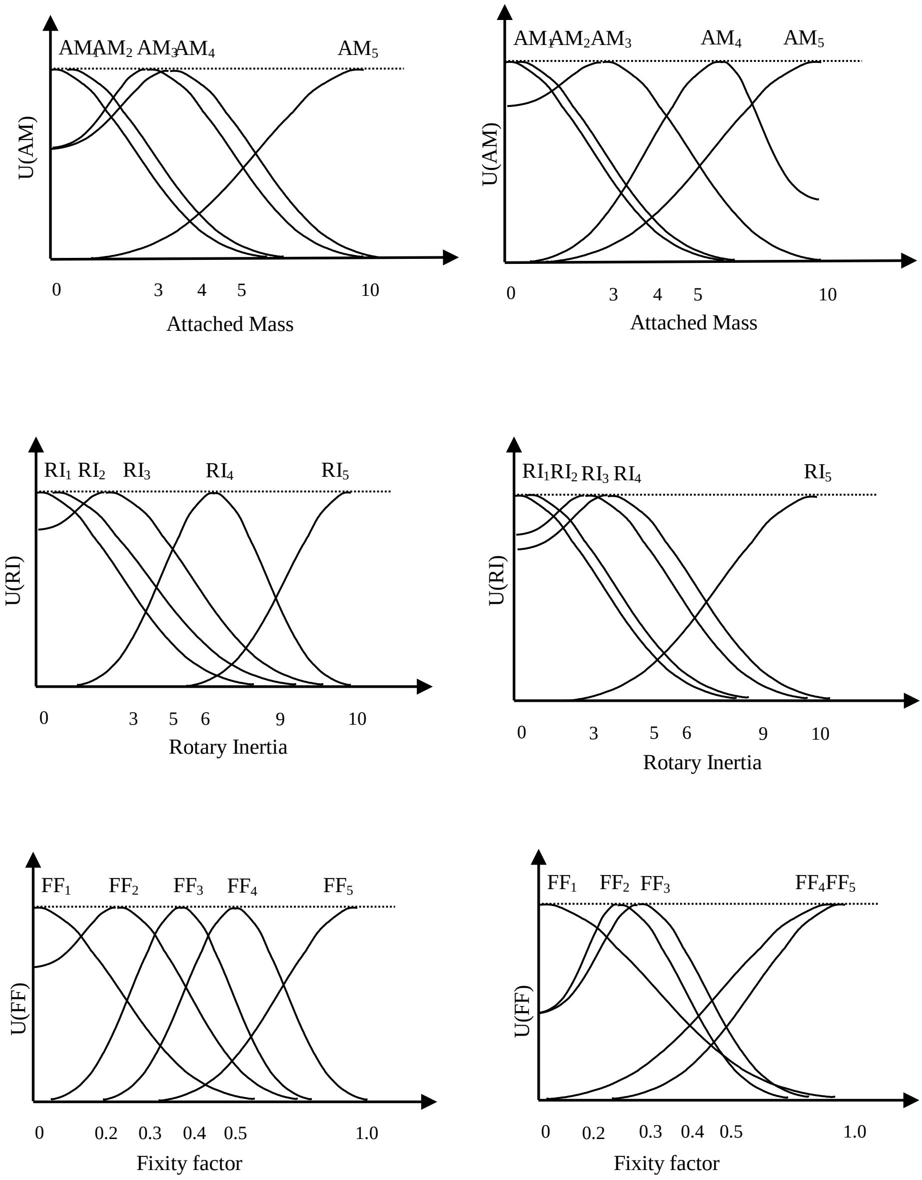


Fig. 8 Membership functions of the NFFE1 and NFFE4 models

Table 1 Output membership functions coefficients and intercepts of NFFE4 model

Output Membership Function	p_i	q_i	s_i	r_i
f_1	-39.83	-5.97	114.00	113.30
f_2	-9.27	0.07	191.10	45.27
f_3	-6.07	-0.75	46.87	81.04
f_4	-6.43	4.62	98.64	2.60
f_5	-2.28	0.15	67.62	37.14

The variables of the antecedent part of the fuzzy inference rules are set up based on evaluation of characteristics of the input data set. The rule base of the model is formed considering membership functions of the variables. In NFFE models, the rule base is ascertained by ANFIS approach. Sample rule bases for NFFE4 model are given in Table 2.

The weighted average method is used for defuzzification in the NFFE models. The mean absolute error (MAE), mean squared error (MSE) and average relative error (ARE) rates of the nine neuro-fuzzy models are presented in Table 3.

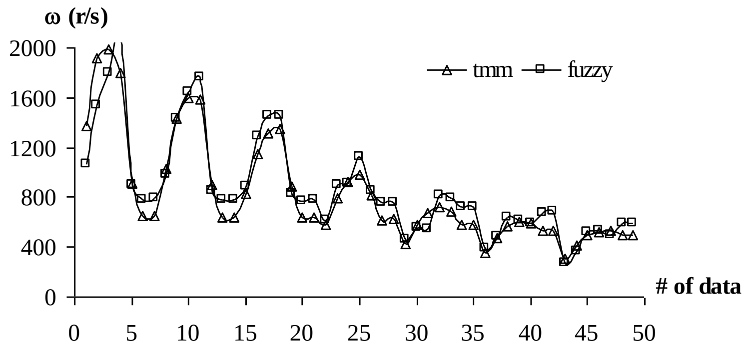
Table 2 NFFE4 model rule bases

1.	IF AM is AM_1 and RI is RI_1 and FF is FF_1 THEN O is f_1
2.	IF AM is AM_2 and RI is RI_2 and FF is FF_2 THEN O is f_2
3.	IF AM is AM_3 and RI is RI_3 and FF is FF_3 THEN O is f_3
4.	IF AM is AM_4 and RI is RI_4 and FF is FF_4 THEN O is f_4
5.	IF AM is AM_5 and RI is RI_5 and FF is FF_5 THEN O is f_5

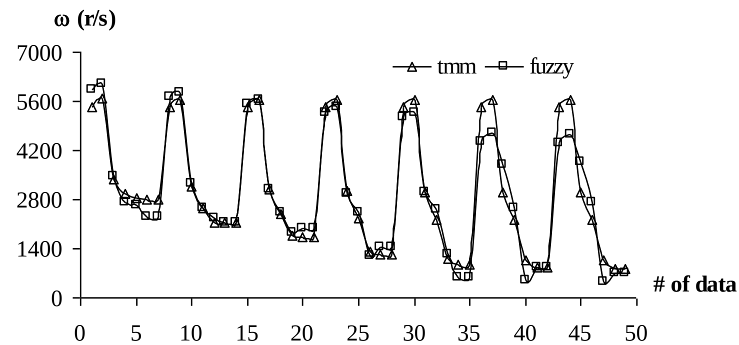
Table 3 Error rates

Models	Errors					
	Train Errors			Test Errors		
	MAE	MSE	ARE (%)	MAE	MSE	ARE (%)
NFFE1	35.73	3610.57	4.22	99.64	16335.40	12.55
NFFE2	238.57	115495.92	11.07	300.85	171364.31	12.39
NFFE3	256.26	101422.78	3.57	351.45	180805.52	5.00
NFFE4	5.61	62.34	6.54	6.18	79.66	6.65
NFFE5	9.08	181.48	1.70	18.50	1032.96	3.54
NFFE6	34.71	3283.39	2.95	43.68	5891.18	3.58
NFFE7	0.73	0.99	3.44	1.18	3.59	4.29
NFFE8	3.53	24.49	2.09	6.00	92.24	3.58
NFFE9	7.82	128.37	1.81	17.27	539.95	4.25

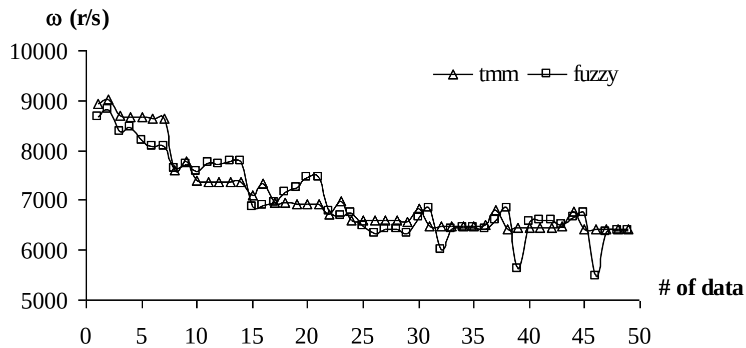
The frequency values of the elastically supported Timoshenko column with 1, 5 and 10 attached masses are computed by TMM and estimated by Neuro-Fuzzy. The comparison graphs of the frequency values obtained for the models with 1, 5 and 10 attached masses are presented, respectively, in Figs. 9-11 for the first, second and third modes. The data in the *x* axis of the graphs is the number of natural frequency values used for the testing phase of neuro-fuzzy models.



a) First mode

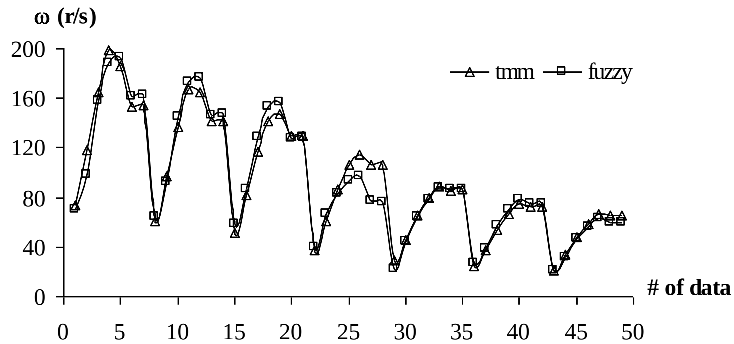


b) Second mode

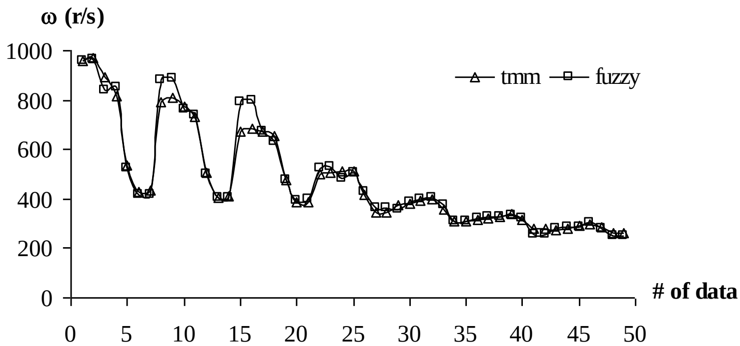


c) Third mode

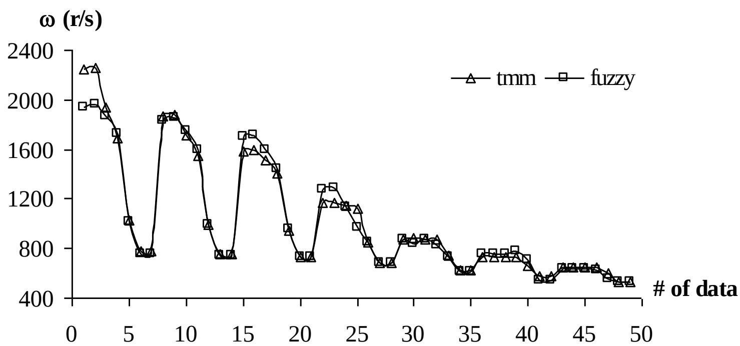
Fig. 9 Comparing frequency values of the model with 1 attached mass obtained from TMM and fuzzy



a) First mode

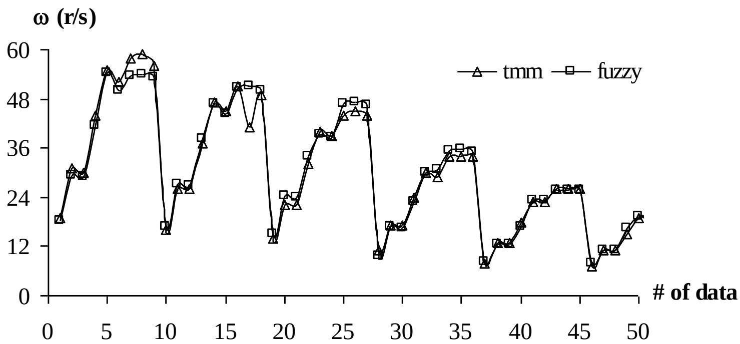


b) Second mode

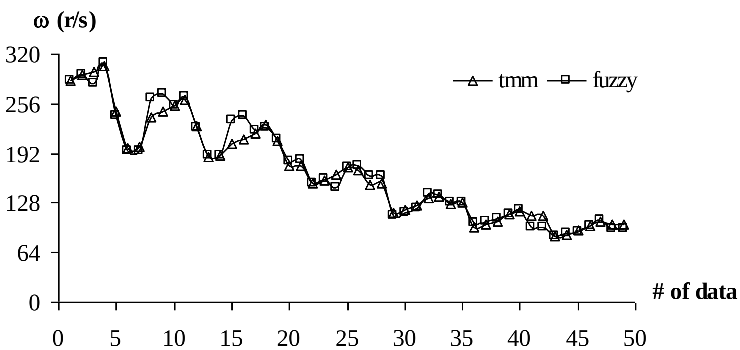


c) Third mode

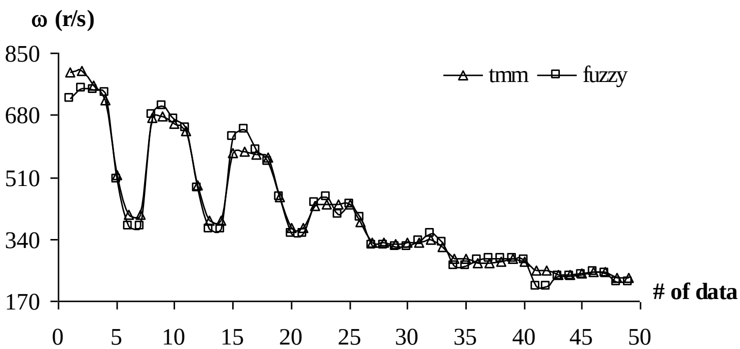
Fig. 10 Comparing frequency values of the model with 5 attached mass obtained from TMM and fuzzy.



a) First mode



b) Second mode



c) Third mode

Fig. 11 Comparing frequency values of the model with 10 attached mass obtained from TMM and fuzzy.

6. Conclusions

In this study, elastically supported Timoshenko column with attached masses is under consideration to obtain its free vibration natural frequencies using two different algorithm; transfer matrix method and fuzzy neural approach.

For one or two span models it is easy to obtain the frequency equation in explicit form by equating the determinant of coefficient matrix written according to boundary conditions of the entire system to zero, however, for large number of spans frequency equation will be extremely complex, therefore, the transfer matrix method will be more computationally efficient for these kind of models. In addition, another effective method -neuro-fuzzy approach- that reduces the computational effort and time is also used to obtain the free vibration frequencies of the model in the study.

The results of TMM and the ANFIS models are compared in training and test sets; the comparison of the test sets with TMM is given in graphs, and errors of the training and test sets are given in tables. From the comparing graphs in Figs 9-11, it can be concluded that neuro-fuzzy approach give values generally close to the values obtained from TMM, thus, ANFIS can be applied for vibration frequency estimation. It is seen from Table 3 that MAE value is decreasing as the number of attached mass is increasing, it means that neuro-fuzzy approach give better results for the model with five attached masses than with one and for the model with ten attached masses than with five. Thus, neuro-fuzzy approach may give encouraging results for these kinds of models having great number of attached masses.

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