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VEHICLE TRAJECTORY MODELING UNDER THE INFLUENCE OF LATERAL SLIDING

Summary. This paper presents an analysis of vehicle trajectory on curved path, in the presence of lateral sliding. The pure rolling motion is not always possible especially where working conditions are rough and not predictable. To take sliding effects into account, the variables which characterize sliding effects are introduced into the mathematical model (steering angle, vehicle speed, tire cornering stiffness and etc.). This mathematical model is linear with two freedom degrees. From the study based on the verification of force in the contact zone tire/ground, we conclude that speed exceeding 60 km/h and small steering angles can destabilize the vehicle.
Keywords: Steering angle, vehicle speed, tire cornering stiffness, vehicle trajectory

MODELOWANIE TRAJEKTORII POJAZDU POD WPŁYWEM PRZESUNIĘCIA BOCZNEGO

Streszczenie. W artykule przedstawiono analizę trajektorii ruchu pojazdu na zakrzywionym torze, w obecności przesunięcia bocznego. Czysty ruch toczny nie jest zawsze możliwy, zwłaszcza gdy warunki pracy są niebezpieczne i nie do przewidzenia. By wziąć pod uwagę zmienne efekty przesuwne, charakteryzujące działanie przesuwne, do modelu matematycznego są wprowadzone kąt kierownicy, prędkość pojazdu, sztywność opon na zakrętach itp. Ten model matematyczny jest liniowy o dwóch stopniach swobody. Z badań opierających się na weryfikacji obowiązujących w strefie styku opony/ziemia, możemy stwierdzić, że prędkość przekraczająca 60 km/h oraz małe kąty skrętu mogą zdestabilizować ruch pojazdu.
Słowa kluczowe: Kąt skrętu, prędkość pojazdu, sztywność opon, trajektoria ruchu pojazdu

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1. INTRODUCTION

The behavior of the vehicles represents the results of the interactions among the driver, the vehicle and the environment. The problem of vehicle motion on a curved path represents a subject of high interest and it is important part of vehicle safety. The motion is influenced by many external factors such as road roughness, lateral aerodynamics, and tire construction [2, 5, 7]. Examination of the vehicle trajectory needs all this factors to be evaluated. With the development of the electronics and mechatronics applied in the automotive industry there are always new solutions how to keep the vehicle stable and how to control the vehicle trajectory.

For the vehicle safety, one of the most complex components is the tire and the road interaction problem. While moving in a curve, forces appear at the contact surfaces between the wheel and the road. Under these forces, the tires are deformed and the velocity on the wheel is deviated from the wheel plane under a certain angle, depending on the tire lateral rigidity and force magnitude (see Fig. 1).

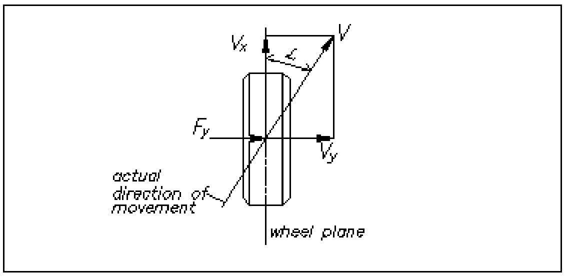

Fig. 1. Tire slip angle
Rys. 1. Kąt poślizgu opony

The tire plays an important role in the performance of the vehicle model. The modeling of vehicle dynamic behavior has to take into account the tire elastic in the contact zone tire/road (see Fig. 2). This will give the possibility of better predict and control of the vehicle trajectories. More or less complicated variants of the tire model can be found in literature [3, 5].

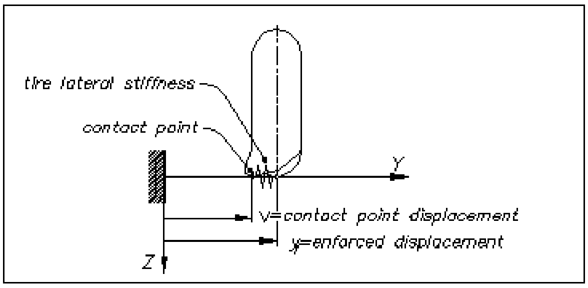

Fig. 2. Contact zone tire/road
Rys. 2. Miejsce kontaktu opony z drogą

The movement on a curved trajectory has been treated in numerous papers [1, 3, 6, 9, 10]. In general, these authors use the fundamental principles of dynamics. For example, to describe the lateral dynamics, Segal [10] presents a vehicle model with three degrees of freedom in order to describe lateral movements. If roll movement is ignored, a simple model known as the "*Bicycle Model*" is obtained. This model is currently used for studies of lateral vehicle dynamics (yaw, lateral speed and slip angle).

This paper is consecrated of the vehicle movement on curved trajectory using a model of two degree of freedom. The main objective is to study the influence of some vehicle properties such as vehicle speed, position of vehicle center of gravity, tire cornering stiffness and steering angle while the vehicle is turning. In this context, a mathematical-mechanical model is developed to describe the vehicle behavior in large interval of driving conditions from normal to the limits of controllability. This model has two degrees of freedom (df): translation around the axis O_y and rotation in axis O_z. The complete vehicle is considered as suspended mass related to the wheels. This simple model is currently used in the literature to describe the lateral acceleration, yaw and slip angle. In fact, these parameters permit to describe a vehicle during the turning maneuver. The study aims is to define the criteria for the detection of critical situations.

The paper is structured as follows: *Section 1* provides introductive elements, notations and motivations. *Section 2* introduces the linear vehicle model, used for the simulation. *Section 3* describes the simulation method and the indicator proposed to determine the risk of tire lateral slipping. In *Section 4*, the results are analyzed and shown that the vehicle speed is the most critical for vehicle stability while cornering. Conclusions and discussions are given in *Section 5*.

2. THE MATHEMATICAL MODEL

Modeling the vehicle dynamic behavior in all is a complex subject and requires good knowledge of the components involved and their physics [1,2]. The first step in the study of the vehicle lateral behavior is to create a mathematical model that have to represent the physical system with good approximation. The formulation of the following model takes into account these assumptions:

- The vehicle and his model are symmetrical to the axis O_x;
- The dynamical process (displacement around to the axis O_z) isn't exanimated;
- The laterals forces are due of centrifuge force;
- The tire lateral force varies linearly with the slip angle;
- The camber angle is neglected;
- The tire angles (τ) are small ($\cos \tau = 1$et $\sin \tau = \tau$).

Fig. 3 shows the vehicle model used in this research. The vehicle model in Fig. 3 has two degrees of freedom. The vehicle motion is defined by its translation around the axis O_y and its rotation in the axis O_z. The vehicle is considered as a rigid body (sprung mass) related to the wheels. This model is completed with a linear model of force in the contact zone tire/road [7].

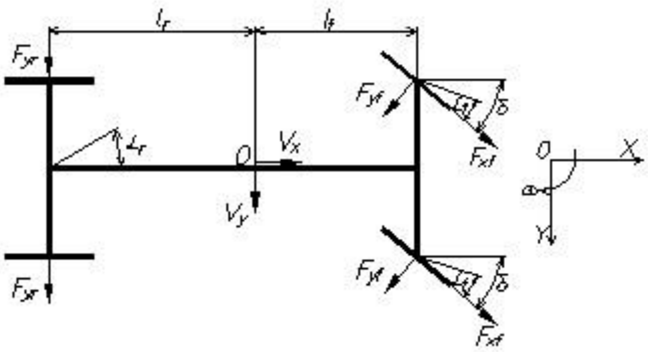

Fig. 3. „Four wheels model”
Rys. 3. „Model czterech kół”

In this research, we have assumed the front and rear lateral forces to be proportional to the tire slips angles. This functional link is expressed into the following relation:

$$
\begin{aligned}
F_{YF} &= C_{\alpha F} . \alpha_F \\
F_{YR} &= C_{\alpha R} . \alpha_R
\end{aligned}
\tag{1}
$$

The following equations define the slip angles of front and rear tires:

$$
\begin{aligned}
\alpha_F &= \delta - \tan^{-1}\left(\frac{V_Y + l_F \dot{\psi}}{V_X}\right) \\
\alpha_R &= \tan^{-1}\left(\frac{l_R \dot{\psi} - V_Y}{V_X}\right)
\end{aligned}
\tag{2}
$$

Where Cα represents the tire cornering stiffness witch depends on road adherence μ, on the tire internal pressure p and the tire vertical force F_z. This parameter is essential in the evaluation of the potential of tire used [8, 11].

Finally, we obtain a linear model with four varying parameters:

- The longitudinal speed (V);
- The steering angle (δ);
- The rigidity of the tire (C_α);
- The position of vehicles center of gravity (l_f and l_r).

For the differentials equations describing the system are valid:

$$
\left|
\begin{aligned}
m\ddot{y} &= F_{YR} + F_{YF}\cos(\delta) + F_{XF}\sin(\delta) \\
I_Z\ddot{\psi} &= l_F F_{YF}\cos(\delta) - l_R F_{YE} + l_F F_{XF}\sin(\delta)
\end{aligned}
\right.
\tag{3}
$$

$$
\left|
\begin{aligned}
\ddot{y} &= [F_{YR} + F_{YF}\cos(\delta) + F_{XF}\sin(\delta)] : m \\
\ddot{\psi} &= [l_F F_{YF}\cos(\delta) - l_R F_{YR} + l_F F_{XF}\sin(\delta)] : I_Z
\end{aligned}
\right.
\tag{4}
$$

Fig. 4 shows the block diagram of modeling.

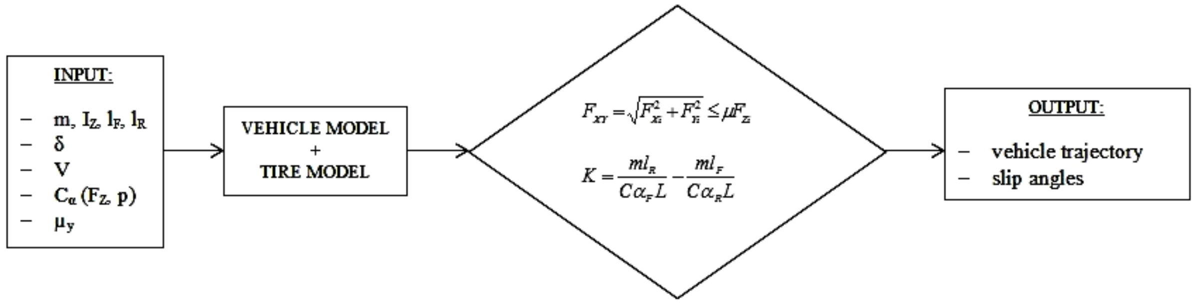

Fig. 4. Blog diagram for simulation
Rys. 4. Diagram symulacji

The block "*Input*" supplies the necessary simulation data. The parameters: m, I_z, l_f and l_r characterize the chosen vehicle. The steering angle δ simulates the driver actions, while the tire cornering stiffness is chosen as a function of the tire vertical load F_z and the tire internal pressure p.

The block "*Vehicle Model+ Tire Model*" uses the data from the first block to solve differential equations 3 and 4 to obtain lateral acceleration, yaw rate and slip angle.

The *third* block evaluates the vehicle dynamic state and detects the critical situation – the saturation limit of the efforts in the contact zone tire/road.

Finally, the block "*Output*" shows the vehicles trajectory and tires slip angles. At the same time, it indicates if the tires are sliding and as a consequence if the limit of controllability is achieved.

3. SIMULATION

Vehicle model was built in MATLAB in order to analyze the state and predict the vehicle behavior under the different initials conditions of vehicle speed, steering angel, tire cornering stiffness and vehicles position of center of gravity.

Lateral instability may result from slippery road conditions or excessive speed in a curve. The poor-handling is another reason for lateral instability and which represents a significant proportion of the vehicle accidents. In this context and to judge the vehicles behavior while cornering is adopted a test called "*Angular dynamic*". The aim of this test is to keep the vehicle at a constant speed on a constant radius turn with a constant steering angle [2]. This steering behavior of the vehicle is estimated with the gradient K:

$$K = \frac{ml_R}{C\alpha_F L} - \frac{ml_F}{C\alpha_R L} \tag{5}$$

Three different steady-states can be identified:

$K = 0 \rightarrow \alpha_F = \alpha_R \rightarrow$ the vehicle is neutral;
$K > 0 \rightarrow \alpha_F > \alpha_R \rightarrow$ the vehicle shows understeer;
$K < 0 \rightarrow \alpha_F < \alpha_R \rightarrow$ the vehicle shows oversteer.

Equation 5 shows the importance of the vehicle position of the center of gravity l_f/l_r and tire cornering stiffness C_α in terms of maneuverability.

In this work, we are used a pneumatic tire type "*175/70R13 T 82*" with internal pressure of 2 bar and variation of tire vertical load F_z from 3 kN to 8 kN. Table 1 shows the parameters used for the vehicles simulation. These values come from commercial specifications from a standard European vehicle.

Table 1

Vehicle characteristics and numerical parameters

№	Parameter	Symbol	Value
1.	Total masse of vehicle [kg]	m	1603
2.	Total yaw inertia of vehicle [kgm2]	J_z	3156
3.	Distance between CG and front axle [m]	l_f	1,050
4.	Distance between CG and rear axle [m]	l_r	1,525
5.	Tire front cornering stiffness [kN/rad]	$C_{\alpha F}$	(30÷50)
6.	Tire rear cornering stiffness [kN/rad]	$C_{\alpha R}$	(30÷50)
7.	Front steering angle [grad]	δ	(0-32)

4. CASE STUDY

The figures presented in this section show the trajectories obtained for different initial conditions of the system. The effects of the vehicle speed, the position of vehicle center of gravity, the tire cornering stiffness and the steering angle are studied.

4.1. Vehicles center of gravity is located near to the front axle

When the vehicles center of gravity is located near to the front axle, the vehicle shows understeer (K>0), it means that the slip angle of the front tires is greater than the slip angle of the rear tires. Fig. 4 presents the vehicles simulated trajectories for a vehicles speed of 5 m/s. The steering angle has been varied between 5° and 30° and the curves are plotted for different coefficients of tire cornering stiffness. It can be seen from these figures that the augmentation of the steering angle decreases the radius of the trajectory curve. This represents a risk when the slip angles are high. Also, we can observe that the slip angle decrease with augmentation of tire cornering stiffness.

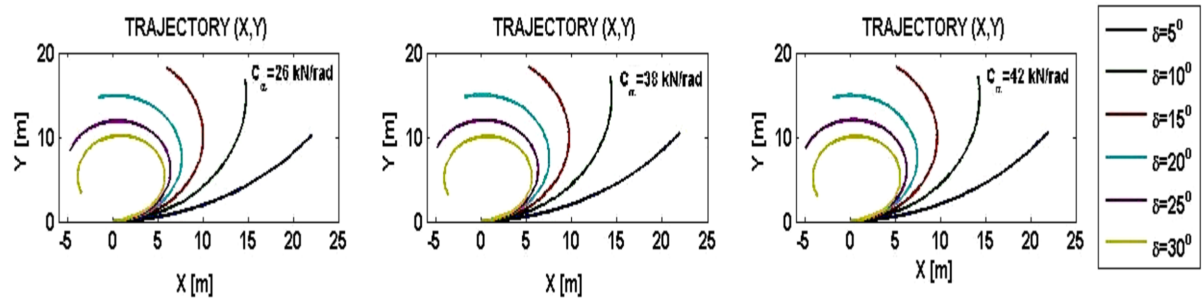

Fig. 5. Vehicle trajectory estimated for speed V = 5 m/s, K > 0, pure motion
Rys. 5. Trajektoria ruchu pojazdu dla prędkości V = 5 m/s, K > 0, czysty ruch

In a second simulation, the vehicles speed is increased to 10 m/s. The objective is to estimate the effect of speed on the vehicle path. The results obtained are present in Fig. 6. By comparison with Fig. 5, it can be seen that the displacement is increased as a consequence of the augmentation of vehicles speed. Also, the augmentation of vehicle speed doesn't permit to attack the turn with steering angle greater than 10°.

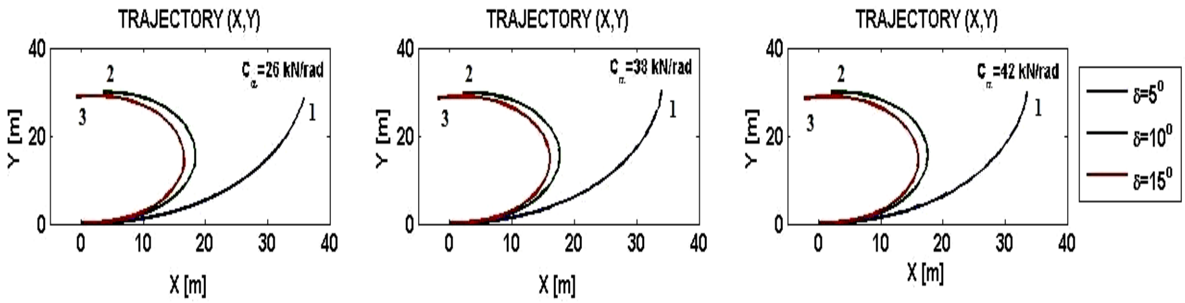

Fig. 6. Vehicle trajectory estimated for speed V = 10 m/s, K > 0
1 – motion without sliding; 2 – motion + sliding; 3 – pure sliding
Rys. 6. Trajektoria ruchu pojazdu dla prędkości V = 10 m/s, K > 0,
1 – ruch bez poślizgu; 2 – ruch + poślizg; 3 – czysty poślizg

When the vehicle speed is increased to 15 m/s as can be seen from Fig. 6, the displacement are increased too and confirming the previous conclusions concerning the effect of vehicles speed. At this speed, the maximum steering angle allowed is 5°. We can conclude that the vehicle speed controls the maximum steering angle with witch a turn can be taken.

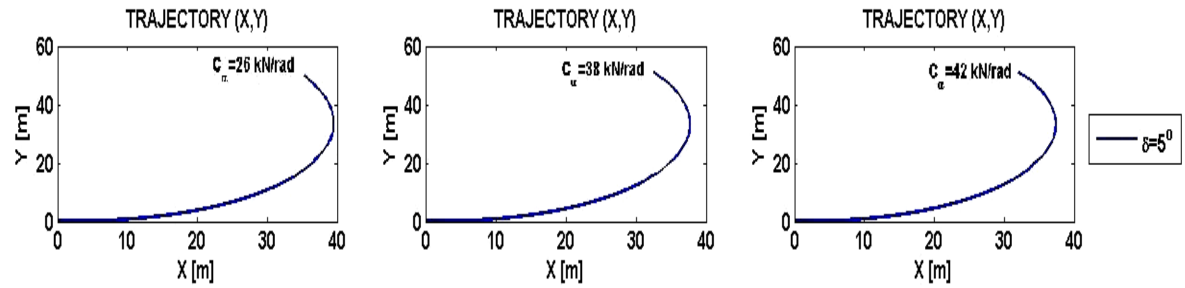

Fig. 7. Vehicle trajectory estimated for speed V = 15 m/s, K > 0, pure sliding
Rys. 7. Trajektoria ruchu pojazdu dla prędkości V = 15 m/s, K > 0, czysty poślizg

4.2. Vehicles center of gravity is located near to the rear axle

When the vehicles center of gravity is located near to the rear axle, the vehicle shows oversteer (K<0), it means that the slip angle of the rear tires is greater than the slip angle of the front tires. Here the effect of vehicle speed gives a very large influence because the vehicle is oversteer and the effects are much more severe.

Fig. 8 presents the vehicle simulated trajectories for a vehicles speed of 5 m/s. The steering angle has been varied between 5° and 30° and the curves are plotted for different coefficients of tire cornering stiffness, again. The steering angles effect on the curve radius is confirmed.

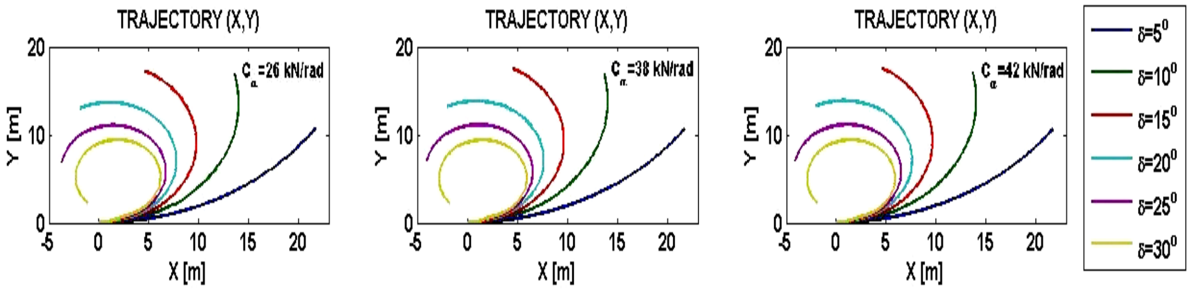

Fig. 8. Vehicle trajectory estimated for speed V = 5 m/s, K < 0, pure motion
Rys. 8. Trajektoria ruchu pojazdu dla prędkości V = 5 m/s, K < 0, czysty ruch

Fig. 9 shows the trajectories when the speed is augmented to 10 m/s. Once again, the vehicle displacement increases with the augmentation of vehicle speed and the turn may be attacked with a maximal steering angle of 10°.

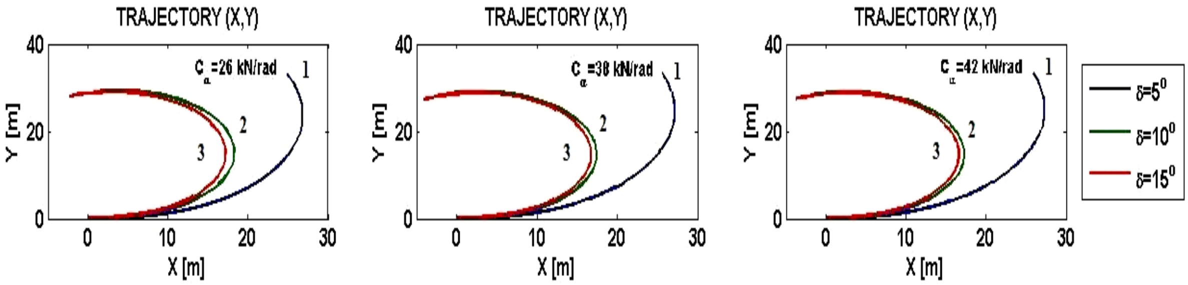

Fig. 9. Vehicle trajectory estimated for speed V = 10 m/s, K < 0
1 – motion without sliding; 2 – motion + sliding; 3 – pure sliding
Rys. 9. Trajektoria ruchu pojazdu dla prędkości V = 10 m/s, K < 0,
1 – ruch bez poślizgu; 2 – ruch + poślizg; 3 – czysty poślizg

When vehicle speed is increased to 15 m/s as can be seen from Fig. 10, the displacement are increased too and confirming the previous conclusions concerning the effect of vehicles speed. At this speed, the maximum steering angle allowed is 5°. We can conclude that the vehicle speed controls the maximum steering angle with witch a turn can be taken.

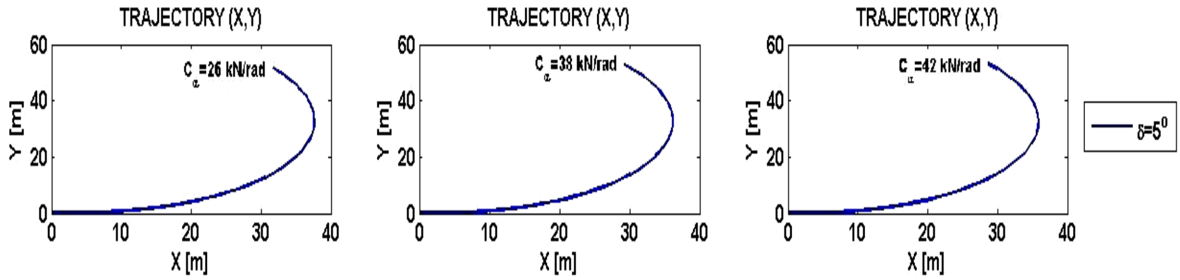

Fig. 10. Vehicle trajectory estimated for speed V = 15 m/s, K <0, pure sliding
Rys. 10. Trajektoria ruchu pojazdu dla prędkości V = 15 m/s, K < 0, czysty poślizg

4.3. The center of gravity is in the middle of the vehicle

To illustrate the influence of the tires cornering stiffness of the vehicles trajectory we fix the center of gravity in the middle of the vehicle. Some of simulations results are shown in Fig. 11. In this case, the simulation is made for vehicles speed of 10 m/s and steering angles from 5° to 10°. When the tire cornering stiffness of front axis $C_{\alpha f}$ is greater than the tire cornering stiffness of rear axis $C_{\alpha r}$ the vehicle has the tendency to entre too in the turning. This means that the slip angle of the rear tires is greater than the slip angle of the front tires.

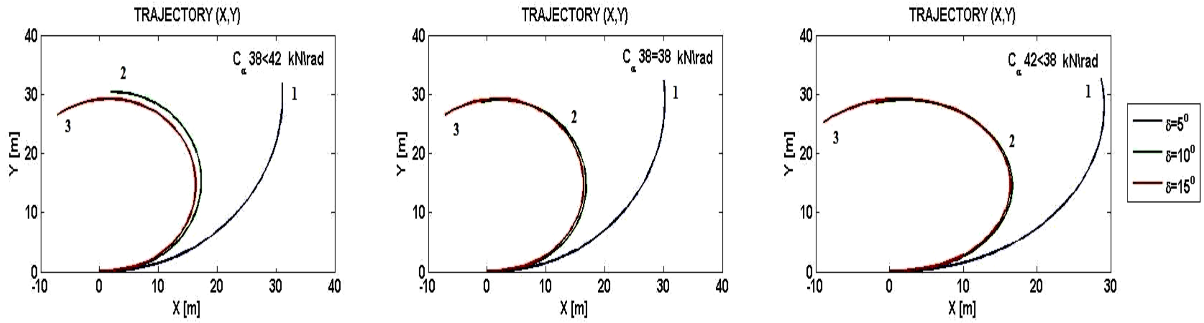

Fig. 11. Vehicle trajectory estimated for speed V = 10 m/s, K > 0, K = 0, K < 0;
1 – motion without sliding; 2 – motion + sliding; 3 – pure sliding
Rys. 11. Trajektoria ruchu pojazdu dla prędkości V = 10 m/s, K > 0, K = 0, K < 0,
1 – ruch bez poślizgu; 2 – ruch + poślizg; 3 – czysty poślizg

The conclusion that can be drawn from this study is that for speeds which exceed 60 km/h, relatively small steering angles can destabilize the vehicle in his movement on a curved path. Also, for steering angles greater than 10 the vehicle speed has to decrease under 60 km/h in order to maintain a stable course.

5. CONCLUSION

One of the objectives of this work has been to develop vehicle model that could be used to investigate the vehicle lateral behavior. The dynamics for the vehicle have been presented with assumptions. The purpose of this model is to characterize the vehicle stability during the cornering follow the dynamic state of the system and steering angle applied to the wheel. A method is proposed to determine the risk of tire lateral slipping. The choice of this indicator is based on the efforts estimation of in the contact zone tire / road. This choice is justified by the fact that the saturation of efforts in the contact zone shows that the wheel is no longer able to ensure the stability of the vehicle.

The analysis of the results shows that the vehicle speed has an important influence on the vehicle stability. This can be explained by the fact that the steer angle needed to follow a circular turn depends largely on the vehicle speed. When the vehicle speed is increased, the tire transversals reactions rise and therefore the slip angles also increase. The results show that the coefficient of under/oversteer K is related to the location of the center of mass and the stiffness value of the tire. The start of sliding isn't at the same time for each wheel.

Future work will be to improve vehicle stability by implementing load transfers during the turning maneuver [2-4].

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Nomenclature:

m – total masse of vehicle, kg,
I_z – total yaw inertia of vehicle, kgm^2,
V_x – vehicle speed around O_x, m\s ,
l_f & l_r – distance between CG and front & rear axle, m,
$C_{\alpha f}$ & $C_{\alpha r}$ – tire front & rear cornering stiffness, N/rad,
δ – steer front angle,
α_f & α_r – front & rear tire slip angle, degree,
y – displacement around lateral axis O_y, m,
ψ – yaw motion (rotation in axis O_z), degree,
F_{xn} – representing the longitudinal efforts in the contact zone tire/road, N,
F_{yn} – representing the lateral efforts in the contact zone tire/road, N,
$\ddot{y}$ – lateral acceleration, m/s^2,
$\ddot{\psi}$ – yaw acceleration, rad/s^2,
K – coefficient understeer or oversteer.