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FOURTH ORDER COMPACT FINITE DIFFERENCE SCHEME FOR SOYBEAN HYDRATION MODEL WITH MOVING BOUNDARY

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ABSTRACT. A numerical solution of the nonlinear diffusion model for soybean hydration with moving boundary is obtained using fourth order compact finite difference method (CDF4). CFD4 is applied in spatial coordinates and explicit finite difference is used in time coordinates. Velocity of radius is calculated by CFD4. The results were compared with explicit finite difference method which is used in [**7**] and demonstrated that the present method has high accuracy with minimal computational effort for soybean hydration model.

1. Introduction

Measurement of moisture content during soaking is one of the most important analyses performed on food product such as soybean-derived foods. Water absorbation has significant influence on grain texture which affects grinding and protein extraction [**1**]. Pre-soaking brings down the cooking time required to achieve the desired softness making the cooking process convenient. Also, this process prior to cooking of soybean eliminates the toxic factors contained in the raw seed and reduces firmness of cooked soybeans [**2**]. The problems which include soybean hydration are described by emprical models [**3**] and by phenomenological models [**4, 5**]. Emprical models include simple mathematical correlation but do not describe diffusion steps. In phenomenological models, elementary steps of diffusion and/or convection mass transfer is considered [**5**] and this model are represent lumped or distributed parameters. Lumped parameters system which do not take into account concentration variations inside the grain are modeled by ordinary differential equations. On the other hand, distributed parameter systems are represent

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concentration gradients inside the grain over time. Since these problems are time-dependent problems, they are modeled by parabolic partial differential equation with an initial condition and two boundary condition, are class of initial-boundary value problems. Boundary conditions are adopted for center and surface of the grain. In such problems, the increase in size of the grain occurs when the water diffuses enters the system [**6, 7, 8, 9, 10**]. The problems involving increase or decrease in the size of systems such as swelling of grease, grain and polymers, drying of potatoes and grapes are known as moving boundary problems or Stefan problems, with reference to work of J.Stefan was studied in the melting of the polar ice cap. After the Stefan's work, Stefan problems were mainly developed for tracking the motion of front that occurs in the phase change for water-ice problems.

Due to diffuculties in obtaining analytical solution of Stefan problems several authors have dedicated to numerical solutions of Stefan problem by applying various numerical methods including finite differences, finite elements and integral methods. Earlier studies are found in Ref.[**11**].

Very recently, Sadoun et al. [**12**] proposed a modified variable space grid method for heat conduction problem and compared their solution with other solution in the literature. Yiğit [**13**] used finite difference method with variable space grid and variable time step for one dimensional solidification problem to position of the moving front and its velocity. Also, he developed an analytical method for limiting case and compared with numerical results. Reutskiy [**14, 15**] presents a new messles method for one dimensional Stefan problem and Stefan type problem with moving boundaries in spherical coordinates. Mitchell et al.[**16**] used the Keller box finite difference method with boundary immobilization method for the Stefan problem of evaporiation of spherical droplets. Lee et al. [**17**] proposed a finite difference moving mesh method for moving boundary solution and apply their method to the porous medium equation, Richard's equation and the Crank-Gupta problem.

The swelling problems as Stefan Problem taken into account by Davey et al. [**8**], McGuinness et al. [**9**] and Barry and Caunce [**10**]. They proposed for the moisture diffusion model where diffusivity is exponential function of moisture content. In these papers, the models include two moving boundaries simultaneously. One of them expresses movement of the radius and other expresses the movement of the water inside the material, in hand. These studies propose exact solution for the model but they do not extended to analyzing moisture profiles obtained during modeling.

Viollaz et al. [**18**] and Viollaz and Rovedo [**19**] used boundary immobilization method for problem with volume change due to drying or swelling.

Nicolin et al. [**7**] present a model which considers volume variation to analyzes the moisture profiles inside the grains. The model has one moving boundary which represent behaviour of the grain by a differential equation based on a global mass balance over the spherical soybean. They used Variable Space Grid Method (VSGM) to represent the movement of the system grid and solved the problem explicit finite difference method by validating experimental data.

In this work, we aimed to apply a combination of explicit finite difference and fourth order compact finite difference method to obtain solution of the soybean hydration model proposed by Nicolin et al. [**7**]. Our numerical results is compared by Nicolin et al. [**7**] and calculated computational times for both of method, explicit finite difference method used Nicolin et al. [**7**] and compact finite difference method.

2. Mathematical Modeling

The model was obtained by transient mass balance on differential volume element of soybean grains. Since the geometry of soybeans are assumed spherical, the Eq.(2.1) that represents water absorption by soybean is developed in spherical coordinates based on Fick's Law of Diffusion. It is assumed that diffusion takes place only in radial direction.

$$\frac{\partial X}{\partial t} = D\left(\frac{2}{r}\frac{\partial X}{\partial r} + \frac{\partial^2 X}{\partial r^2}\right) \tag{2.1}$$

where X represents moisture content, r is the radial position which is a function of time changes during soaking. D is the diffusion coefficient and is constant.

Eq.(2.1) is second order partial differential equation. Therefore, one initial condition and two boundary conditions are required for the solution. These boundary conditions are adopted for the center and the surface of the grain. Eq.(2.2) which shows the initial condition is uniform throughout the dry solid at time t=0,

$$X(r,0) = X_0, \quad \forall r \tag{2.2}$$

The boundary conditions are

$$\frac{\partial X}{\partial r} = 0 \quad at \quad r = 0, \quad t > 0 \tag{2.3}$$

$$X(R(t),t) = X_{eq}, \quad t > 0 \tag{2.4}$$

Eq.(2.3) defines symmetry of the problem in the center of the grain in any instant of time and Eq.(2.4) represents moisture content on the solid-fluid interface ($r = R(t)$) and it reaches equilibrium moisture content at the beginning of the soaking.

$$\frac{dR(t)}{dt} = \alpha\frac{\partial X}{\partial r}, \quad r = R(t) \tag{2.5}$$

$$R(0) = R_0 \tag{2.6}$$

Eq.(2.5) represents Stefan condition presenting the motion of the front. For soybean hydration model $\alpha = D\frac{\rho_{DS}}{\rho_{water}}$ is obtained by Nicolin et al. [**7**].

The boundary condition is defined by Eq.(2.3) causes an indetermination in the Eq.(2.1) since the Eq.(1) is not defined at the center of the grain due to term $(2/r)$ which becomes infinite when r approaches to zero. Therefore, L'Hospital rule was applied to Eq.(2.1) to obtain the solution for the center. Eq.(2.7) is the valid for the center of the grain [**7**].

$$\frac{\partial X}{\partial t} = 3D\frac{\partial^2 X}{\partial r^2} \tag{2.7}$$

3. The Compact Finite Difference Scheme

Compact finite difference schemes can be deal with two kind of categories. These are explicit compact finite differences which computes the numerical derivatives at each grid by using large stencils and implicit compact finite differences which evaluates the numerical derivatives through solving a system of linear equation and by using smaller stencil[**20, 21, 22**]. In this work,we used implicit compact finite differences for spatial discretization.

3.1. Spatial Discretization. Spatial derivatives are computed by the compact finite difference scheme [**23**]. For simplicity, a uniform 1D mesh consisting of N points: $r_1 < r_2 < ... < r_N$. The mesh size $\Delta r = r_{i+1} - r_i$ is equal at any instant of time. The first derivatives are for all interior points $\left(r_i, t^j\right)$, $2 \leqslant i \leqslant N-1$ is given by Eq. (8) [**20**].

$$\alpha X'(r_{i+1}, t^j) + X'(r_i, t^j) + \alpha X'(r_{i-1}, t^j) = b\frac{X(r_{i+2},t^j)-X(r_{i-2},t^j)}{4\Delta r^j} + a\frac{X(r_{i+1},t^j)-X(r_{i-1},t^j)}{2\Delta r^j} \tag{3.1}$$

which provides an α-family of fourth order tridiagonal schemes with

$$a = \frac{2}{3}(\alpha+2), \quad b = \frac{1}{3}(4\alpha-1) \tag{3.2}$$

For $\alpha = \frac{1}{4}$, it is obtained fourth order tridiagonal scheme

$$\frac{1}{4}X'_{i-1} + X_i + \frac{1}{4}X'_{i+1} = \frac{3}{4}\frac{X_{i+1} - X_{i-1}}{2\Delta r} \tag{3.3}$$

where, for simplicity, X_i was taken instead of $X(r_i, t^j)$.

The derivatives of the points near the boundaries for non-periodic problems is given by one-sided schemes. For the first derivatives at the boundary r_1, fourth order formulae can be given as

$$X'_i + 3X'_{i+1} = \frac{1}{\Delta r}\left(-\frac{17}{6}X_i + \frac{3}{2}X_{i+1} + \frac{3}{2}X_{i+2} - \frac{1}{6}X_{i+3}\right) \tag{3.4}$$

For the first derivatives at the boundary r_2, fourth order formulae can be given as

$$\frac{1}{4}X'_{i-1} + X'_i + \frac{1}{4}X_{i+1} = \frac{1}{\Delta r}\left(-\frac{3}{4}X_{i-1} + \frac{3}{4}X_{i+1}\right) \tag{3.5}$$

By symmetry, for the first derivatives at the boundaries r_{N-1} and r_N the fourth order formulas are similar to Eq.(3.5),(3.4),respectively. For r_{N-1} and r_N, fourth order formulas are given by Eq.(3.6)-(3.7).

$$\frac{1}{4}X'_{i-1} + X'_i + \frac{1}{4}X_{i+1} = \frac{1}{\Delta r}\left(\frac{3}{4}X_{i+1} - \frac{3}{4}X_{i-1}\right) \tag{3.6}$$

$$3X'_{i-1} + X'_i = \frac{1}{\Delta r}\left(\frac{17}{6}X_i - \frac{3}{2}X_{i-1} - \frac{3}{2}X_{i-2} + \frac{1}{6}X_{i-4}\right) \tag{3.7}$$

The scheme (10)-(14) can be written as

$$AX' = \frac{1}{\Delta r}BX \tag{3.8}$$

where $X = (X_1, X_2, ..., X_N)^T$.The second order derivative terms are obtained by applying the first operator twice,

$$AX'' = \frac{1}{(\Delta r)^2}BX' \tag{3.9}$$

where,

$$A = \begin{bmatrix} 1 & 3 & & & & & \\ \frac{1}{4} & 1 & \frac{1}{4} & & & & \\ & \frac{1}{4} & 1 & \frac{1}{4} & & & \\ & \ddots & \ddots & \ddots & & & \\ & & & \frac{1}{4} & 1 & \frac{1}{4} & \\ & & & & \frac{1}{4} & 1 & \frac{1}{4} \\ & & & & & 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{17}{6} & \frac{3}{2} & \frac{3}{2} & -\frac{1}{6} & & & \\ \frac{3}{4} & 0 & \frac{3}{4} & & & & \\ & \frac{3}{4} & 0 & \frac{3}{4} & & & \\ & \ddots & \ddots & \ddots & & & \\ & & & \frac{3}{4} & 0 & \frac{3}{4} & \\ & & & & \frac{3}{4} & 0 & \frac{3}{4} \\ & & & & & 3 & 1 \end{bmatrix}$$

First and second derivatives at all grid points are obtained by solving the tridiagonal linear system.

3.2. Application of Soybean Hydration Model. In numerical solution of the problem (1)-(6), three-point fourth order compact finite difference scheme and explicit finite difference scheme are used for derivatives of radial coordinate where the interval $[0, R(t)]$. Time derivatives are dicretized by forward finite difference scheme.

4. Numerical Solution

To express the volume variation Nicolin et al.[**7**] used Variable Space Grid method(VSGM) which was proposed Murray and Landis [**24**] in Eq.(2.1). In this method, the number of space intervals between a fixed boundary $x = 0$ and moving boundary $x = R(t)$ is kept constant and equal to N. Thus, the moving boundary lies on the Nth grid. The model analyzes the time partial derivation by tracking a given line instead of a constant r. For the line ith grid point, we have

$$\frac{\partial X}{\partial t}|_i = \frac{\partial X}{\partial r}|_t \frac{dr}{dt} + \frac{\partial X}{\partial t}|_t \tag{4.1}$$

General grid point at r moved according to Eq.(18). The authors who used the method studied on Cartesian coordinates [**12, 25**]. Eq.(2.1) is in spherical coordinate but since the diffusion takes place only radial direction, mass transfer is similar to in Cartesian coordinates. Therefore Eq.(4.2) can be used for this model.

$$\frac{dr_i}{dt} = \frac{r_i}{R(t)} \frac{dR(t)}{dt} \tag{4.2}$$

Substituting the Eq.(4.1) and Eq.(4.2) into Eq.(2.1), we get

$$\frac{\partial X}{\partial t} = \frac{r_i}{R(t)} \frac{dR(t)}{dt} \frac{\partial X}{\partial r} + D\left(\frac{2}{r}\frac{\partial X}{\partial r} + \frac{\partial^2 X}{\partial r^2}\right) \tag{4.3}$$

To solve the model the radial coordinate was divided into N points ($i = 1, 2, .., N$). The number of time intervals is determined by amount of absorbation water. When the whole grain reaches %99 of the equilibrium moisture content, the process is cut off.

Discretization of Eq. (2.2) is given by Eq. (20),

$$X_i^1 = X_0, \quad i = 1, 2, ..., N \tag{4.4}$$

To discretization of the boundary condition Eq.(2.3) for the center of the grain, we used explicit finite difference approximation.

$$X_2^j = X_0^j \tag{4.5}$$

We used central finite difference approximation in Eq.(7) which represents moisture content at the central of the grain ($r = 0$).

$$X_1^{j+1} = X_1^j + \frac{6D\Delta t}{(\Delta r^j)^2}\left(X_2^j - 2X_1^j + X_0^j\right) \tag{4.6}$$

To eliminate the term X_0^j we used Eq.(4.5) and obtain Eq.(4.7)

$$X_1^{j+1} = X_1^j + \frac{6D\Delta t}{(\Delta r^j)^2}(X_2^j - X_1^j) \tag{4.7}$$

Discretization of the moisture diffusion equation for internal nodes ($2 \leqslant i \leqslant N$) is given by Eq.(3.3) with the position of each grid point defined by $r_i^j = (i - 1)\Delta r^j$. First and second derivatives $(X_r)_i^j$ and $(X_{rr})_i^j$ in Eq.(4.8),respectively, are constructed row-by-row by using fourth order compact finite difference scheme explained above, i.e.,$(X_r)_i^j = \frac{1}{\Delta r^j} A^{-1} B X_i^j$

$$X_i^{j+1} = X_i^j + \left(\frac{\Delta t r_i^j}{R^j \Delta r^j} + \frac{2D\Delta t}{r_i^j} \right) X_{ri}^j + \frac{D\Delta t}{(\Delta r^j)^2} X_{rri}^j \tag{4.8}$$

Since radial coordinate increases in each time step due to soaking, the mesh size is defined by $\Delta r^j = R^j/N$.

The term v^j which appears in Eq.(4.8) represents motion of the boundary, is radius. The velocity of motion of the radius is represented by Eq.(2.5) and discretization of it is given by Eq.(4.9).

$$v^j = \left(\frac{dR}{dt} \right)^j = \frac{\rho_{DS}}{\rho_{water}} D X_{rri}^j \tag{4.9}$$

The boundary condition on the spherical grain surface is presented by Eq.(4.10).

$$X_N^j = X_{eq} \tag{4.10}$$

The position of radius at the next time step is calculated by the following approximation.

$$R^{j+1} = R^j + \Delta t v^j$$

5. Results

We performed the computations using the software MATLAB R2012a on ASUS machine Intel Core i7 2.4 GHz 6 GB memory. The computational domain for space is considered $0 \leqslant r \leqslant R(t)$, is evaluated by over time and different numbers of uniform mesh point are used for numerical calculations. Constant in model $D = 3.277.10^{-11}(m^2/s)$, $\rho_{DS} = 1.057(kg_{DS}/m^3)$, $\rho_{water} = 1.000(kg_{water}/m^3)$ at temperature of 10^0C is taken as [**7, 26**]. $X_{eq} = 1.651(kg_{water}/kg_{DS})$, $X_0 = 0.126(kg_{water}/kg_{DS})$ and $R_0 = 0.003$ m [**7**].

In explicit method stability criterion

$$k = \frac{D\Delta t}{(\Delta r(t))^2} < \frac{1}{2} \tag{5.1}$$

is valid for all instant of time when $\Delta t = 1$ [**27**].

Explicit finite difference solutions and CFD4 solutions are listed in Table 1 for different N values. Table 2 shows the position of the radius and its velocity for explicit finite different method and CFD4 method at different time values and different N values.Equilibrium times (t_{eq}) and CPU times at different N values for both method are given at Table 3.

Absolute error and relative error which is listed in Table 4,5 and 6, is calculated as in below, respectively,

$$\epsilon = \mid u_i^{j+1} - u_i^j \mid \tag{5.2}$$

$$\eta = \frac{\epsilon}{u_i^j} \tag{5.3}$$

TABLE 1. Comparison of results at different radius and times for different N values

		$N=60$		$N=80$		$N=100$	
r(m)	*t(s)*	Nicolin, et al.[**7**]	CFD4	Nicolin,et al.[**7**]	CFD4	Nicolin, et al. [**7**]	CFD4
0.001	27	0.126000	0.125999	0.126000	0.126000	0.126000	0.126000
	4548	0.126978	0.126867	0.126904	0.126844	0.126868	0.126830
	19113	0.385871	0.385696	0.376661	0.376682	0.380595	0.380697
	45511	0.966678	0.967705	0.963192	0.963987	0.961144	0.961798
	91026	1.401123	1.402032	1.401134	1.401790	1.401129	1.401645
	t_{eq}	1.636197	1.636197	1.636183	1.636183	1.636047	1.636047
0.002	27	0.126000	0.1260000	0.126000	0.126000	0.126000	0.126000
	4548	0.227550	0.225594	0.228806	0.227693	0.229668	0.228959
	19113	0.789807	0.789924	0.785076	0.785273	0.782290	0.782502
	45511	1.199005	1.199710	1.192376	1.192923	1.188438	1.188889
	91026	1.473970	1.474602	1.474928	1.475382	1.475491	1.475847
	t_{eq}	1.639979	1.640330	1.640112	1.640112	1.640191	1.640191
0.003	27	0.778263	0.754335	0.993099	0.9746491	1.133495	1.109210
	4548	1.247416	1.246271	1.248858	1.248221	1.249776	1.249379
	19113	1.363186	1.363268	1.347651	1.347755	1.358715	1.358816
	45511	1.467596	1.467877	1.472715	1.472923	1.465850	1.466028
	91026	1.569826	1.570110	1.570300	1.570502	1.570578	1.570737
	t_{eq}	1.645592	1.645592	1.645430	1.645429	1.645558	1.645558

TABLE 2. Comparison of position of radius and velocity of radius at different times for different N values

		Position of Radius		Velocity of Radius	
N	*t(s)*	Nicolin, et al.[**7**]	CFD4	Nicolin, et al. [**7**]	CFD4
60	27	0.003027	0.003032	7.188910e-07	6.581855e-07
	4548	0.003341	0.003337	3.347523e-08	3.352256e-08
	19113	0.003626	0.003623	1.283695e-08	1.284040e-08
	45511	0.003854	0.003851	5.888267e-09	5.885257e-09
	91026	0.004023	0.004019	2.206632e-09	2.202524e-09
	t_{eq}	0.004129	0.004124	1.377649e-10	1.378431e-10
80	27	0.003029	0.003031	6.479504e-07	4.585342e-07
	4548	0.003340	0.003337	3.347523e-08	3.352446e-08
	19113	0.003626	0.003623	1.283695e-08	1.284037e-08
	45511	0.003854	0.003851	5.888267e-09	5.885222e-09
	91026	0.004023	0.004019	2.206632e-09	2.202490e-09
	t_{eq}	0.004129	0.004124	1.377649e-10	1.378418e-10
100	27	0.003029	0.003029	5.882738e-07	4.082022e-07
	4548	0.003339	0.003337	3.349053e-08	3.352564e-08
	19113	0.003625	0.003623	1.283846e-08	1.284039e-08
	45511	0.003853	0.003851	5.887775e-09	5.885213e-09
	91026	0.004021	0.004019	2.205708e-09	2.202477e-09
	t_{eq}	0.004127	0.004124	1.377806e-10	1.378441e-10

Table 3. Equilibrium times and CPU times for different N values

	Nicolin,et al.[7]		CFD4	
N	teq	CPU	teq	CPU
60	234662	293.124552	234148	96.316208
80	234507	329.169541	234146	120.966301
100	234423	368.56395	234144	125.140448

Table 4. Absolute and Relative Error at different time for $N = 60$

		Nicolin, et al.[7]		CDF4	
$r(m)$	$t(s)$	Absolute Error	Relative Error	Absolute Error	Relative Error
0.001	27	0.0	0.0	1.387779e-16	1.101412e-15
	4548	1.513467e-06	1.191930e-05	1.403080e-06	1.113555e-05
	19113	2.723904e-05	7.059605e-05	2.731884e-05	7.083498e-05
	45511	1.642773e-05	1.699429e-05	1.644101e-05	1.698997e-05
	91026	5.297395e-06	3.780835e-06	5.288743e-06	3.772211e-06
	t_{eq}	2.819543e-07	1.723229e-07	2.826316e-07	1.727369e-07
0.002	27	0.0	0.0	1.882302e-10	1.493890e-09
	4548	5.433395e-05	2.388354e-04	5.438817e-05	2.411463e-04
	19113	2.828561e-05	3.581458e-05	2.834531e-05	3.588488e-05
	45511	1.098746e-05	9.163900e-05	1.099167e-05	9.162031e-06
	91026	3.696163e-06	2.507631e-06	3.690468e-06	2.502694e-06
	t_{eq}	2.029807e-07	1.237414e-07	2.101808e-07	1.281607e-07
0.003	27	0.015013	0.019669	0.015990	0.021657
	4548	5.250877e-05	4.209578e-05	5.296802e-05	4.250302e-05
	19113	1.111918e-05	8.156831e-06	1.113617e-05	8.168800e-06
	45511	4.377335e-06	2.982666e-06	4.378097e-06	2.982613e-06
	91026	1.650910e-06	1.051652e-06	1.648666e-06	1.050035e-06
	t_{eq}	1.030740e-07	6.263720e-08	1.029289e-07	6.254821e-08

Fig.1 shows moisture content profiles as a function of radial position,is calculated by CFD4 scheme, for different N values at various times (right) and comparison of CFD4 and explicit finite difference scheme (left). It is seen that explicit finite difference solution and CFD4 solutions are in good agrement.

Fig.2 shows moisture content profiles with respect to time. In Fig.3, the increase of size of the grain calculated by CFD4 is shown(right) and it is compared with explicit finite difference solution (left). Nicolin, et al. [7] demonstrated numerically R_{max} has $\approx$ %37.4 increasing and experimentally $\approx$ %40.6 increasing. In this work, the increase in radius of size is calculated $\approx$ %37.46. Hence, the result is obtained by CFD4 approximates the result of Nicolin et. al [7] very well.

Table 5. Absolute and Relative Error at different time for $N = 80$

			Nicolin, et al.[7]		CFD4
r(m)	*t(s)*	Absolute Error	Relative Error	Absolute Error	Relative Error
0.001	27	0.0	0.0	0.0	0.0
	4548	1.432193e-05	1.128576e-05	1.371378e-06	1.081166e-05
	19113	2.699398e-05	7.167155e-05	2.704816e-05	1.081166e-05
	45511	1.651780e-05	1.714931e-05	1.652489e-05	1.714253e-05
	91026	5.300768e-06	3.783212e-06	5.294189e-06	3.776748e-06
	t_{eq}	2.824334e-07	1.726172e-07	2.828976e-07	1.729009e-07
0.002	27	0.0	0.0	6.222828e-12	4.938752e-11
	4548	5.502261e-05	2.405351e-04	5.507663e-05	2.419488e-04
	19113	2.838508e-05	3.615713e-05	2.842108	3.619390e-05
	45511	1.115482e-05	9.355210e-06	1.115643e-05	9.352270e-06
	91026	3.677841e-06	2.493579e-06	3.673519e-06	2.489882e-06
	t_{eq}	2.072986e-07	1.263930e-07	2.076437e-07	1.266033e-07
0.003	27	0.013695	0.013983	0.013584	0.014134
	4548	5.244936e-05	4.199963e-05	5.272227e-05	4.223969e-05
	19113	1.172348e-05	8.699271e-06	1.173304e-05	8.705694e-06
	45511	4.253775e-06	2.888399e-06	4.253762e-06	2.887980e-06
	91026	1.642150e-06	1.045757e-06	1.640427e-06	1.044525e-06
	t_{eq}	1.004790e-07	6.105501e-08	1.006464e-07	6.115677e-08

Table 6. Absolute and Relative Error at different time for $N = 100$

		Nicolin, et al.[7]		CFD4	
r(m)	*t(s)*	Absolute Error	Relative Error	Absolute Error	Relative Error
0.001	27	0.0	0.0	0.0	0.0
	4548	1.390885e-06	1.096337e-05	1.352781e-06	1.066619e-05
	19113	2.713013e-05	7.128857e-05	2.717108e-05	7.137700e-05
	45511	1.656995e-05	1.724013e-05	1.657417e-05	1.723278e-05
	91026	5.302804e-06	3.784680e-06	5.297441e-06	3.779458e-06
	t_{eq}	2.851582e-07	1.742971e-07	2.855206e-07	1.745186e-07
0.002	27	0.0	0.0	2.013140e-12	1.597730e-11
	4548	5.543744e-05	2.414386e-04	5.548694e-05	2.424036e-04
	19113	2.844106e-05	3.635749e-05	2.846550e-05	3.637864e-05
	45511	1.125378e-05	9.469479e-06	1.125424e-05	9.466270e-06
	91026	3.666935e-06	2.485237e-06	3.663415e-06	2.482251e-06
	t_{eq}	2.058661e-07	1.255135e-07	2.061279e-07	1.256731e-07
0.003	27	0.010978	0.009780	0.010615	0.009662
	4548	5.239481e-05	4.192512e-05	5.257732e-05	4.208453e-05
	19113	1.130235e-05	8.318473e-06	1.130781e-05	8.321875e-06
	45511	4.424064e-06	3.018096e-06	4.423744e-06	3.017512e-06
	91026	1.636938e-06	1.042253e-06	1.635525e-06	1.041248e-06
	t_{eq}	9.916444e-08	6.025364e-08	9.929071e-08	6.033036e-08

FIGURE 1. Solution of the model by CFD4 for different N values at various time and explicit finite difference scheme and CFD4 solutions at $N = 60$ at various times

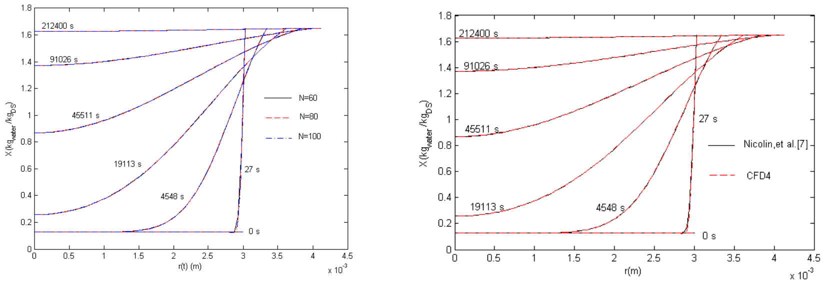


FIGURE 2. Solutions of the model as a function of time by CFD4 for different N values and explicit finite difference scheme and CFD4 solutions at $N = 60$ at various radial positions

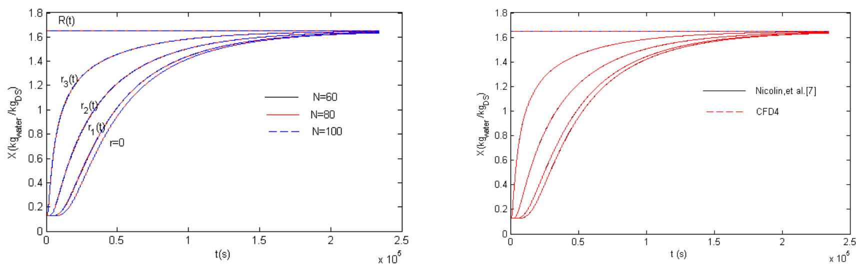
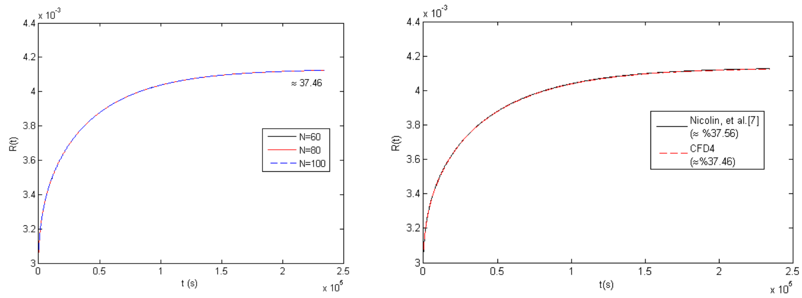


FIGURE 3. Grain radius as a function of time is calculated by CFD4 for different N values (right) and the radius sizes for both of method (left)



6. Conclusion

In this work, CDF4 scheme was applied to soybean hydration model as considered Stefan Problem succesfully. Numerical results are compared Nicolin,et al. [**7**] results and demonstrated that CFD4 solutions are in good agree with Nicolin, et al. [**7**]. CDF4 reaches equilibrium time faster than explicit solutions with high accuracy and CDF4 method has minimal computational effort.

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