АВТОМАТИЧНІ ТА АВТОМАТИЗОВАНІ СИСТЕМИ УПРАВЛІННЯ ТЕХНОЛОГІЧНИМИ ПРОЦЕСАМИ

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TWO-DIMENSIONAL FLAME INSTABILITY AND CONTROL OF BURNING IN THE HAlF-OPEN FIRE-CHAMBER

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Abstract

The flame stability with regard to two-dimensional exponential perturbations both for the combustion in the half-open fire-chamber and the flame propagating in half-open channels is investigated. It is proved that only instability is possible for the combustion process. At the same time the one-dimensional flame instability is guaranteed near the front wall of the fire-chamber where the fuel supply is realized. Possibilities for the control of combustion in the half-open fire-chamber and diminishing of intensity of pulsations are discussed.

Keywords

Flame, combustion, vibratory combustion, instability, laminarity, turbulence, fire-chamber.

Introduction

One-dimensional flame instability is investigated analytically [1] for the closed fire-chamber. Such kind of instability does not lead to distortion of the flame front [1,2]. But the case of multidimensional (two-dimensional) flame instability is more interesting and more complicated from the mathematical point of view. Such instability causes cellular structure of flame and turbulent combustion [4]. Multidimensional flame instability can be also the reason of deflagration-to-detonation transition (DDT) [5]. Realization of one-dimensional instability is impossible under development of multidimensional (two-dimensional) perturbations in case of the instable flame front.

Aim

Aim of the research is to investigate two-dimensional stability of flames in half-open channels and chambers and to ground mathematically control of burning in fire-chambers.

Main body

Mathematical model. The following mathematical model of combustion is considered (Fig. 1). Along x-axis, at $x < 0$, the ideal inviscid gas moves at a stationary subsonic velocity u_1 (u_1 is much less than the sonic speed a_1; velocity u_1 equals the burning rate or the fuel supply rate). Plane $x = 0$ corresponds to the flame front. Planes $x = -L_1$ and $x = L_2$ correspond to front and back ends of chamber or channel accordingly. Plane $x = -L_1$ corresponds the hard wall (this wall can be regarded as the fire-chamber wall or as the closed end of channel). Plane $x = L_2$ is not a physical surface and corresponds the open end of chamber or channel. Planes $y = -d$ and $y = d$ correspond to hard walls (these walls can be regarded as the side walls of fire-chamber or as the walls of channel). Zone "1" $(-L_1 < x < 0,\ -d < y < d)$ is occupied by combustible gas mixture, whereas zone "2" $(0 < x < L_2,\ -d < y < d)$ is occupied by the combustion products. The combustion products are the homogeneous inviscid gas moving at a stationary subsonic velocity u_2 (u_2 is also much less than the sonic speed a_2 in products). It is obvious that $L = L_1 + L_2$, where L is the total chamber length, and $2d$ is the width (diameter) of the chamber or channel. It is assumed that all physical and chemical transformations occur in a moment on the flame front $x = 0$. This assumption is correct if the width of the flame zone is much less than the total chamber length L. It is not necessarily to mean by the flame only laminar flame with plane front because of the small physical width of this kind of flame (0,1…10 mm). It may be also laminar flame with slightly distorted front but with effective width (including its distortion) much less than L. The turbulent flame also satisfies this model if its fire zone with fuzzy front (which is by a lot of physical reasons much wider than the laminar flame zone) is much more narrow than the chamber extent L. Another assumption is incompressibility either of combustible mixture or of combustion products; this assumption is correct because of inequalities $u_1 \ll a_1,\ u_2 \ll a_2$ (in most cases the burning rate or the fuel supply rate u_1 ranges from 0.2 mm/s to 10m/s.

This model is fit either for the combustion in fire-chamber or for the flame propagating in channel (with the frame of reference connected to the flame front). In the last case the ends of tube or channel must also move in the frame of reference connected to the flame, but this movement is negligible regarding to comparison of the low velocity u_1 with velocities of small ("acoustic") perturbations mentioned below.

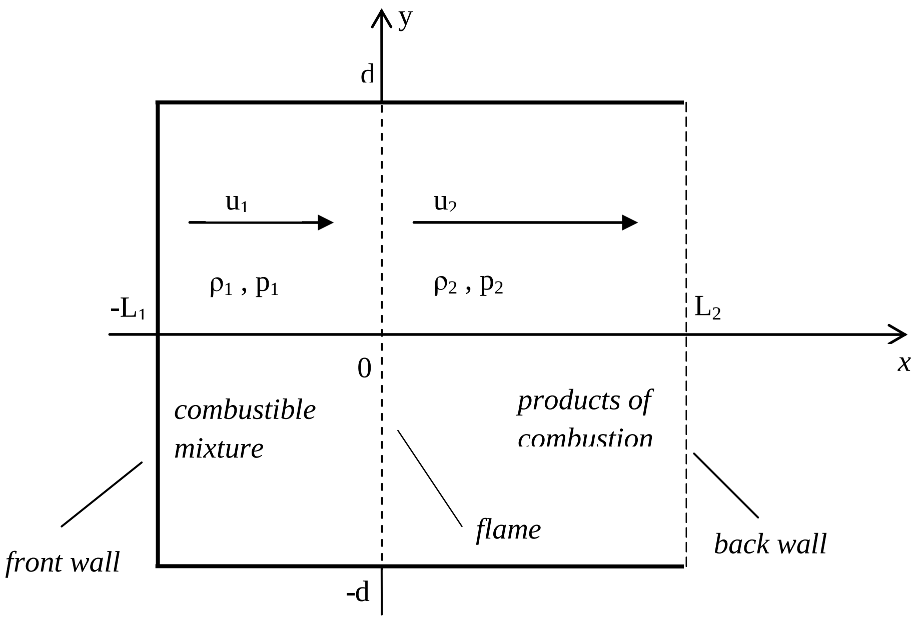

Fig. 1. Scheme of flame in half-open fire-chamber or channel

The parameters of combustible and products of combustion are related to each other by the conservation laws of mass, momentum and energy

$$\begin{cases} \rho_1 u_1 = \rho_2 u_2 \\ p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \end{cases}, \tag{1}$$

where ρ_j is density, p_j is pressure, $j = 1, 2$.

Fundamental equations and their linearization. The flow field is governed by a set of two-dimensional hydrodynamic equations

$$\begin{cases} \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \\ \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0 \\ \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \end{cases} , \tag{2}$$

where ρ is density, p is pressure, u_x, u_y are projections of the velocity vector on the coordinate axes, t is time.

Let us consider that the flame front obtains small (infinitesimal) displacement $\varepsilon(t) = A_{00} L \exp(ihy - i\omega t)$ as a result of accidental processes inside the flame zone. So equation of the disturbed flame front is

$$x = \varepsilon(y,t) = A_{00} L \exp(ihy - i\omega t) \ , \tag{3}$$

where

i is unit imaginary number ($i^2 = -1$),

A_{00} is indefinite constant,

ω is complex number (main component of eigen-value),

$h = \frac{2\pi}{\lambda}$ is wave number ($h > 0$),

λ is the wavelength of perturbation.

Such a choice for the form of perturbations is connected with possibility to present (by spatial coordinate y and by time coordinate t) every linearized perturbation as a Fourier series or a Fourier integral, that is to get this perturbation as superposition of the elementary waves of the exponential type $\exp(ihy - i\omega t)$.

Stationary flows of combustible gas (in zone "1") and products of combustion (in zone "2") are also disturbed, that is

$$u_x = u_j + u'_{jx}(x,y,t),\ u_y = u'_{jy}(x,y,t), p = p_j + p'_j(x,y,t), \tag{4}$$

where $u'_{jx}(x,y,t)$, $u'_{jy}(x,y,t)$, $p'_j(x,y,t)$ are small (infinitesimal) perturbations of the velocity vector projections and pressure accordingly.

Let us substitute expressions (4) into equations (2) and neglect all infinitesimals of second infinitesimal order (that is the essence of linearization). Set of linearized equations is

$$\begin{cases} \frac{\partial u'_{jx}}{\partial t} + u_j \frac{\partial u'_{jx}}{\partial x} + \frac{1}{\rho_j}\frac{\partial p'_j}{\partial x} = 0 \\ \frac{\partial u'_{jy}}{\partial t} + u_j \frac{\partial u'_{jy}}{\partial x} + \frac{1}{\rho_j}\frac{\partial p'_j}{\partial y} = 0 \\ \frac{\partial u'_{jx}}{\partial x} + \frac{\partial u'_{jy}}{\partial y} = 0 \end{cases} \tag{5}$$

Particular solutions of equations (5) are

$$\begin{cases} \dfrac{u'_{jx}}{u_1} = \left[-\dfrac{1}{\dfrac{z}{\delta_j}+1} A_{j1} e^{hx} + \dfrac{1}{\dfrac{z}{\delta_j}-1} A_{j2} e^{-hx} + A_{j3} e^{-\frac{z}{\delta_j}hx} \right] \exp(ihy - i\omega t) \\ \dfrac{u'_{jy}}{u_1} = \left[-\dfrac{i}{\dfrac{z}{\delta_j}+1} A_{j1} e^{hx} - \dfrac{i}{\dfrac{z}{\partial_j}-1} A_{j2} e^{-hx} - i\dfrac{z}{\delta_j} A_{j3} e^{-\frac{z}{\delta_j}hx} \right] \exp(ihy - i\omega t), \\ \dfrac{p'_j}{\rho_1 u_1^2} = \left[A_{j1} e^{hx} + A_{j2} e^{-hx} \right] \exp(ihy - i\omega t) \end{cases} \quad (6)$$

where

$$k_{j1} = 1,\ k_{j2} = -1,\ k_{j3} = -\frac{z}{\delta_j}, \quad (7)$$

$$\delta_j = \frac{\rho_1}{\rho_j} = \frac{u_j}{u_1} \ \ (\delta_1 = 1), \quad (8)$$

$$z = -\frac{i\omega L}{u_1}, \quad (9)$$

and A_{jl} $(j = 1,2;\ l = 1,2,3)$ are indefinite constants.

Thus particular solutions of equations (5) present themselves as superposition of the three types of perturbations corresponding to three indefinite constants A_{jl} $(l = 1,2,3)$. Perturbations "1" (corresponding to A_{j1} ,$k_{j1} = 1$) and "2" (corresponding to A_{j2} ,$k_{j2} = -1$) are so-called "acoustic"(more precisely, quasi-acoustic) perturbations. For such kind of perturbations $p'_j \neq 0$.

Perturbation "3" (corresponding to A_{j3} ,$k_{j3} = -\dfrac{z}{\delta_j}$) is in accordance with $p'_j = 0$ and is so-called vortex perturbation, because exactly for this perturbation the velocity vortex Ω_j is not equal zero:

$$\Omega_j = \frac{\partial u'_{jy}}{\partial x} - \frac{\partial u'_{jx}}{\partial y} \quad (10)$$

It is obvious from (10) that the vortex perturbation is carried together with medium (this result was predictable because of the well known Helmholtz theorem).

It is supposed that there are no vortices in zone "1" (combustible mixture), that is $\Omega_j = 0$ and $A_{13} = 0$. This supposition is justified either for the combustion in fire-chamber (if fuel supply is realized without intentional swirling) or for the flame propagating in channel (if combustible gas mixture was not previously disturbed). Taking this supposition into account perturbations in zone "1" are described by such equations

$$\begin{cases} \dfrac{u'_{1x}}{u_1} = \left[-\dfrac{1}{z+1} A_{11} e^{hx} + \dfrac{1}{z-1} A_{12} e^{-hx} \right] \exp(ihy - i\omega t) \\ \dfrac{u'_{1y}}{u_1} = \left[-\dfrac{i}{z+1} A_{11} e^{hx} - \dfrac{i}{z-1} A_{12} e^{-hx} \right] \exp(ihy - i\omega t), \\ \dfrac{p'_1}{\rho_1 u_1{}^2} = \left[A_{11} e^{hx} + A_{12} e^{-hx} \right] \exp(ihy - i\omega t) \end{cases} \tag{11}$$

Boundary conditions. There are three groups of boundary conditions:

1) boundary conditions at the flame front, that is with $x = \varepsilon(y,t)$;

2) boundary conditions at the ends of the chamber or the channel, that is with $x = -L_1$ or $x = L_2$;

3) boundary conditions at the side walls of the chamber or the channel, that is with $y = \pm d$.

Boundary conditions at the flame front are laws of conservation of mass and momentum (in two projections on the coordinate axes) for the disturbed flow. In the linear approximation these laws are given by:

$$\rho_1 (u'_{1x} - \frac{\partial \varepsilon}{\partial t})\Big|_{x=0} = \rho_2 (u'_{2x} - \frac{\partial \varepsilon}{\partial t})\Big|_{x=0}, \tag{12}$$

$$p_1'\Big|_{x=0} + 2\rho_1 u_1 u'_{1x}\Big|_{x=0} = p_2'\Big|_{x=0} + 2\rho_2 u_2 u'_{2x}\Big|_{x=0}, \tag{13}$$

$$u'_{1y}\Big|_{x=0} + u_1 \frac{\partial \varepsilon}{\partial y} = u'_{2y}\Big|_{x=0} + u_2 \frac{\partial \varepsilon}{\partial y} \tag{14}$$

The additional boundary condition at the flame front is well-known condition of Landau

$$u'_{1x}\Big|_{x=0} - \frac{\partial \varepsilon}{\partial t} = 0, \tag{15}$$

which reduces equation (12) to

$$u'_{2x}\Big|_{x=0} - \frac{\partial \varepsilon}{\partial t} = 0 \tag{16}$$

Taking into account equalities (1) and equation (16) boundary conditions (12)–(15) reduce to

$$\frac{u'_{1x}}{u_1}\Big|_{x=0} - \frac{1}{u_1}\frac{\partial \varepsilon}{\partial t} = 0, \tag{17}$$

$$\frac{u'_{2x}}{u_1}\Big|_{x=0} - \frac{1}{u_1}\frac{\partial \varepsilon}{\partial t} = 0, \tag{18}$$

$$\frac{p'_1}{\rho_1 u_1{}^2}\Big|_{x=0} = \frac{p'_2}{\rho_1 u_1{}^2}\Big|_{x=0}, \tag{19}$$

$$\frac{u'_{1y}}{u_1}\Big|_{x=0} + \frac{\partial \varepsilon}{\partial y} = \frac{u'_{2y}}{u_1}\Big|_{x=0} + \frac{1}{\delta}\frac{\partial \varepsilon}{\partial y}, \tag{20}$$

where $\delta \equiv \delta_1 = \dfrac{\rho_1}{\rho_2} = \dfrac{u_2}{u_1}$.

Boundary condition on the closed end of the channel (front end of the chamber) is $u'_{1x} = 0$, that is

$$\frac{u'_{1x}}{u_1}\Big|_{x=-L_1} = 0, \tag{21}$$

Boundary condition on the open end of the channel (back end of the chamber) is $p_2' = 0$, that is

$$\frac{p_2'}{\rho_1 u_1^2}\Big|_{x=L_2} = 0 \tag{22}$$

Boundary conditions at the side walls of the chamber or the channel are $\mathrm{Re}\, u'_{1y}\, |_{y=\pm d} = 0$, $\mathrm{Re}\, u'_{2y}\, |_{y=\pm d} = 0$, that is

$$\mathrm{Re}(\frac{u'_{1y}}{u_1})\,|_{y=\pm d} = 0,\ \mathrm{Re}(\frac{u'_{2y}}{u_1})\,|_{y=\pm d} = 0 = 0\ , \tag{23}$$

where Re means real part.

Boundary conditions (17)–(23) are all in the dimensionless (unitless) form.

Eigen-value problem. Substitution of particular solutions (6), (11) into boundary conditions (17)–(22) leads to the set of six linear algebraic homogeneous equations for six indefinite constants $A_{00}, A_{11}, A_{12}, A_{21}, A_{22}, A_{23}$. This set of equations has untrivial solution if and only if its determinant equals zero. And so this is the eigen-value problem for z, that leads to characteristic (secular) equation

$$Az^3 + Bz^2 + Cz + D = 0\,, \tag{24}$$

where

$$A = \frac{sh(\xi \tilde{L}_1)}{\delta^2} + \frac{e^{\xi} ch(\xi \tilde{L}_2)}{\delta} + (\frac{1}{\delta^2} - \frac{1}{\delta}) e^{\xi \tilde{L}_2} sh(\xi \tilde{L}_1) ch(\xi \tilde{L}_2)\,, \tag{25}$$

$$B = e^{\xi}[\frac{ch(\xi \tilde{L}_2)}{\delta} + sh(\xi \tilde{L}_2)] + e^{\xi \tilde{L}_2} sh(\xi \tilde{L}_1) sh(\xi \tilde{L}_2)(\frac{1}{\delta} - 1)\,, \tag{26}$$

$$C = sh(\xi \tilde{L}_1)[(\frac{1}{\delta} - 1) e^{\xi \tilde{L}_2} ch(\xi \tilde{L}_2) - 1] + e^{\xi} sh(\xi \tilde{L}_2)\,, \tag{27}$$

$$D = (1-\delta) e^{\xi \tilde{L}_2} sh(\xi \tilde{L}_1) sh(\xi \tilde{L}_2)\,, \tag{28}$$

$$\xi = hL\,, \tag{29}$$

$$\tilde{L}_1 = \frac{L_1}{L},\ \tilde{L}_2 = \frac{L_2}{L},\ \tilde{L}_1 + \tilde{L}_2 = 1 \tag{30}$$

It is enough to know only signs of real parts of roots for the equation (24) to solve the stability problem.

If equation (24) has a root with positive real part (that is $\mathrm{Re}\, z > 0$) then instability takes place. If all the roots of equation (24) have a negative real part (that is $\mathrm{Re}\, z < 0$) then the process is stable to perturbations of the exponential type (5). But this fact is not a guarantee of the absolute stability for the flow and the flame front.

Function in the left part of the equation (24) is polynomial of the 3rd degree (for z).

It is known that the combustible gas density is much more than the density of products, that is

$$\delta_2 = \frac{\rho_1}{\rho_2} = \frac{u_2}{u_1} > 1 \tag{31}$$

Using inequality (31) it is easy to prove that

$$A > 0,\ D < 0 \tag{32}$$

By Stodola theorem the necessary condition for the stability of the polynomial with real coefficients and positive leading coefficients is positiveness of all its coefficients. But in this case this necessary condition is not fulfilled, so the polynomial in the left part of the equation (24) is instable, that is it has roots with positive real parts. This result takes place for every value of ξ, that is for any value of the wavelength λ.

Consideration of the boundary conditions at the side walls. The above result does not consider boundary conditions (23) at the side walls of the chamber or the channel. Those conditions lead to

$$h = n\pi / 2d (n = 1, 2, ...) \tag{33}$$

or

$$\lambda = 4d / n (n = 1, 2, ...) \tag{34}$$

In other words there are integer lengths of the half waves along the diameter of the channel.

Results and discussion. Theoretical conclusions are in good agreement with experimental data and classic analytical results. For example in the extreme case $\xi \to \infty,\ \tilde{L}_1 \equiv \frac{L_1}{L} \to \infty,\ \tilde{L}_2 \equiv \frac{L_2}{L} \to \infty$ instable root of the characteristic equation (24) is

$$z_* = \frac{\delta}{1+\delta}(-1 + \sqrt{1 + \delta - \frac{1}{\delta}}),\ z_* > 0 \tag{35}$$

This root coincides with the well-known Landau root in the problem of stability for the flame in the unlimited space. Such facts prove correctness of the suggested theory.

Conclusions

1. Two-dimensional instability of the flame front in the plane channel mathematically explains the nature of turbulent combustion. But turbulent combustion is caused by the development of perturbations with the wave length λ_m that corresponds to the fastest growth rate of amplitude. Only if λ_m is more than the channel width *2d* turbulence takes place. Otherwise one-dimensional instability develops, that leads sometimes to the vibratory combustion [1].
2. The main stabilizing factor for flames is viscosity [6]. But this factor is not taken into account in present investigation.
3. Combustion in engines needs control: in some cases to avoid turbulent combustion, in other cases – to get turbulent regime. It is possible to avoid turbulent combustion either by increase of the combustible mixture viscosity or by decrease of the channel width. It is necessary to act contrary to get turbulence in the chamber. Possibilities and methods of such control differs greatly for various engines and fire-chambers. Such possibilities and methods exceed the limits of the present investigation.

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