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On the Study of Dynamical Model of Moods pertaining to etching behavioural patterns

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Abstract: Often times, we experience ups and downs in our spirit, sometimes we have no vivid explanation to what we really feel within. We normally classify these feeling as "bad mood". At times the situation is the other way round, we feel elated, "positive" and confident to go about our normal roles. During these periods in our lives, we are normally inclined to doing certain things that interest us. The question we ask ourselves is; is there a way to explain this inevitable feeling and how to manage them? Are there negative or positive effects associated with these feelings? In this research, we delve deeper into things that cause these ups and downs in our emotional feelings and their effects on our lives being it positive or negative. Upon further investigations and research, we present a model of second order ordinary linear differential equation to measure and describe the various changes in mood with time. The rapid and periodic changes between extreme moods forms (mood swings) is further elaborated with a mathematical model. The trend in the changes of mood is demonstrated graphically with elaborative commentary.

Keywords: mood, Differential equations, mathematical model,

Introduction:

Our model is to explain the linkage between ones internal influence (Cognitive, emotions, mental states) and their environment that affect their performance or satisfaction.

The theory proposes that positive mood as well as negative moods are distinguishable and have significant psychological impact upon individual's satisfaction. To that end, when one experience uplift (positive) or hassles (negative), their intensions to continue doing that activity or to quit depends upon the emotions, mood and thoughts associated with the satisfaction they derive from the activity.

Moods are dynamic phenomenon. The interaction between mood and behavior necessarily causes changes over time. Yet this dynamic and evolving nature of mood and behavior largely has been ignored in previous studies. In time past, emotions were left out of consideration in the areas of cognitive (the mental processes such as memory, language, perception, reasoning and problem solving), and user modeling. Only a few computational models of mood have been developed. It is of interest to measure the relationship between an individual's mood and his action and to what degree mood affects life. We further seek to develop a possible mathematical model to predict the mood of an individual given certain factors and this is by far the goal of our work.

Mood as already discussed is an emotional state. Emotions are treated indifferent from mood, but the small difference between the two is that, emotions are the individual strong feelings such as joy, regret, anger, love and the like. One or more of these feeling combined yield the mood of a person. Mood generally has either a positive or negative valence. That is whether they are pleasant or unpleasant.

The idea of modeling of mood may sound uninteresting or unimportant with regards to improvement of productivity, for this reason it has for some time now lost its stand in terms of research studies. The subject matter have been overlooked or underrated, but integrating knowledge on the mood and emotional wellbeing of workers in productivity has diverse importance and worth an attention.

Objectives

1. We seek to address the various forms of mood and outline the underlying components or factors which contribute to each form of mood.

2. To address some of the clinical and psychological disorders associated with mood and to come up with suggestion to curb some of these problems.
3. Finally, to propose a mathematical model to measure or predict the mood of an individual.

Types of Mood

Positive/ Good mood/High mood

Negative/Bad mood/Low mood

Mood Swings

A **mood swing** is an extreme or rapid change in mood. In other words, the basic physical dynamics in state of mood can be termed mood swings. Many people suffer from mood swings, being happy at one moment, sad the next moment. Periodic changes in mood of women during pregnancy are a clear demonstration of mood swings. Causes of mood swings vary from person to person; biological, psychological, medical and environmental factors are all potential causes of mood swings.

Mood Regulation Strategy

People actively attempt to create and maintain positive moods and to escape from negative moods by engaging in various consumption activities. The principle of homeostasis explains the essence of this mood-management behavior: that is, people adjust their mood and activities to preserve constant the conditions of life. Since moods are fluctuated at any point in time, there is a need for mood regulation strategies. The deliberate use of strategies to influence moods and emotions is known as controlled affect regulation and this can either be cognitive strategies or behavioral strategies.

Mood Disorders

Intense and prolonged bad mood can cause certain disorders to a person which is more intensive and severe than the bad mood itself. Relatively, mood disorders are severe and last longer than mood. Among the common mood disorders are Major Depression Disorder, Dysthymic Disorder (a chronic or mild depression) Bipolar Disorder (manic depression). Bipolar disorder is characterized by extreme mood swings, usually between mania and depression.

Treatment To Depressive And Bad Mood

The impact of mood and its effects are manageable by employing certain techniques. These include the coping ability of a person helps to manage and optimize the impact of bad mood as effective as possible and to limit the damage of bad mood, Having enough rest and Adopting good nutritional habit.

Sample projects on mood

- In [1], Fiemke Both et al., did a project on modelling the dynamics of mood and depression. It stated that depression is a form of mood value below a certain maximum for a certain time period. Their analysis was brought to a conclusion that the model can describe how stress factors under some conditions can lead to a depression, while it won't lead to a depression under other conditions. Some of the factors include coping, thoughts and situations. With further elaboration, bad coping person gets depressed after negative situation and a good coping person does not get depressed after one negative situation. Also negative thoughts result in lower subjective situations which results in depression and finally, increasing more positive situations avoids depression.

- In [2] the study was to examine whether the trainees can more successfully improve their mood by the use of different types of mood-regulation strategy. The strategy used was both cognitive and engagement (behaviour) strategy. It concluded that the engagement group (behaviour) reported significantly higher level of cheerfulness.

- In [3], Mike sellers and Mirjam Palosaari also did a study of Mood in Dynemotion and the Mind Module. With a case study of affecting the mood of a character using Affective Actions in order to explore how players would take to the use of mood in a social game player context. The context for the test was World of Minds (WoM), a phenotype game where the personalities of the inhabitants are the basis for the game mechanism [4]. Guided play test shows that despite issues of naming moods, users find their graphical and textual display useful in games play situations that uses metaphor of social interactions.
- In [5], Holbrook and Gardner's (2000) studied to examine how people's mood changed over time as a result of listening to music.
- In [6], Eliashberg and Sawhney's (1994) studied to explore the dynamic effects between initial mood state and the emotional characteristics of scenes from a movie on participants' subsequent mood state and overall enjoyment of the movie.

- In [7], Baumgartner, Sujan, and Padgett (1997) examined students' moment-to-moment emotional responses to television adverts in a single-variant correlational analysis.

Methodology

In this paper, we present a model to measure the mood of a person in present time and the near future. We intend to achieve this by employing differential equations as our major mathematical tool. Stability and instability conditions of linear systems are used to explain solutions and their face portraits.

Introduction to Differential Equations

The subject of Differential Equation (DE) is solving problems and making predictions. With the systematic study of differentials equations, the calculus of functions of a single variable reaches a state of completion. The use of differential equation provides a way to simplify many areas of application to mathematical analysis.

Differential Equation can simply be defined as an equation with derivatives, an example of differential equation that almost everybody know is the equation of Newton's second law of motion, where he expresses the force applied on a body as a function of mass and acceleration. $\mathrm{F} = \mathrm{Ma}$. Where **"a"** is the acceleration and **M** is mass of the body; velocity so $F(v,t) = M\frac{dv}{dt}$. This is an example of a differential equation.

Types of Differential Equations

There are two types of differential equation; these are **Ordinary Differential Equation (ODE)** and **Partial differential equation (PDE).**

- ODE is an equation involving an unknown function of a single variable together with one or more of its derivatives.

  Example: $\frac{dy}{dt} = y - t$ is an ODE. Here, $\mathrm{y} = \mathrm{y(t)}$ is the unknown function, t is an input variable or the independent variable.

- PDE is an equation involving a function with two or more independent variables.

Example: $\frac{\partial y}{\partial t} = C^2\frac{\partial y}{\partial x}$ is a PDE. Here, $\mathrm{y} = \mathrm{y(t,x)}$ is the unknown function, (x , t) are the independent variables

Basic Definitions and Criteria for Classifying Differential Equation

Given a DE, it can fall under several categories, below are some basic criteria for classifying a differential equation

ORDER**:**

Order of DE is defined as the highest derivative that occurs in the equation. Example; $y' = y^2 - t$ This is ODE of **first order.**

DEGREE:

Degree of DE is the power to which the highest ordered derivative is raised to after it is cleared of all fractions and decimals. Example : $y'' + (y')^3 + ay.$ This is also an ODE of the *second order* with *degree of 1.* Since the highest derivative (y'') is of index 1.

Some Applications of Differential Equations

We use differential equations to model and solve real-life problems. Few examples are;

1. The discovering of the laws of mechanics. Example is Newton's second law which state that forces acting on a mass equal to the rate of change of momentum with respect to time. Momentum is the product of mass and velocity (MxV). Thus force equals the derivative of the momentum, if mass is constant.

   $F = \frac{d}{dt}mv = m\frac{dv}{dt}\ ma.\ hence\ f = ma.$

2. Modeling Advertising Awareness. This is to know the rate at which a population(y) of potential customers gets to hear about a particular product through advertisement at a given time (t).

Mathematical Analysis Of The Model

This section describes a mathematical model which attempts to estimate the mood of a person based on factors. These factors are the awareness level and attitude of the person towards mood. Mood is described over time and it differs from person to person. Likewise the awareness level and attitude, they all vary over time and differ from person to person.

Let;

M(t) = be the desired level of mood or the mood level at any time (t)

B(T) = be attitude /behaviour of the subject under study

C(t) = be awareness level of subject.

Behaviour and attitude shall be used interchangeable as per this model. We therefore write a simple differential equation for these variables

$$\frac{dm(t)}{dt} = b(B - \beta m) \dots\dots\dots\dots\dots\dots\dots 1$$

$$\frac{dB(t)}{dt} = a(m - \mu B) + \gamma c \dots\dots\dots\dots\dots\dots\dots\dots 2$$

Where μ, β, a, b are parameters.

Equation (1) expresses the time rate of change of mood as a function of the difference between the behaviour/attitude and the mood level of an individual.

Similarly equation (2) expresses the rate of change of behaviour as a function of the difference between mood and behaviour.

Parameters

b= is the responsiveness of mood to attitude. The character varies across individuals. $0 \leq b \leq 1$

a= is responsiveness of attitude to mood.

β= measures the motivation or effort of a subject to manage mood (β ϵR)

Equation (1) and (2) is a system of linear ordinary differential equations [8].

From (1) $B = \frac{1}{b}\frac{dm}{dt} + \beta m \dots\dots\dots\dots\dots\dots 3$

Substituting this into equation (2), we have1

$$\frac{d}{d}\left(\frac{1}{b}\frac{dm}{dt} + \beta m\right) = a\left(m - \mu\left(\frac{1}{b}\frac{dm}{dt} + \beta m\right)\right) + \gamma c$$

=

$$\frac{1}{b}\frac{d^2m}{dt^2} + \beta\frac{dm}{dt} = am - \left(\frac{a\mu}{b}\frac{dm}{dt} + a\mu\beta m\right) + \gamma c$$

$$\frac{d^2m}{dt^2} + \beta b\frac{dm}{dt} = abm - a\mu\frac{dm}{dt} - ab\mu\beta m + b\gamma c$$

$$\frac{d^2m}{dt^2} + (a\mu + b\beta)\frac{dm}{dt} + (ab\mu\beta - ab)m = b\gamma c$$

$$\frac{d^2m}{dt^2} + (a\mu + b\beta)\frac{dm}{dt} + ab(\mu\beta - 1)m = b\gamma c \dots\dots\dots\dots\dots\dots 4$$

This gives us a non-homogeneous 2nd order differential equation. The solution to equation (4) will be of the form $M = M_C + M_p$. M_C= complementary solution which depends on the homogeneous part of equation (4). M_p= particular solution depending on the non-homogeneous part.

Solving for the homogeneous part such that

$$\frac{d^2m}{dt^2} + (a\mu + b\beta)\frac{dm}{dt} + ab(\mu\beta - 1)m = 0 \dots\dots\dots\dots\dots\dots\dots\dots *$$

$x_{1,2} = \frac{-(a\mu + b\beta) \pm \sqrt{(a\mu - b\beta)^2 - 4ab}}{2}$ by the quadratic equation.

Given that all parameters are positive, x_1 and x_2 will be real. And $x_2 < 0$. The sign of x_1 depends on the value of $(\mu\beta - 1)$. If $\mu\beta > 1$ then $x_1 < 0$ whereas if $\mu\beta < 1$ then , $x_1 > 0$. Therefore $m_c = Ae^{x_1 t} + Be^{x_2 t}$; Where A, B are constants

Interpretation

Considering the complementary solution as a system of trajectories moving on the x , y plane.

1. For all parameters positive such that $x_2 < 0$, the trajectory $Ae^{x_2 t}$ decays exponentially and gradually approaches zero as time (t) approaches infinity

   $Be^{x_2 t} \to 0 \; as \; t \to \infty$

2. For $\mu\beta < 1$ such that $x_1 > 0$ then the trajectory $Ae^{x_1 t}$ grows exponentially. Hence $Ae^{x_1 t}$ will explode. Condition (1) and (2) will result in an unstable form of mood constantly changing.

   This is an unstable situation

3. On the other hand if $\mu\beta > 1$ then $x_1 < 0$ and similarly $Ae^{x_2 t} \to 0 \; as \; t \to \infty$

The system becomes stable if $x_1 + x_2 < 0$.

Solving the non-homogeneous part m_p : Surely, the kind of solution depends on the awareness function c(t)considering the scenario when c(t) is a constant. Say $c(t) = C$. Let $m_p = k$ then, $\frac{dm_p}{dt} = 0$, $\frac{d^2m_p}{dt} = 0$. Substituting into equation (4) gives $k = \frac{b\gamma c}{ab(\mu\beta - 1)} = \frac{\gamma c}{a(\mu\beta - 1)}$

For instance:

From the 2nd order differential equation

$$\frac{d^2m}{dt^2} + (a\mu + b\beta)\frac{dm}{dt} + ab(\mu\beta - 1)m = 0 \dots\dots\dots\dots\dots\dots\dots\dots *$$

For the constants a, b positive $0 \leq a \leq 1 \; and \; 0 \leq b \leq 1$ say a$= 0.5 = b$ that is moderate responsiveness between mood and behaviour

For $\mu > 0, \beta > 0$

Let the term $(a\mu + b\beta) = 5. \; \mu = 4, \beta = 6. \; ie \; 0.5((4) + (6)) = 5$

And

$ab(\mu\beta - 1) = 6\ ie\ \ 0.5(0.5)(4.6 - 1) = 5.75 \cong 6$

Hence the equation $m'' + 5m' + 6m = 0 \ldots\ldots 1$ with initial condition $m(0) = 2, m'(0) = 3$

Assume $m = e^{xt}$ a solution to equation (1)

$\Longrightarrow m' = xe^{xt}, m'' = x^2e^{xt}$

Substituting into equation (1)

We get

$x^2e^{xt} + 5xe^{xt} + 6e^{xt} = 0$

$(x^2 + 5x + 6)e^{xt} = 0$

$(x^2 + 5x + 6)e^{xt} = 0$

But $e^{xt} \neq 0$ hence $x^2 + 5x + 6 = 0$

Solving this quadratic equation

$$x_{1,2} = \frac{-5 \pm \sqrt{5^2 - 4.6}}{2}$$

$x_1 = -2, x_2 = -3$

Hence the solution

$m = Ae^{-2t} + Be^{-3t}$

$m(0) = 2 = Ae^0 + Be^0 \ldots\ldots\ldots 2$

$2 = A + B \ldots\ldots 1$

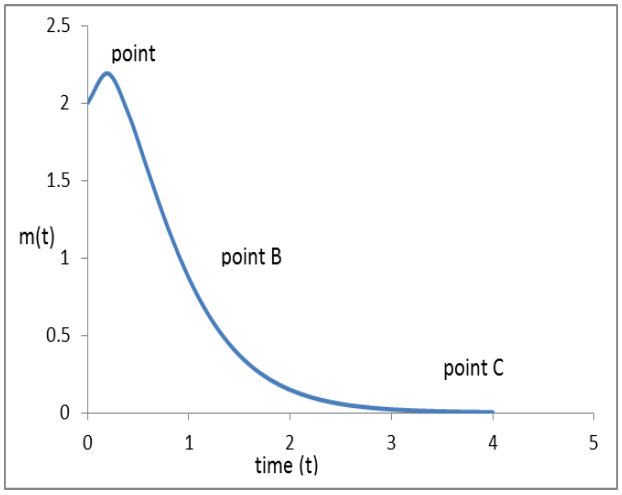
$m' = -2Ae^{-2t} - 3Be^{-3t}$

$M'(0) = 3 = -2A - 3B \ldots\ldots\ldots 3$

Solving equation (1) and (2) simultaneously

$A = 9, \qquad B = -7$

$m = 9e^{-2t} - 7e^{-3t}$



Assuming equation (1) above represent the mood of a person who has received a valuable gift, both roots of the equation has negative real roots. The negative nature of the roots implies that both part of the solution decays with time.

On the other hand, let us assume values that will yield complex roots Let $(a\mu + b\beta) = -\frac{8}{16}$.

$$(\mu\beta - 1) = \frac{145}{16}$$

$\Longrightarrow$

$m'' - \frac{8}{16}m' + \frac{145}{16}m = 0 \ldots\ldots\ldots\ldots (4). m(0) = -2, m'(0) = 1$

Let $m = e^{xt}$ a solution to

$m' = xe^{xt}, m'' = x^2e^{xt}$

$\Longrightarrow$

$$x^2e^{xt} - \frac{8}{16}xe^{xt} + \frac{145}{16}e^{xt} = 0$$

$$(x^2 - \frac{8}{16}x + \frac{145}{16})e^{xt} = 0$$

$$(x^2 - \frac{8}{16}x + \frac{145}{16})e^{xt} = 0$$

$(16x^2 - 8x + 145)e^{xt} = 0$

$16x^2 - 8x + 145 = 0$

Solving this gives us

$$x = \frac{8 \pm \sqrt{64 - 9280}}{32}$$

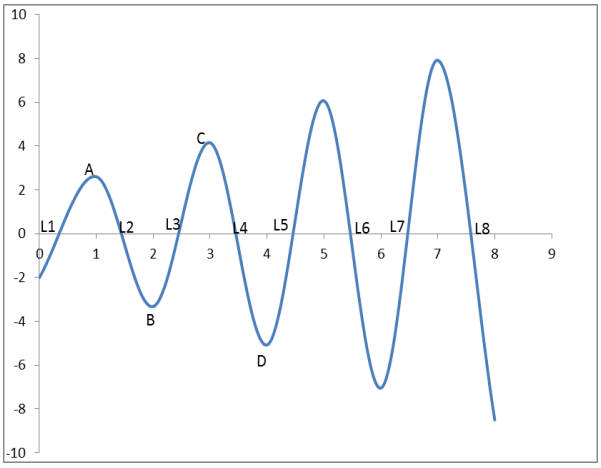
$$x = \frac{1}{4} \pm 3i$$

Hence $m(t) = e^{\frac{1}{4}t}(A\cos 3t + iB\sin 3t)$

$m(0) = -2, m'(0) = 1$

$A = -2, \quad B = -1/3$

$$m(t) = e^{\frac{1}{4}t}(-2\cos 3t + i\frac{1}{3}\sin 3t)$$



The roots of equation (4) above are a complex conjugate with positive real part. From stability, we saw that when the real part of a complex root is positive, this makes the system unstable. The trajectory of such a solution explodes as time increases.

Let the parameters of equation (4) satisfy the mood of a contestant in say a musical contest which comprises many stages, before he is selected to be part of the contestants at the auditioning stage, anxiety and the fear of being denied a chance causes a drop in his mood below normal level (that is between the point 0 and L1). If he is nominated, joy and contentment causes his mood to rise above the normal level through L1 to point A. At the end of the first round in the competition, when judges are about to pass judgement and make an eviction, panic sets in and the subject's mood level drop drastically through L2 to point B due to the fear of getting evicted. If he makes it to the next round, he becomes excited with his mood level rising through L3 up to point C. This cycle repeats itself until the end of the contest with increasing amplitude at each stage; the increase in amplitude of the graph at each stage can be attributed to the level of importance attached to the stages as we near the end of the contest. In this case, the subject considers an eviction in the first stage less painful than the final stage and likewise a success in the higher stages carries more joy and fulfilment than in the early stages.

As the contest continues, Stage after Stage, the mood of the subject increases and decreases at greater amplitudes and finally explodes irrespective of him winning or losing. This up and down nature makes his mood unstable

Conclusion

The paper addressed two forms of mood, being it good or bad with a condition which gives rise to each form. The factors are said to be psychological and biological and other peripheral factors such as sound sleep, completing a task and hearing of a bad news.

Furthermore we address some of the clinical and psychological disorders associated with moods. Mood disorders come as a result of mood swings that are the rapid change in mood. The main cause of this is psychiatric and non -psychiatric. This leads to common disorders such as depression and dysthymic disorders.

Treatment to such disorders includes one's ability to cope with issues at hand, making all efforts to avoid bad mood and promote good mood, having enough rest and adopting good nutritional habit.

We have also used second order linear differential equations to model moods in different settings. The variations were seen in the graphs. These were based on individual's awareness level and attitude towards mood.

Recommendation

We recommend that people learn to control their emotions and behaviors in times of uncertainty.

- Mood affects life and productivity and it is very important to adopt conscious efforts to manage mood.
- It is also important to consult a psychologist or specialist on depressive moods if the need arises to avoid further complications.

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