International Journal of Academic Research in Accounting, Finance and Management Sciences
Vol. 4, No.1, January 2014, pp. 397–408
E-ISSN: 2225-8329, P-ISSN: 2308-0337
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A Model and its Solution Method for a Two-Item Newsvendor Supply Chain with Return Policy and Demand Leakage

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Abstract *We study a one-manufacturer and one-retailer type of supply chain that the manufacturer manufactures two newsvendor-type items and offers a buy-back contractual commitment to the retailer who sells the items in a stochastic demand market with various prices, allowing demand leakage from high-priced item to low-priced one. The objective of this study is to coordinate the chain by jointly determining wholesale prices, buy-back prices, retail prices and order sizes. We first derive a succinct model for the chain in which the manufacturers expected profit subject to the retailer's optimal expected profit will be explored. And a solution method to the case of uniformly distributed error demand is subsequently proposed; accordingly, a series of examples along with graphical concavity and satisfying constraint of the manufacturer's expected profit are conducted to validate our solution method.*

Key words Supply chain; Returns policy; Demand leakage; Inventory

DOI: 10.6007/IJARAFMS/v4-i1/697 URL: http://dx.doi.org/10.6007/IJARAFMS/v4-i1/697

1. Introduction

Recently, a Taipei-based Smartphone company, HTC, promoted a series of distinct classes of Smartphone's in the European market, aiming at drawing more different consumer groups to enlarge market shares. In that case, his downstream retailer will face a problem on how to set retail prices in response to the marketing strategy. If the high-class phones are overpriced, some of potential buyers, who originally intend to purchase the phones, might choose the low-class but relatively cheap ones instead; contrarily, those who initially prefer the inexpensive low-class phones could opt to purchase the expensive high-class ones due to the insignificant price gap if the high-class phones are underpriced. Of these two situations the former damages the retailer's profit because of demand leakage from high-priced phones to low-priced phones who possess meager profit by comparison; and the latter also impairs his profit because of the high-priced phone's diminishing profit margin. This study therefore tackles this problem from the viewpoint of supply chain that a manufacturer manufactures two newsvendor-type items and offers a buy-back commitment to a retailer, who helps sell the items in stochastic demand environment with various prices allowing demand leakage from high-priced item to low-priced one.

A number of extant publications in the literature have termed revenue management as one of the most pressing topic in the field of management science and operation research since large revenue generally yields large profit (Bell, 1998); and a commonly used strategy to increase revenue is to differentiate a single market into multiple sub-market segments through various prices mainly enticing more buyers, which accounts for why many firms usually offer discount prices for earlier purchases and online purchases alike. Gerchak *et al.* (1985) primarily dealt with the relevant problem whether a limited supply of bagels should be sold as a single item with higher price or as part of other combination with lower price; and later, consumers are partitioned into two sub-groups by Pfeifer (1989) with price-sensitivity and price-insensitivity to investigate airline discount fare allocation problem. Shi and Chen (2007) analyzed Pareto-optimal contracts for a newsvendor supply chain with satisfying objectives. Lately, Zhang and Bell (2007) and Zhang *et al.* (2010) initiated a model of two demand classes associated with demand dependency allowing demand leakage across the segments, from where the result of total expected profit of two variously priced sub-market segments always outstrips that from a single market segment is identified. Wang *et al.* (2013) also discussed this two demand class's model. They developed a critical level rationing policy in which a threshold mechanism is adopted to allocate backorders when multiple outstanding orders exist.

A supply chain is called to be coordinated if it reaches a possible maximal channel profit as a whole by virtue of contractual terms negotiating among the chain members. And a traditional price-only contract is a trade whereby manufacturer does not offer any incentives to retailers; wholesale prices are the only decision variables between them (Lariviere and Porteus, 2001). A return policy contract, however, is a commitment provided by upstream member to accept unsold stock of downstream members at the end of selling season; in practice, it actually helps ease nervousness on being overstocked, especially in a market full of demand uncertainty. Unlike the Larivieve (1999) who reported that the priced-only contract fails to coordinate supply chain, a benchmark paper by Pasternack (1985) expressed that channel coordination could be accessible by means of the return policy implementation. Thus two existing types of return policy are extensive discussed: the first is a complete return policy that promises to refund a buy-back price (smaller than wholesale price to avoid arbitrage opportunity) for each returned unit; the second is a partial return policy that reimburses retailer wholesale price only for part of unsold stock, usually a certain percentage of order size also known as the quantity flexibility (Tsay, 1999). Based on the policies, Lariviere (1999) accordingly derived a mathematical equivalence between the two return policies under stochastic demand environment. Also, Bose and Anand (2007) handled the pertinent issue by first adopting the partial return policy and then extending to the complete return policy with same approach in a framework of single-period problem.

In competitive newsvendor environment, ordering decisions are assumed to be set simultaneously- there is no priority or time sequence among competitors; each competes with others by satisfying a market with substitutable products. Parlar (1988) verified the existence of unique Nash solution for two vendors in this situation; Netessine and Rudi (2003) sequentially generalized Parlar's result to the case of any number of vendors. In contrast with the competitive environment, a Stackelberg game includes a leader and other followers; the leader first sets contractual terms and then makes optimal decisions after knowing the followers' responses, aiming to earn higher profit than that in competitive game. Bose and Anand (2007) coped with the game focusing on one manufacturer and one vendor supply chain with exogenously fixed wholesale price, where the manufacturer maximizes his expected profit subject to participation constraint of the retailer's optimal profit; and with aid of numerical examples, channel coordination of the constrained Nash equilibrium is claimed. Later, Yao *et al.* (2008) conducted a string of examples regarding the game in light of the price-only and the return policy contracts, along with a conclusion that the return policy contract actually improves channel profit; in the case of high demand variability, they further suggested that manufacturer is supposed to split some profit to continue the game, a similar result appeared in Lau and Lau (1999), Lau *et al.* (2000) and Tsay (2001). Besides that, Serin (2007) stressed on two-vendor problem with initial and reallocated demand considerations, core value of which is that, under certain condition on profit function, problems of two-player type (Nash game) and leader-follower type (Stackelberg game) share common optimal solutions in inventory management. Unlike the papers above, Serel (2008) studied a single-period problem assuming that retailer could place his order from a reliable supplier, a risky supplier or the both, in which the retailer is making optimal ordering decision on how to divide his order between the reliable but high-cost supplier and the unreliable but low-cost supplier.

In this study we zero in on a 1-leader-and-1-follower type of newsvendor supply chain that the manufacturer, who is a leader offering a complete return policy, maximizes his expected profit by determining wholesale prices and buy-back prices under the constraint of the retailer's optimal expected profit, who is a follower determining optimal retail prices and optimal order sizes in a stochastic demand market. The contributions of our study are twofold. Firstly, we successfully derive a succinct model for the two-item supply chain that is rather complicated since demand leakage and buy-back policy are taken into account altogether. Secondly, due to interdependence of the two leaking demands, widely used software packages are unable to directly solve the problem; we thus develop a solution method of iteration that will be proven valid by means of a string of examples, accompanied with graphical concavity and satisfying constraint of the manufacturer's expected profit.

The remainder of this study is organized as follows. Assumptions and notation are made in Section 2 incorporating with pertinent models formulation and corresponding theoretical analyses. In Section 3, an iterative solution procedure to optimize the case of uniform distribution of error demand is developed so as that many numerical examples are conducted to show the validity of our solution method. Finally, remarks on the study and possible directions for further research are presented in Section 4 to complete the study.

2. The model

The problem of this study is defined as follows. A manufacturer manufactures two newsvendor-type items and offers a complete returns policy contract to a retailer who sells the items in a stochastic demand market. For item *i* , *i*=1,2, wholesale price, buy-back price, retail price and order size are w_i , b_i , p_i and Q_i , respectively; unit production cost c_i and unit shortage cost s_i for unsatisfied demand are also incurred. Following Zhang and Bell (2007), the deterministic parts of the items' demands allowing leakage from high-priced item to low-priced one are defined by

$$\begin{aligned} D_1 &= \alpha_1 - \beta_1 p_1 - \gamma(p_1 - p_2) \\ D_2 &= \alpha_2 - \beta_2 p_2 + \gamma(p_1 - p_2) \end{aligned} \tag{1}$$

Where, for *i*=1,2, α_i >0 is a primary demand; β_i >0 is a consumer's price-sensitivity; γ >0 is a leakage rate. Then, based on Mills (1959), the items' stochastic demands are given by

$$x_i = D_i + \varepsilon_i \quad i=1,2 \tag{2}$$

Where, for item *i*, ε_i is an error demand defined on a range $[A_i, B_i]$ with pdf $f_i(\varepsilon_i)$ and cdf $F_i(\varepsilon_i)$.

According to the previous assumptions, for *i*=1, 2, the retailer's profit from the item *i* is calculated by

$$\begin{cases} p_i x_i - w_i Q_i + b_i(Q_i - x_i) & x_i \le Q_i \\ (p_i - w_i)Q_i - s_i(x_i - Q_i) & x_i > Q_i \end{cases} \tag{3}$$

Applying the scheme in Thowsen (1975) and Petruzzi and Dada (1999), we define $z_i = Q_i - D_i$ and substitute $x_i = D_i + \varepsilon_i$ into equation (3), then it becomes

$$\begin{cases} p_i(D_i + \varepsilon_i) - w_i(D_i + z_i) + b_i(z_i - \varepsilon_i) & \varepsilon_i \le z_i \\ (p_i - w_i)(D_i + z_i) - s_i(\varepsilon_i - z_i) & \varepsilon_i > z_i \end{cases} \tag{4}$$

Thus the retailer's total expected profit is

$$\begin{aligned} E[\pi_r] = \sum_{i=1}^{2} (\int_{A_i}^{z_i} (p_i(D_i + \varepsilon_i) - w_i(D_i + z_i) + b_i(z_i - \varepsilon_i)) f_i(\varepsilon_i) d\varepsilon_i \\ + \int_{z_i}^{B_i} ((p_i - w_i)(D_i + z_i) - s_i(\varepsilon_i - z_i)) f_i(\varepsilon_i) d\varepsilon_i) \end{aligned} \tag{5}$$

Now, let $\Lambda_i(z_i)=\int_{A_i}^{z_i}(z_i-\varepsilon_i)f_i(\varepsilon_i)d\varepsilon_i$, $\Theta_i(z_i)=\int_{z_i}^{B_i}(\varepsilon_i-z_i)f_i(\varepsilon_i)d\varepsilon_i$ and the mean $\mu_i=\int_{A_i}^{B_i}\varepsilon_i f(\varepsilon_i)d\varepsilon_i$, the retailer's expected profit can be re-written as follows.

$$E[\pi_r]=\sum_{i=1}^{2}(p_i-w_i)D_i+\sum_{i=1}^{2}(p_i-w_i)\mu_i-\sum_{i=1}^{2}(w_i-b_i)\Lambda_i(z_i)$$
$$-\sum_{i=1}^{2}(p_i-w_i+s_i)\Theta_i(z_i) \tag{6}$$

Notice that each term of $E[\pi_r]$ owns managerial meaning itself. The first term implies a profit coming from the deterministic part of the demand; the second term is an expected profit associated with the error demand; the third term is an expected cost due to overstock; and the last term means an expected shortage cost and an expected profit loss due to unsatisfied demand.

Theorem 1. The retailer's expected profit $E[\pi_r]$ is concave in p_i and z_i for *i*=1,2, as long as $\beta_i(p_i+s_i-b_i)f_i(z_i)>\frac{1}{2}$ holds.

Proof of the concavity of $E[\pi_r]$

$$E[\pi_r]=\sum_{i=1}^{2}(p_i-w_i)D_i+\sum_{i=1}^{2}(p_i-w_i)\mu_i-\sum_{i=1}^{2}(w_i-b_i)\Lambda_i(z_i)-\sum_{i=1}^{2}(p_i-w_i+s_i)\Theta_i(z_i)$$

For *i*=1,2 , *j*=3-*i*, we have

$$\Lambda_i^{'}(z_i)=F_i(z_i),\qquad \Theta_i^{'}=F_i(z_i)-1$$
$$\frac{\partial E[\pi_r]}{\partial z_i}=-(w_i-b_i)F_i(z_i)+(p_i-w_i+s_i)(1-F_i(z_i))$$
$$\frac{\partial E[\pi_r]}{\partial p_i}=-2(\beta_i+\gamma)p_i+2\gamma p_j+\alpha_i+(\beta_i+\gamma)w_i-\gamma w_j+\mu_i-\Theta_i(z_i)$$
$$\frac{\partial^2 E[\pi_r]}{\partial z_i^{\,2}}=-(p_i+s_i-b_i)f_i(z_i),\quad \frac{\partial^2 E[\pi_r]}{\partial p_i^{\,2}}=-2(\beta_i+\gamma)$$
$$\frac{\partial^2 E[\pi_r]}{\partial z_i\partial z_j}=0,\quad \frac{\partial^2 E[\pi_r]}{\partial p_i\partial z_i}=1-F_i(z_i),\quad \frac{\partial^2 E[\pi_r]}{\partial p_i\partial z_j}=0,\quad \frac{\partial^2 E[\pi_r]}{\partial p_i\partial p_j}=2\gamma$$

Then corresponding Hessian matrix is

$$H=\begin{pmatrix}-(p_1+s_1-b_1)f_1(z_1) & 0 & 1-F_1(z_1) & 0\\ 0 & -(p_2+s_2-b_2)f_2(z_2) & 0 & 1-F_2(z_2)\\ 1-F_1(z_1) & 0 & -2(\beta_1+\gamma) & 2\gamma\\ 0 & 1-F_2(z_2) & 2\gamma & -2(\beta_2+\gamma)\end{pmatrix}$$

The first principal minor of *H* is

$$|H_{11}|=-(p_1+s_1-b_1)f_1(z_1)<0$$

The second principal minor of *H* is

$$|H_{22}|=(p_1+s_1-b_1)(p_2+s_2-b_2)f_1(z_1)f_2(z_2)>0$$

The third principal minor of *H* is

$$|H_{33}|=(p_2+s_2-b_2)f_2(z_2)\left(-2(\beta_1+\gamma)(p_1+s_1-b_1)f_1(z_1)+(1-F_1(z_1))^2\right)$$

$$< (p_2+s_2-b_2)\,f_2(z_2)\left(-2\beta_1(p_1+s_1-b_1)\,f_1(z_1)+(1-F_1(z_1))^2\right)$$

<0 if $\beta_1(p_1+s_1-b_1)\,f_1(z_1)>\dfrac{1}{2}$ holds

The fourth principal minor of H is

$$|H_{44}|=\left(2(\beta_1+\gamma)(p_1+s_1-b_1)\,f_1(z_1)-(1-F_1(z_1))^2\right)\left(2(\beta_2+\gamma)(p_2+s_2-b_2)\,f_2(z_2)-(1-F_2(z_2))^2\right)$$
$$-4\gamma^2(p_1+s_1-b_1)(p_2+s_2-b_2)\,f_1(z_1)\,f_2(z_2)$$
$$>\left(2\beta_1(p_1+s_1-b_1)\,f_1(z_1)-(1-F_1(z_1))^2\right)\left(2\beta_2(p_2+s_2-b_2)\,f_2(z_2)-(1-F_2(z_2))^2\right)$$

>0 if $\beta_2(p_2+s_2-b_2)\,f_2(z_2)>\dfrac{1}{2}$ holds.

Combining with the above inequalities, we now complete the proof.

Since the concavity of $E[\pi_r]$ is identified, the optimal value z_i and p_i can be obtained by solving its first-order necessary conditions $\dfrac{\partial E[\pi_r]}{\partial z_i}=0$ and $\dfrac{\partial E[\pi_r]}{\partial p_i}=0$, i=1,2.

As to the manufacturer's expected profit, we have the following outcome.

Theorem 2. The manufacturer's expected profit is in the form of

$$E[\pi_m]=\sum_{i=1}^{2}(w_i-c_i)D_i+\sum_{i=1}^{2}(w_i-c_i)\mu_i+\sum_{i=1}^{2}(w_i-c_i-b_i)\Lambda_i(z_i)$$
$$-\sum_{i=1}^{2}(w_i-c_i)\Theta_i(z_i) \tag{7}$$

Derivation of the manufacturer's expected profit

The manufacturer's profit from the two items is

$$\pi_m=\begin{cases}(w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2)-b_1(z_1-\varepsilon_1)-b_2(z_2-\varepsilon_2) & \varepsilon_1\le z_1,\varepsilon_2\le z_2\\ (w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2)-b_2(z_2-\varepsilon_2) & \varepsilon_1> z_1,\varepsilon_2\le z_2\\ (w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2)-b_1(z_1-\varepsilon_1) & \varepsilon_1\le z_1,\varepsilon_2> z_2\\ (w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2) & \varepsilon_1> z_1,\varepsilon_2> z_2\end{cases}$$

Then the manufacturers expected profit is

$$E[\pi_m]=\int_{A_1}^{z_1}\int_{A_2}^{z_2}\left((w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2)-b_1(z_1-\varepsilon_1)-b_2(z_2-\varepsilon_2)\right)f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_1$$
$$+\int_{z_1}^{B_1}\int_{A_2}^{z_2}\left((w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2)-b_2(z_2-\varepsilon_2)\right)f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_1$$
$$+\int_{A_1}^{z_1}\int_{z_2}^{B_2}\left((w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2)-b_1(z_1-\varepsilon_1)\right)f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_1$$
$$+\int_{z_1}^{B_1}\int_{z_2}^{B_2}((w_1-c_1)(D_1+z_1)+(w_2-c_2)(D_2+z_2))\,f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_1$$
$$=\sum_{i=1}^{2}(w_i-c_i)D_i\;+$$
$$\int_{A_1}^{z_1}\int_{A_2}^{z_2}((\underbrace{w_1-c_1-b_1)(z_1-\varepsilon_1)}_{(1)}+\underbrace{(w_2-c_2-b_2)(z_2-\varepsilon_2)}_{(2)}+\underbrace{(w_1-c_1)\varepsilon_1}_{(5)}+(w_2-c_2)\varepsilon_2)\,f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_2$$
$$+\int_{z_1}^{B_1}\int_{A_2}^{z_2}(\underbrace{-(w_1-c_1)(\varepsilon_1-z_1)}_{(3)}+\underbrace{(w_2-c_2-b_2)(z_2-\varepsilon_2)}_{(2)}+\underbrace{(w_1-c_1)\varepsilon_1}_{(5)}+(w_2-c_2)\varepsilon_2)\,f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_2$$

$$+\int_{A_1}^{z_1}\int_{z_2}^{B_2}(\underset{(1)}{(w_1-c_1-b_1)(z_1-\varepsilon_1)}-\underset{(4)}{(w_2-c_2)(\varepsilon_2-z_2)}+\underset{(5)}{(w_1-c_1)\varepsilon_1}+(w_2-c_2)\varepsilon_2)\,f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_2$$

$$+\int_{z_1}^{B_1}\int_{z_2}^{B_2}(\underset{(3)}{-(w_1-c_1)(\varepsilon_1-z_1)}-\underset{(4)}{(w_2-c_2)(\varepsilon_2-z_2)}+\underset{(5)}{(w_1-c_1)\varepsilon_1}+(w_2-c_2)\varepsilon_2)\,f_2(\varepsilon_2)\,f_1(\varepsilon_1)d\varepsilon_2 d\varepsilon_2$$

$$=\sum_{i=1}^{2}(w_i-c_i)D_i+\underset{(5)}{\sum_{i=1}^{2}(w_i-c_i)\mu_i}+\underset{(1)}{(w_1-c_1-b_1)\Lambda_1(z_1)}+\underset{(2)}{(w_2-c_2-b_2)\Lambda_2(z_2)}$$

$$-\underset{(3)}{(w_1-c_1)\Theta_1(z_1)}-\underset{(4)}{(w_2-c_2)\Theta_2(z_2)}$$

$$=\sum_{i=1}^{2}(w_i-c_i)D_i+\sum_{i=1}^{2}(w_i-c_i)\mu_i+\sum_{i=1}^{2}(w_i-c_i-b_i)\Lambda_i(z_i)-\sum_{i=1}^{2}(w_i-c_i)\Theta_i(z_i)$$

That way we complete the proof.

Likewise, each term of $E[\pi_m]$ has its own managerial meaning. The first and the second terms have the same meanings as those in $E[\pi_r]$; the third term implies either an expected profit from returns if $w_i > c_i + b_i$ or an expected cost due to returns if $w_i < c_i + b_i$; and the last term is an expected profit loss due to the retailer's unsatisfied demand.

Ultimately, the manufacturer's objective is to determine w_i and b_i so as to maximize $E[\pi_m]$ subject to the constraint of $\frac{\partial E[\pi_r]}{\partial z_i}=0$ and $\frac{\partial E[\pi_r]}{\partial p_i}=0$, *i*=1,2.

We note that the proposed models are applicable to any type of distribution of error demand. But, because of difficulty in analyzing the problem, we plan to explore it with the assumption of uniformly distributed error demand, and then provide a simple solution procedure to conduct many examples, aiming at showing the validity of our solution method.

3. The numerical examples

As mentioned earlier, we assume that the error demand ε_i is uniformly distributed with $f_i(\varepsilon_i)=\frac{1}{\xi_i}$, where $\varepsilon_i \in [-\frac{\xi_i}{2},\frac{\xi_i}{2}]$ and ξ_i >0 for *i*=1,2. That way we have μ_i=0, and the constraint of $\frac{\partial E[\pi_r]}{\partial z_i}=0$ and $\frac{\partial E[\pi_r]}{\partial p_i}=0$ yields the following equations. For *i*=1,2

$$z_i = F_i^{-1}\left(\frac{p_i-w_i+s_i}{p_i-b_i+s_i}\right)=\frac{\xi_i(p_i-w_i+s_i)}{p_i-b_i+s_i}-\frac{\xi_i}{2} \tag{8}$$

$$Q_i = D_i+\frac{\xi_i(p_i-w_i+s_i)}{p_i-b_i+s_i}-\frac{\xi_i}{2} \tag{9}$$

$$\Lambda_i(z_i)=\frac{\xi_i(p_i-w_i+s_i)^2}{2(p_i-b_i+s_i)^2} \tag{10}$$

$$\Theta_i(z_i)=\frac{\xi_i(b_i-w_i)^2}{2(p_i-b_i+s_i)^2} \tag{11}$$

$$-2(\beta_i+\gamma)p_i+2\gamma p_j+\alpha_i+(\beta_i+\gamma)w_i-\gamma w_j-\Theta_i(z_i)=0 \quad j=3\text{-}i \tag{12}$$

Meanwhile, the retailer's and manufacturer's expected profits to the case of uniformly distributed error demand respectively become

$$E[\pi_r] = \sum_{i=1}^{2}(p_i - w_i)D_i - \sum_{i=1}^{2}(w_i - b_i)\Lambda_i(z_i) - \sum_{i=1}^{2}(p_i - w_i + s_i)\Theta_i(z_i) \tag{13}$$

$$E[\pi_m] = \sum_{i=1}^{2}(w_i - c_i)D_i + \sum_{i=1}^{2}(w_i - c_i - b_i)\Lambda_i(z_i) - \sum_{i=1}^{2}(w_i - c_i)\Theta_i(z_i) \tag{14}$$

Finally, the optimization problem turns into

$$\underset{w_i, b_i}{Maximize} \quad E[\pi_m]$$

Subject to the equation (8) ~ (12)

Still, its complexity prevents us from straightforwardly solving the problem via computer software packages, let alone an analytical closed-form solution. However, after a numerous trial-and-error, we find that the constraint of equation (8) ~ (12), which is a system of nonlinear equations in p_i , *i*=1,2, is a key factor to keep computer software from solving the problem directly; and this inspires us to develop a solution-finding method with iteration of p_i , mainly generating the linear equations (11) and (12) of p_i in the constraint. Experimentally, four repeats of the iteration will converge to values that are believed to be the optimal values of our optimization problem because not only the constraint of the equations (11) and (12) will be satisfied, but graphical concavity of $E[\pi_m]$ will also be illustrated in our subsequent examples.

Before proceeding to our numerical examples, four notifications are clarified as follows. First, in order to validate our solution method, a number of examples with various parameters values have been completed; but due to limited pages, only three examples will be offered; sensitivity analysis and managerial insights with respect to our supply chain are also omitted. Second, only the first example will be provided with the detailed solution procedure, while the other two's can also be acquired by substituting the expressions of p_1 and p_2 obtained in *Step* 1 into the first example's solution procedure if needed. Third, since $E[\pi_m]$ is a function of w_1, w_2, b_1, b_2, we therefore graphically illustrate its concavity as b_1, b_2 and w_1, w_2 are the optimal values, respectively; this is mainly to identify the obtained w_1, w_2, b_1, b_2 are indeed the optimal values that maximize the $E[\pi_m]$. Finally, equation (11) ~ (12) of the constraint are then examined by showing that differences between both sides of the expression of p_i , *i*=1,2, in *Step* 1 are nearly close to zero; this is mainly to identify that the obtained p_1 and p_2 really satisfy $\frac{\partial E[\pi_r]}{\partial p_i} = 0$,*i*=1,2, in the equation (12). Once the w_1, w_2, b_1, b_2, p_1 and p_2 are finalized, the optimal z_i, optimal Q_i ,*i*=1,2, and optimal $E[\pi_r]$ could be respectively gained by equation (8),(9) and (13).

Example 1. Parameters values: α_1=300, β_1=6, α_2=200, β_2=5, γ=5, c_1=5, c_2=3, s_1=3, s_2=2, $\varepsilon_1 = \varepsilon_2$=50

Solution procedure for Example 1:

Step 1 Solve the equation (12) to obtain $p_1 = \frac{1}{34}(800 + 17w_1 - 2\Theta_1(z_1) - \Theta_2(z_2))$ and $p_2 = \frac{1}{170}(3700 + 85w_2 - 5\Theta_1(z_1) - 11\Theta_2(z_2))$

Step 2 Let $p_1 = \frac{1}{34}(800 + 17w_1)$, $p_2 = \frac{1}{170}(3700 + 85w_2)$

Step 3 Find $\Lambda_i(z_i) = \dfrac{\xi_i(p_i - w_i + s_i)^2}{2(p_i - b_i + s_i)^2}$ and $\Theta_i(z_i) = \dfrac{\xi_i(b_i - w_i)^2}{2(p_i - b_i + s_i)^2}$, *i*=1,2

Step 4 Find $p_1 = \dfrac{1}{34}(800 + 17w_1 - 2\Theta_1(z_1) - \Theta_2(z_2))$ and

$$p_2 = \frac{1}{170}(3700 + 85w_2 - 5\Theta_1(z_1) - 11\Theta_2(z_2))$$

Step 5 Find $D_1 = \alpha_1 - \beta_1 p_1 - \gamma(p_1 - p_2)$ and $D_2 = \alpha_2 - \beta_2 p_2 + \gamma(p_1 - p_2)$

Step 6 Solve $\underset{w_i, b_i}{Maximize} \quad E[\pi_m]$ and obtain the corresponding w_i, b_i, *i*=1,2. If the absolute value of difference between two consecutive $E[\pi_m]$ is under a tolerant error, then stop.

Step 7 Repeat *Step* 2 ~ *Step* 4

Step 8 Clear w_i, b_i, *i*=1,2

Step 9 Goto *Step* 3

Then the optimal values are as follows: b_1=14.72, b_2=14.39, w_1=24.62, w_2=21.80, p_1=35.49, p_2=32.33, $E[\pi_m]$=2353.72, $E[\pi_r]$=1084.34, Q_1=75.45, Q_2=60.57

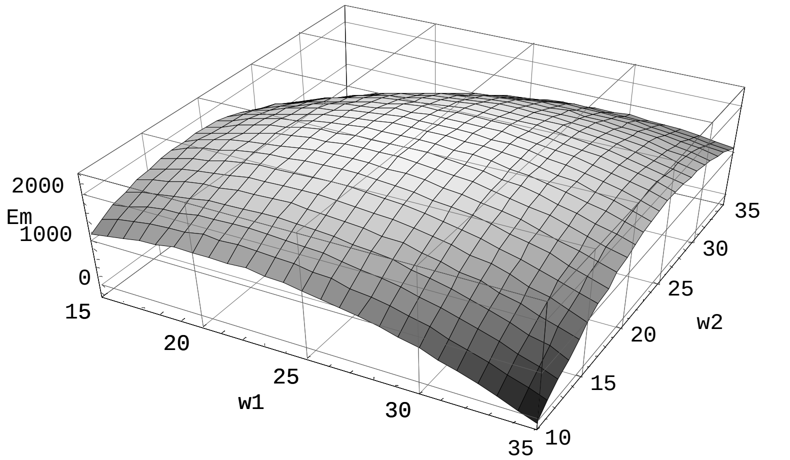

Figure 1.1. Surface graphics of $E[\pi_m(w_1, w_2 \mid b_1 = 14.72, b_2 = 14.39)]$

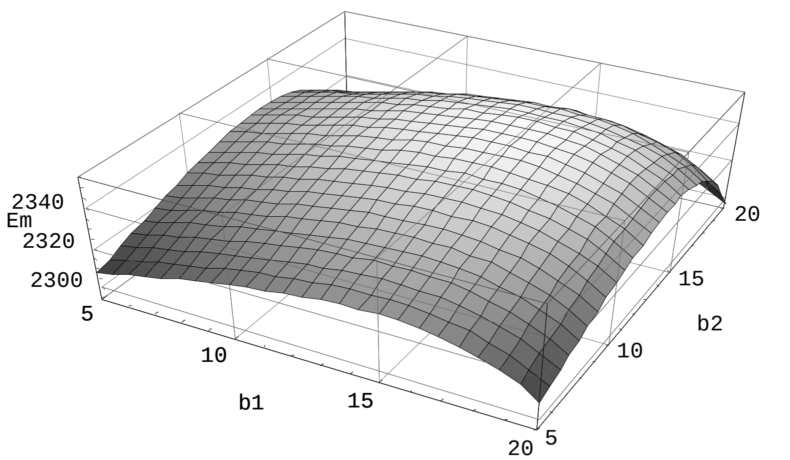

Figure 1.2. Surface graphics of $E[\pi_m(b_1, b_2 \mid w_1 = 24.62, w_2 = 21.80)]$

Check the constraint:

$$\Theta_1(z_1) = \frac{\xi_1(b_1 - w_1)^2}{2(p_1 - b_1 + s_1)^2} = 4.331$$

$$\Theta_2(z_2) = \frac{\xi_2(b_2 - w_2)^2}{2(p_2 - b_2 + s_2)^2} = 3.451$$

$$p_1 - \frac{1}{34}(800 + 17w_1 - 2\Theta_1(z_1) - \Theta_2(z_2)) = 0.007$$

$$p_2 - \frac{1}{170}(3700 + 85w_2 - 5\Theta_1(z_1) - 11\Theta_2(z_2)) = 0.016$$

Example 2. Parameters values: α_1 =1000, β_1=5, α_2 =1200, β_2 =8, γ =5, c_1 =20, c_2 =12, s_1 =10, s_2 =8, $\varepsilon_1 = \varepsilon_2$ =100

Step 1 Solve the equation (12) to obtain $p_1 = \frac{1}{210}(19000 + 105w_1 - 13\Theta_1(z_1) - 5\Theta_2(z_2))$

and $p_2 = \frac{1}{42}(3400 + 21w_2 - \Theta_1(z_1) - 2\Theta_2(z_2))$

Then the optimal values are as follows: b_1 =61.73, b_2 =57.56, w_1 =97.72, w_2 =84.73, p_1 =138.65, p_2 =122.79, $E[\pi_m]$ =38690.5, $E[\pi_r]$ =18703.7, Q_1 =236.04, Q_2 =309.88

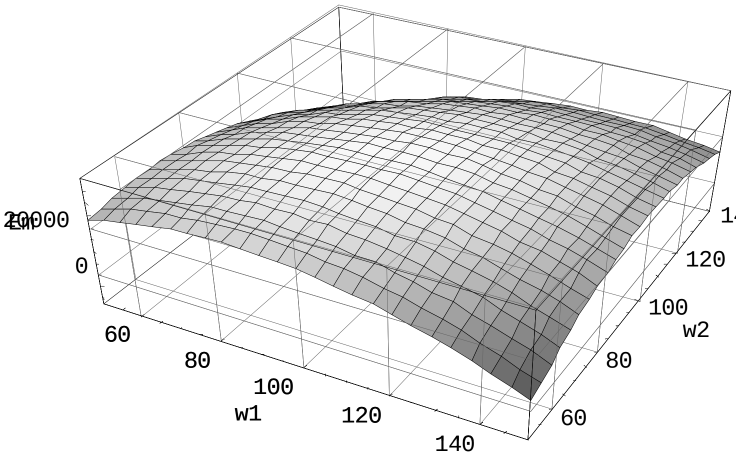

Figure 2.1. Surface graphics of $E[\pi_m(w_1, w_2 \mid b_1 = 61.73, b_2 = 57.56)]$

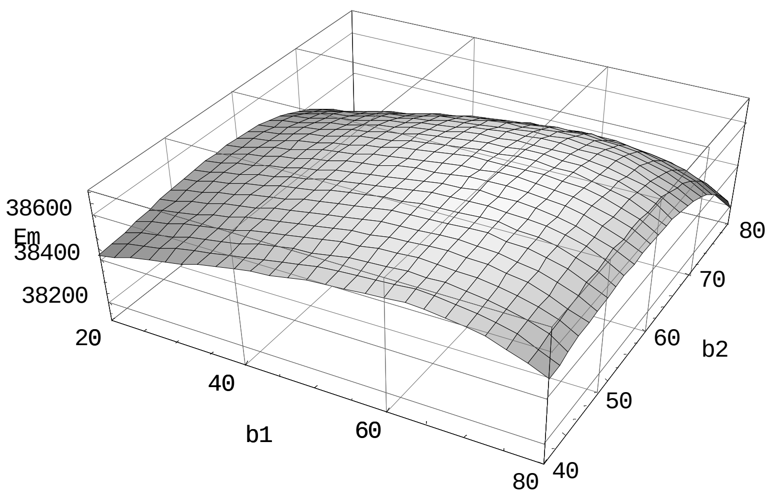

Figure 2.2. Surface graphics of $E[\pi_m(b_1, b_2 \mid w_1 = 97.72, w_2 = 84.73)]$

Check the constraint:

$$\Theta_1(z_1) = \frac{\xi_1(b_1 - w_1)^2}{2(p_1 - b_1 + s_1)^2} = 8.572$$

$$\Theta_2(z_2) = \frac{\xi_2(b_2 - w_2)^2}{2(p_2 - b_2 + s_2)^2} = 6.883$$

$$p_1 - \frac{1}{210}(19000 + 105w_1 - 13\Theta_1(z_1) - 5\Theta_2(z_2)) = 0.008$$

$$p_2 - \frac{1}{42}(3400 + 21w_2 - \Theta_1(z_1) - 2\Theta_2(z_2)) = 0.004$$

Example 3. Parameters values: α_1 =5000, β_1=15, α_2 =5500, β_2 =30, γ =5, c_1 =50, c_2 =30, s_1 =20, s_2 =10, $\varepsilon_1 = \varepsilon_2$ =500

Step 1 Solve the equation (12) to obtain $p_1 = \frac{1}{270}(40500 + 135w_1 - 7\Theta_1(z_1) - \Theta_2(z_2))$

and $p_2 = \frac{1}{270}(27000 + 135w_2 - \Theta_1(z_1) - 4\Theta_2(z_2))$

Then the optimal values are as follows: b_1 =94.18, b_2 =67.10, w_1 =169.66, w_2 =111.72, p_1 =233.21, p_2 =154.90, $E[\pi_m]$ =226520.0, $E[\pi_r]$ =108324.0, Q_1 =1123.01, Q_2 =1266.44

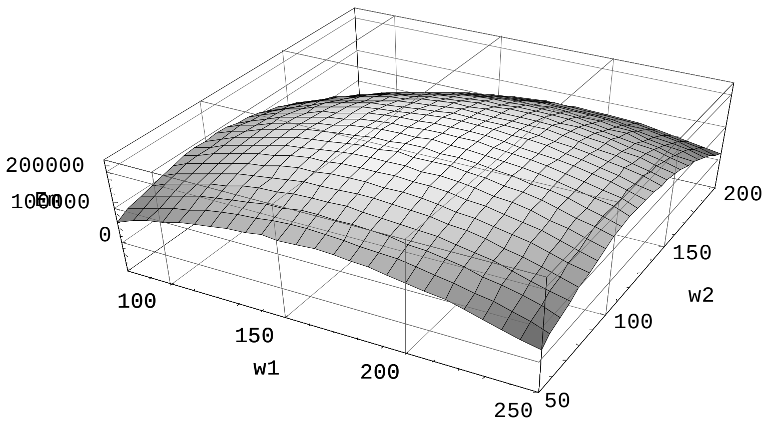

Figure 3.1. Surface graphics of $E[\pi_m(w_1, w_2 \mid b_1 = 94.18, b_2 = 67.10)]$

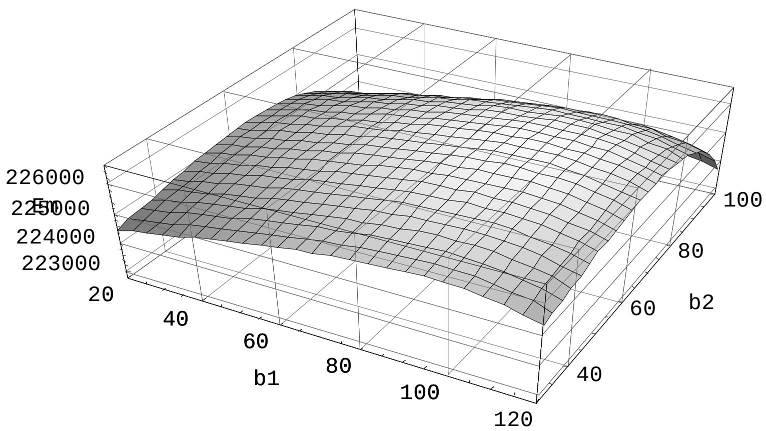

Figure 3.2. Surface graphics of $E[\pi_m(b_1, b_2 \mid w_1 = 169.66, w_2 = 111.72)]$

Check the constraint:

$$\Theta_1(z_1) = \frac{\xi_1(b_1 - w_1)^2}{2(p_1 - b_1 + s_1)^2} = 56.308$$

$$\Theta_2(z_2) = \frac{\xi_2(b_2 - w_2)^2}{2(p_2 - b_2 + s_2)^2} = 52.034$$

$$p_1 - \frac{1}{270}(40500 + 135w_1 - 7\Theta_1(z_1) - \Theta_2(z_2)) = 0.035$$

$$p_2 - \frac{1}{270}(27000 + 135w_2 - \Theta_1(z_1) - 4\Theta_2(z_2)) = 0.019$$

4. Conclusions

This study is devoted to a one-leader-and-one-follower type of supply chain with two newsvendor-type items that are variously priced for market share concerns, in which the demand is allowed to leak from high-priced item to low-priced one, and a complete return policy contract is offered by manufacturer to the purpose of coordinating the supply chain. Two contributions of the study are as follows. First, as shown in Appendix B, a two-item supply chain with return policy and demand leakage is far complex, but we successfully derived a pertinent and succinct model for the chain, and this could provide an exemplar for further researches. Second, due to the complex nonlinear constraint, we found that neither a theoretically analytical solution nor a computerized numerical solution of our problem is accessible; we therefore developed an efficient solution method and made every effort to prove its validity by conducting three examples in association with their graphical concavity illustrations and satisfying constraint, and this could be a reference direction for other related solution-finding approaches.

Yet, we are unable to theoretically support our solution method although it has been validated by a number of examples; so attempting to present a theory-supported interpretation in response to the solution method will be a top priority in our further researches. Also, sensitivity analysis and managerial insights in the frame of our chain are still necessary. Meanwhile, to modify our two-item model into a multi-item one and to extend our one-leader-and-one-follower type of supply chain into a one-leader-and-multi-follower one are two worthwhile issues for deeper explorations.

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