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ON SOME LINEAR AND POSITIVE OPERATORS ON SLOT

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Abstract: *The aim of this note is to discuss about the behavior and the properties of some linear and positive operators on SLOT*

§ 1. Introduction

- Let _ be a locally compact Hausdorff space and (G,V), a locally convex cone.
- Let $\psi : X \to R_+^*$, a weight on _ and $V_\psi = \left\{ v_\psi \middle| v_\psi = \frac{v}{\psi},\ \mathrm{v} \in \mathrm{V} \right\}$.
- Let consider the following set: $C_S(X;G) = \{ f : X \to G | f \text{ continuă în raport cu topologia simetrică pe G} \}$
- Endowed with the topology of uniform convergence determined by: $f \le g + \bar{v} \overset{\det}{\Leftrightarrow} f(x) \le g(x) + v,\ (\forall) x \in X,$ unde $\bar{\mathrm{v}} : \mathrm{X} \to \mathrm{G}, \bar{\mathrm{v}}(\mathrm{x}) = v,\ \mathrm{v} \in \mathrm{V}$, $(C_S(X;G), \bar{V})$ becomes a locally convex cone.
- Then, $C^\psi(X;G) = \{ f \in C_S(X;G) | (\forall) v \in V, (\exists) Y \subset X \text{ compactly a.i. } \mathrm{f} \le \mathrm{v}_\psi$ and $0 \le \mathrm{f} + \mathrm{v}_\psi\ pe\ \mathrm{X} \setminus \mathrm{Y} \}$, with the topology of uniform convergence determined by: $f \le g + v_\psi \Leftrightarrow \psi f \le \psi g + \bar{v}$, is also a locally convex cone, named Nachbin cone relative to the weight ψ.
- Let $M \subset G^*$ and $M_X^\psi = \{ \mu_X | \mu_X \in (C_S(X;G))^*, \mu \in \mathrm{M} \text{ și } \mathrm{x} \in \mathrm{X} \}$, where $\mu_X : C^\psi(X;G) \to R,\ \mu_{\mathrm{X}}(f) = \mu(f(x))$.
- Definition 1: If $G_0 \subset C^\psi(X;G)$ is a sub coneand $\mu_X \in M_X^\psi$, then:

a) $f \in C^\psi(X;G)$ is a G_0-superharmonic in $\mu_X \Leftrightarrow \Leftrightarrow \begin{cases} 1.\ \mu_{\mathrm{X}}(f) < +\infty \\ 2.\ (\forall)\Phi \in (\mathrm{C}^\psi(X;G))^*,\ \Phi \underset{\mathrm{G}_0}{\prec} \mu_X \Rightarrow \Phi(f) \le \mu_X(f) \end{cases}$

b) $f \in C^\psi(X;G)$ is a G_0- subharmonic în $\mu_X \Leftrightarrow \Leftrightarrow \begin{cases} 1.\ \mu_{\mathrm{X}}(f) < +\infty \\ 2.\ (\forall)\Phi \in (\mathrm{C}^\psi(X;G))^*,\ \mu_{\mathrm{X}} \underset{\mathrm{G}_0}{\prec} \Phi \Rightarrow \mu_X(f) \le \Phi(f) \end{cases}$

§2. The main results

Definition 2: $K \subset C^\psi(X)$ is called Korovkin system for $C^\psi(X)$ iff $K_I(G) = C^\psi(X)$, where $G = span(K)$ and $K_I(G) = \{h \in C^\psi(X) | (\forall)(T_\alpha)_\alpha$, an $u - eqtuicontinuous$ net, $T_\alpha : C^\psi(X) \to C^\psi(X)\ linear, T_\alpha(G) \to (\forall) g \in G \Rightarrow T_\alpha(h) \to h\}$ is called the Korovkin cone associated to G.

- The next result gives a characterization of the Korovkin cone associated to a subspace G of $C^\psi(X)$.
- Proposition 3: Let $G \subset C^\psi(X)$, a subspace. Then the followings are equivalent:

1. $f \in K_I(G)$;
2. $f(x) = \sup\limits_{\varepsilon > 0} \inf \left\{ \mathrm{g}(\mathrm{x}) \middle| \underset{(*)}{\mathrm{g}} \in \mathrm{G},\ \mathrm{f} \le \mathrm{g} + \varepsilon_\psi \right\} = \inf\limits_{\varepsilon > 0} \sup \left\{ \underset{(**)}{\mathrm{g}(\mathrm{x})} \middle| g \in G, \mathrm{g} \le \mathrm{f} + \varepsilon_\psi \right\}$, $(\forall) \mathrm{x} \in \mathrm{X}$.

- Note 4:
  $(*) \Leftrightarrow f(x) = \hat{f}(\varepsilon_X) \Leftrightarrow f \in Sup_G(\varepsilon_X)$.
  $(**) \Leftrightarrow f(x) = \breve{f}(\varepsilon_X) \Leftrightarrow f \in Sub_G(\varepsilon_X)$.
- Definition 5: $S \subset C^\psi(X)$ is called Korovkin $C^\psi(X)$ iff $(\forall) \mathrm{f} \in \mathrm{C}^\psi(X)_+,\ \mathrm{f} \in \mathrm{Sup}_{\mathrm{G}_0}(M_X^\psi)$, where G_0 is a sub cone generated by S and $M = \bar{R}^*$.
- Examples 6:

1. $X = [0,1]$ and $\psi = 1 \Rightarrow C^\psi(X) = C[0,1]$.

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$S=\{1,-x,x^2\}$is a Korovkin system$^+$ for $C[0,1]$, because thesub cone generated by S, G_0 contains all the positive constants and all functions, $f(x)=(x-x_0)^2,\ x_0\in[0,1]$.

2. $X\in R,\ \psi=1\Rightarrow C^{\psi}(X)=C_0(R)$.

$S=\left\{e^{-x^2},-xe^{-x^2},x^2e^{-x^2}\right\}$ is a Korovkin system$^+$ for $C_0(R)$.

The following results give characterizations for Korovkin systems and Korovkin system$^+$ for $C^{\psi}(X)$.

- Proposition 7: Let X, be a locally compact Hausdorff space; ψ, a weight on X and $G\subset C^{\psi}(X)$, a subspace. Then, FAE:

1. G is a Korovkin system for $C^{\psi}(X)$.
2. a) $(\exists)k\in G,\ k(x)\neq 0$;

b) $(\forall)\varepsilon>0,\ (\forall)K\subset X$ compact, $(\forall)x\in X$ so that $x\notin K,\ (\exists)k\in G,\ (\exists)u\in C^{\psi}(X)_+$ so that $\|u\|_{\psi}\le\varepsilon,\ 0\le h+u,\ 1\le h+u$ by $K',\ h(x)+u(x)<\varepsilon$ (where $\|\cdot\|_{\psi}:\|f\|_{\psi}\overset{\Delta}{=}\sup_{x\in X}\psi(x)|f(x)|$).

- Proposition 8: Let $S\subset C^{\psi}(X)$. FAE:

1. S is a Korovkin sistem$^+$ for $C^{\psi}(X)$.
2. $(\forall)x\in X,\ (\forall)\mu\in M_b^+(X):\psi\mu(g)\le g(x),\ (\forall)g\in S\Rightarrow(\exists)\lambda\in[0,1]$ so that $\psi\mu=\lambda\varepsilon_X$.
3. $(\forall)x\in X,\ f(x)=\sup_{\varepsilon>0}\inf\{g(x)\mid g\in G_0, f\le g+\varepsilon_{\psi}\},\ (\forall)f\in\left(C^{\psi}(X)\right)_+$.

- Note 9:

1) If we have S a Korovkin sistem$^+$ for $C^{\psi}(X)$ and is S contains $g_0, g_0<0$, then it is a Korovkin system for $C^{\psi}(X)$.

The result form above gives us the possibility to obtain some results for $\left(C^{\psi}(X;G),V_{\psi}\right)$'

- Proposition 10:

Let (G,V), be a locally convex cone and G be a linear space, G_0, a sub cone of $C^{\psi}(X;G)$, (X, locally compact Hausdorff space and ψa weight on X) and S, a Korovkin system$^+$ for $C^{\psi}(X)$.
Iff: (i) $(\forall)a\in G,(\forall)v\in V,(\exists)0\le e\in G$ so that $a\le e+v$ and $\{g\cdot e|g\in S\subset G_0\}$
(ii) $(\forall)a\in G,(\exists)p\in C^{\psi}(X)_+$so that $(p\cdot a)\in G_0$, then, we hawe: $C^{\psi}(X;G)\equiv Sup_{G_0}\left(\left(G^*\right)_X^{\psi}\right)$.

- Corollary 11: If X is a compact space and $B=\bar{S}(0;1)$ in R^n and S is a Korovkin system of positive functions for $C(X)$ then$\tilde{S}=\{f\cdot b|f\in S\cup\{C|C\in Conv(R^n)\}\}$ is a Korovkin system for $C(X;CConv(R^n))$.

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