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ROBUST MIXED H_2/H_∞ ACTIVE VIBRATION CONTROLLER IN ATTENUATION OF SMART BEAM

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Abstract. *The lack of robustness of the mechanical systems due to the unmodeled dynamics and the external disturbances withholds the performance and optimality of the structures. In this paper, this deficiency is obviated in order to reach the desired robust stability and performance on smart structures. For this purpose a multi-objective robust control strategy is proposed for vibration suppression of a clamped-free smart beam with piezoelectric actuator and vibrometer sensor in an LMI framework which is capable of handling weighted exogenous input signals and provides desired pole placement and robust performance at the same time. An accurate model of a homogeneous beam is derived by means of the finite element modal analysis. Then a low order modal system is considered as the nominal model and remaining modes are left as the multiplicative unstructured uncertainty. Next, a robust controller with a regional pole placement constraint is designed based on the augmented plant composed of the nominal model and its accompanied uncertainty by solving a convex optimization problem. Finally, the robustness of the uncertain closed-loop model and the effect of performance index weights on the system output are investigated both in simulation and practice.*

Key Words: *Piezoelectric, Vibration Suppression, Robust Control, Smart Beam, Finite Element Method*

1. INTRODUCTION

The concept of adaptive materials has changed the possibilities for structure design, particularly self-diagnosis and self-controlled arrangements, namely smart structures. This perception is achieved in practice by introduction of multifunctional material based transducers which allows the structure to be sensitive towards the environmental stimuli. Adaptive structures play a crucial role in challenging areas of applied science where high quality performance in extreme environments is an urgent requirement. An active structure

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contains elements such as sensors and actuators, which delivers data in forms of the states of the system and will affect the passive response of the structure.

The evolution of mechanical and aeronautical structures requires them to be lighter and at the same time controllable. Overcoming the defect of these systems, specifically their sensitivity to unwanted disturbances, has attracted many researchers over the past couple of decades in the fields of structural vibration analysis, damage detection, vibration control and noise control [1, 2].

Of various suggested methods of dynamics control, the use of active control techniques in vibration suppression of the light structures is proven to be more effective, where the additional masses of stiffeners or dampers should be avoided. Active techniques are also more suitable in the cases where the disturbance to be cancelled or the properties of the controlled system vary with time [3].

Piezoelectric actuators are broadly employed in many practical applications due to their capability of coupling strain and electric field. In order to control structural vibrations, piezoelectric actuators can be easily bonded on the vibrating structures [4].

In terms of the dynamic performance, the high-efficient dynamic modeling and appropriate control law design are the two key points. For the purposes of dynamical modelling, the finite element method has recognized to be one of the most popular methods. Reviews such as the one presented by Benjeddou [5] provide a condensed overview of the development in the field of the Finite Element Modelling (FEM) modelling of active structures. The development of the FEM tools has proceeded at the same rapid pace in the next decade, followed by the development of active structural control techniques, as reported in the overview by Le Gao *et al.* [6]. Various types of controller design methods such as velocity feedback control [7], high gain feedback regulator [8], linear quadratic regulator (LQR) approach [9], H_2 control [10], H_∞ control [11] have been studied by former scientists. In addition, some others evaluate the performance of control algorithms in vibration suppression of flexible structure experimentally [12]. The authors of this paper have made a contribution to the research field of piezoelectric adaptive structures by dedicating their work to the development of necessary finite elements for piezoelectric coupled-field problems [13], demonstrating advantages of the FEM approach over other methods [14], investigating different aspects of modelling active structures [15], implementing developed tools into commercially available software packages [16], dealing with control techniques for adaptive structures [17], etc.

In this work, an accurate model of a piezolaminated cantilever beam is derived by means of the finite element modal analysis. The derived formulation provides the state space model relating the actuator voltage to sensor voltage. The obtained model is capable of offering a finite order model that shall be considered as nominal system while the remaining high order states are left as multiplicative unstructured uncertainty of modeling. Then, a multi-objective robust controller is designed based on the augmented plant composed of the nominal model and its accompanied uncertainty. In addition, a regional pole placement constraint is included within the Linear Matrix Inequality (LMI) framework to improve closed-loop transient performance. The rest of the paper has the following order. In section 2 the configuration of the experimental setup is described. This will be used to verify the performance of the regulated controller in real time implementation. In section 3 the finite element based modal analysis is performed in order to calculate the eigen frequencies and mode shapes of the coupled electro-elastic system. Then in section 4 the aforementioned robust controller will be introduced and finally the performance of the closed loop system will be evaluated in the next section.

2. EXPERIMENTAL SETUP

The structure of experimental smart beam is presented in Fig. 1. The piezo-laminated beam consists of a cantilever aluminum beam with Young's modulus 70 GPa and density 2.7 g/cm^3. In addition since the ultimate goal is to suppress the vibration two piezoelectric actuators (DuraActTM P-876.A15) are attached to the beam at the same side. (see Fig. 1)

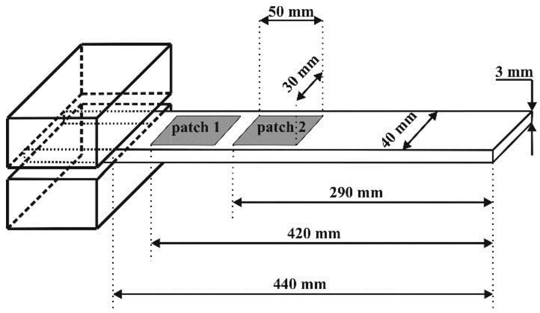

Fig. 1 Geometry of the smart beam

The feedback channel entails the measurement signal namely, the signal measured by a scanning digital laser Doppler vibrometer VH-1000-D. This will provide the measurement of the velocity of the lateral vibration at a point, near the free end of the beam. Schematic configuration of closed-loop vibration control system is presented in Fig. 2.

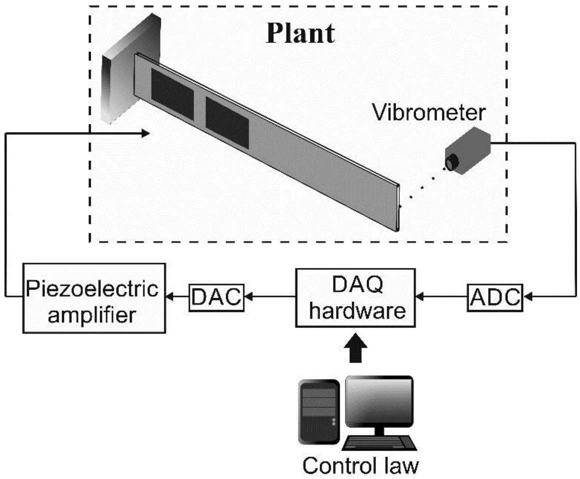

Fig. 2 Sketch of experimental setup

It is worthwhile mentioning that the plant has two inputs: the control input which acts on the actuator piezo-patch and the disturbance signal which excites the system through the disturbance channel. Moreover, the only output of the system is recorded using the previously mentioned vibro-meter.

For implementing the controller in real time, dSPACE digital data acquisition and real-time control system with DS1005 digital signal process board are used. Connection of the digital data acquisition system with the actuators and the computer is provided by an ADC Board DS2004 (Analog to Digital converter) and a DAC Board DS2102 (Digital to Analog converter). To increase the working range of the DAC boards the control input is amplified (PI E-500). The control law, for the active vibration control of the smart beam, is then implemented on MATLAB platform. Finally, the control system is downloaded to the dSPACE digital data acquisition and real-time control system.

3. System Modeling

One should notice that the torsional modes are not considered in controller design because they are not relevant for the bending vibration. It should be mentioned that due to the previous research the dominant mode shape of the flexible beam is the first mode shape [18].

The dynamics of the actuation is addressed by means of the FEM analysis in coupled electro-mechanical domain. This leads to an ordinary differential equation which then will be converted to a linear time invariant (LTI) system since it is a convenient model for the work in the computer aided control system design. It is assumed that the displacements are small enough so that the dynamics of the system remains in linear piezo-elasticity. The finite element method presents the dynamic equation of motion in matrix representation as:

$$M\ddot{q} + C\dot{q} + Kq = F, \tag{1}$$

with M, C and K being the mass, damping and stiffness matrices. Also, q represents the nodal states of displacement $(u_i^T, i = 1,2,...)$ and electric potential $(\varphi_j^T, j = 1,2,...)$:

$$q = [u_1^T \quad \varphi_1 \quad \cdots \quad u_n^T \quad \varphi_n]^T \tag{2}$$

F shows the applied excitation which contains the external forces that is assumed to be zero because the external input disturbance is expected to affect the system from the same channel as the control input. The vector of control forces is therefore:

$$F = Bu(t), \tag{3}$$

B matrix describes the position of the generalized control effort in the finite element structure with u consisting of all modal inputs. For the control design purposes the measurement signal is represented in terms of system states and plant inputs as:

$$y = C_{0q}q + C_{0v}\dot{q}, \tag{4}$$

In which C_{0q} and C_{0v} are the output displacement and output velocity matrices, respectively; they are calculated using the FE procedure and choosing the appropriate sensor location. By applying the conventional harmonic solution of $q = \phi e^{i\omega t}$ one can easily find natural frequencies ω_j and mode shapes ϕ_j (j = 1,2,...,n) solving the determinant of homogenous system of algebraic equations. The solution can be represented in matrix form as:

$$\Omega = \begin{bmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_n \end{bmatrix},$$
$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{21} & \cdots & \phi_{n1} \\ \phi_{12} & \phi_{21} & \cdots & \phi_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1n} & \phi_{2n} & \cdots & \phi_{nn} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 \end{bmatrix}, \quad (5)$$

The nodal model representation (1) can be transformed to modal coordinates by applying the following conversion:

$$q = \Phi q_m, \quad (6)$$

where q_m is the vector of generalized modal displacement. Using the symmetricity of the mass and stiffness matrices one can easily obtain the transformed matrices as [19] :

$$\Phi^T M \Phi = M_m = diag(m_j),$$
$$\Phi^T K \Phi = K_m = diag(m_j \omega_j^2), \quad (7)$$

Similarly, by using the same transformation and the orthogonality of mode shape one can find the modal damping matrices under the assumption of the proportional damping to be:

$$C = \alpha M + \beta K, \quad (8)$$

By selecting the state vector to be $x = \begin{bmatrix} \Omega q_m & \dot{q}_m \end{bmatrix}^T$ the state space model will be:

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du, \quad (9)$$

where:

$$A = \begin{bmatrix} 0 & \Omega \\ -\Omega & 2Z\Omega \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ B_m \end{bmatrix}, \quad C = \begin{bmatrix} C_{mq} & C_{mv} \end{bmatrix}, \quad D = 0, \quad (10)$$

while $\Omega^2 = M_m^{-1} K_m$ and $Z = diag(\zeta_j)$ with ζ_j being the damping ratio of jth mode, $B_m = \Phi^T B$, $C_{mq} = C_{0q}\Phi$, $C_{mv} = C_{0v}\Phi$.

4. CONTROLLER DESIGN

The closed-loop system by considering multiplicative uncertainty will be as shown in Fig. 3.

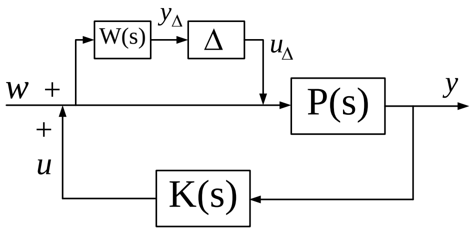

Fig. 3 Closed-loop system with multiplicative uncertainty

where $P(s)$ is the nominal plant, $K(s)$ is desired controller, Δ is a stable transfer function, where $\|\Delta\|_\infty < 1$ and $W(s)$ is the weighting function for multiplicative uncertainty, that satisfies following equation:

$$\frac{P_{real}(s)}{P(s)} - 1 < W(s) \tag{11}$$

where $P_{real}(s)$ is the transfer function of the real system, by considering all or some of the higher modes. Note that, reduction of the order of the nominal plant will hold the designed controller's order in a lower value, but the price will be reduced performance. For robust stability, one should have $\|T_{y_\Delta u_\Delta}\|_\infty < 1$, where $T_{y_\Delta u_\Delta}$ is the transfer function from u_Δ to y_Δ when Δ is removed [20]. However, to handle the stochastic aspects such as measurement noise and random disturbance, despite robust H_∞, only H_2 performance is functional. And finally, for appropriate disturbance rejection and control effort the conventional optimization problem is to minimize $\|y^TQy + u^TRu\|_\infty$, where Q and R are two weighting functions that indicate the relative importance of disturbance rejection and control effort, respectively. For minimizing performance index $\|y^TQy + u^TRu\|_\infty$, we should minimize $\|T_{[y\ u]^T w}\|_2$ instead, where w is a bounded H_2 norm exogenous disturbance. This will be addressed later. The transient response of a linear system is well known to be related to the locations of its closed-loop poles. This is the next issue that has to be addressed.

Since $T_{y_\Delta u_\Delta}$ is equivalent to $T_{uw}W(s)$ [21], the above system can be represented in Fig. 4 with all of the constraints that have to be satisfied in order to reach the predefined H_2 / H_∞ performance and optimal control effort.

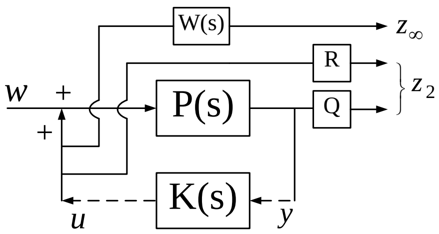

Fig. 4 Desired input and outputs of augmented plant

Now assume that a state space representation of the open-loop system in Fig. 4 (by ignoring K(s)) is:

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z_\infty = C_\infty x + D_{\infty 1} w + D_{\infty 2} u \\ z_2 = C_2 x + D_{21} w + D_{22} u \\ y = C_y x + D_{y1} w \end{cases} \tag{12}$$

where u and w are control input and disturbance, respectively. Our objective is to design a dynamic output-feedback controller with the state space realization:

$$\begin{cases} \dot{\varsigma} = A_K \varsigma + B_K y \\ u = C_K \varsigma + D_K y \end{cases} \tag{13}$$

where ς is the state variable of the controller. Therefore, the corresponding closed-loop system containing the performance and robustness channels will be:

$$\begin{cases} \dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} w \\ z_\infty = C_{cl1} x_{cl} + D_{cl1} w \\ z_2 = C_{cl2} x_{cl} + D_{cl1} w \end{cases} \tag{14}$$

Our three design objectives can be expressed as follows:

***H*$_\infty$ Performance**: the closed-loop RMS gain from w to z does not exceed γ if and only if there exists a symmetric matrix X_∞ such that [22] :

$$\begin{pmatrix} A_{cl} X_\infty + X_\infty A_{cl}^T & B_{cl} & X_\infty C_{cl1}^T \\ B_{cl}^T & -I & D_{cl1}^T \\ C_{cl1} X_\infty & D_{cl1} & -\gamma^2 I \end{pmatrix} < 0 \qquad X_\infty > 0 \tag{15}$$

This LMI constraint is used to minimize $\|T_{z_\infty w}\|_\infty$ (closed-loop H_∞ gain from disturbance to z_∞ output channel).

***H*$_2$ Performance**: the H_2 norm of the closed-loop transfer function from w to z_2 does not exceed v if and only if $D_{cl2} = 0$ and there exist two symmetric matrices X_2 and Q such that [23]:

$$\begin{pmatrix} A_{cl} X_2 + X_2 A_{cl}^T & B_{cl} \\ B_{cl}^T & -I \end{pmatrix} < 0 \qquad \begin{pmatrix} Q & C_{cl2} X_2 \\ X_2 C_{cl2}^T & X_2 \end{pmatrix} > 0 \qquad Trace(Q) < \upsilon^2 \tag{16}$$

Pole placement: the closed-loop poles lie in the LMI region:

$$D = \{ z \in C : L + Mz + M^T \bar{z} < 0 \} \tag{17}$$

with $L = L^T = \{\lambda_{ij}\}_{1\le i,\, j\le m}$ and $M = [\mu_{ij}]_{1\le i,\, j\le m}$ if and only if there exists a symmetric matrix X_{pol} satisfying:

$$\begin{aligned} &\left[\lambda_{ij} X_{pol} + \mu_{ij} A_{cl} X_{pol} + \mu_{ji} X_{pol} A_{cl}^T\right]_{1\le i,j\le m} < 0 \\ &X_{pol} > 0 \end{aligned} \tag{18}$$

For tractability in the LMI framework, we must seek a single Lyapunov matrix:

$$X := X_\infty = X_2 = X_{pol} \tag{19}$$

that enforces all three sets of constraints. Factorizing X as:

$$X = X_1 X_2^{-1}, X_1 := \begin{pmatrix} R & I \\ M^T & 0 \end{pmatrix}, X_2 := \begin{pmatrix} 0 & S \\ I & N^T \end{pmatrix} \tag{20}$$

and, introducing the change of controller variables [25]:

$$\begin{cases} B_K := NB_K + SB_2 D_K \\ C_K := C_K M^T + D_K C_y R \\ A_K = NA_K M^T + NB_K C_y R + SB_2 C_K M^T + S(A + B_2 D_K C_y)R \end{cases} \tag{21}$$

the inequality constraints on X are readily turned into LMI constraints in the variables R, S, Q, A_K, B_K, C_K and D_K [22], [24]. This leads to the suboptimal LMI formulation of our multi-objective synthesis problem, which is defined as:

Minimize $\alpha.\gamma^2 + \beta.trace(Q)$ over variables R, S, Q, A_K, B_K, C_K, D_K and γ^2 satisfying [26]:

$$\begin{aligned} &\begin{pmatrix} AR + RA^T + B_2 C_K + C_K^T B_2^T & A_K^T + A + B_2 D_K C_y & B_1 + B_2 D_K D_{y1} & H \\ H & A^T S + SA + B_K C_y + C_y^T B_K^T & SB_1 + B_K D_{y1} & H \\ H & H & -I & H \\ C_\infty R + D_{\infty 2} C_K & C_\infty + D_{\infty 2} D_K C_y & D_{\infty 1} + D_{\infty 2} D_K D_{y1} & -\gamma^2 I \end{pmatrix} < 0 \\ &\begin{pmatrix} Q & C_2 R + D_{22} C_K & C_2 + D_{22} D_K C_y \\ H & R & I \\ H & I & S \end{pmatrix} > 0 \\ &\left[\lambda_{ij} \begin{pmatrix} R & I \\ I & S \end{pmatrix} + \mu_{ij} \begin{pmatrix} AR + B_2 C_K & A + B_2 D_K C_y \\ A_K & SA + B_K C_y \end{pmatrix}\right. \\ &\left. + \mu_{ij} \begin{pmatrix} RA^T + C_K^T B_2^T & A_K^T \\ (A + B_2 D_K C_y)^T & A^T S + C_y^T B_K^T \end{pmatrix}\right]_{1\le i,j\le m} < 0 \\ &Trace(Q) < \upsilon_0^2 \\ &\gamma^2 < \gamma_0^2 \\ &D_{21} + D_{22} D_K D_{y1} = 0 \end{aligned} \tag{22}$$

Given optimal solutions γ^*, Q^* of this LMI problem, the closed-loop H_∞ and H_2 performances are bounded by:

$$\|T_\infty\|_\infty \leq \gamma^*, \|T_2\|_2 \leq \sqrt{trace(Q^*)} \tag{23}$$

5. Case Study and Discussion

This section presents the vibration damping quality of the proposed method both in simulation and experiment. For implementation of the controller, a structure consisting of an aluminum clamped beam with two piezoelectric patches is used. The patches are attached on the same side of the beam (see Fig. 1). The model of the structure for control design purposes is obtained based on the method described before. Since the actuator placement plays an important role in vibration control performance the optimal placement of the actuator is addressed based on the mixed H_2 / H_∞ method that is described by Nestorović and Trajkov [27].

Firstly, two shape numbers of the clamped beam are considered as nominal model and higher order modes remain as unstructured uncertainty. In addition, a weighting function for multiplicative unstructured uncertainty that satisfies $P_{real}(s)/P(s) = W_{unc}(s) + 1$ is considered. With $P_{real}(s)$, $P(s)$ and $W_{unc}(s)$ being the full order transfer function of the system, nominal transfer function and frequency based appropriate weighting function representing the unstructured uncertainty, respectively. Fig. 5 shows the weighting functions that are considered for modeling unstructured uncertainty, disturbance and H_2 / H_∞ performance.

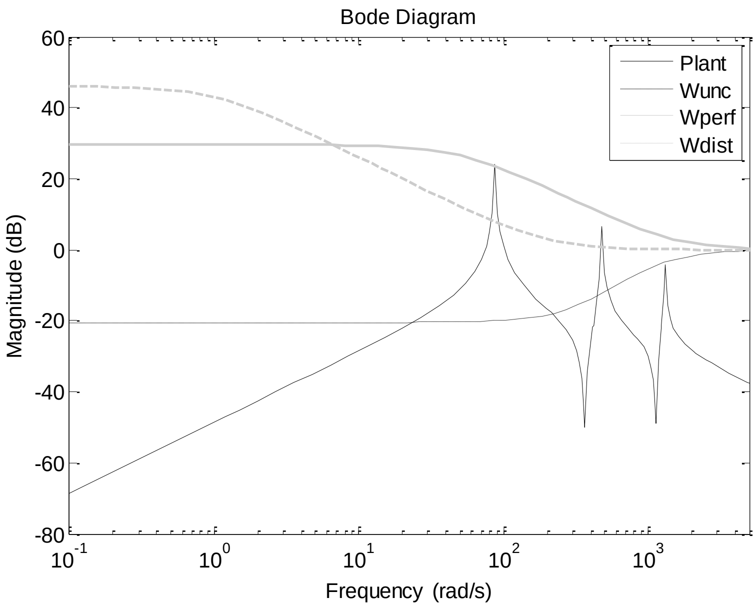

Fig. 5 The relation of weighting function to real system

The desired controller design is carried out by solving convex optimization problem that is formulated in Eq. (22). For obtaining an appropriate H_∞ performance, the magnitude of $|\gamma|$ should be under unit and for increasing the performance one should minimize H_2 norm from exogenous disturbance to performance index. The relative magnitudes of Q and R determine the relative importance of disturbance rejection (vibration suppression) to control effort (actuator saturation). To improve transient performance, as mentioned before, one shall resort to an additional regional pole placement constraint in order to achieve a better closed-loop damping across the uncertainty range. This places the closed-loop poles into a suitable sub-region of the left-half plane that can be expressed as an additional LMI constraint. A typical example of LMI region that is commonly treated in multi-objective synthesis that guarantees H_2 stability is the conic sector centered at the origin and with inner angle $2\theta = 2\cos^{-1}(\zeta)$ [22]. In this work, the closed-loop damping coefficient is assumed to be $\zeta = 0.1$.

The controller is designed by setting $Q = 10$. Comparison of the impulse response of the closed-loop system with this controller and the impulse response of the open-loop system (Fig. 6) shows the performance of the controller in suppressing the vibration.

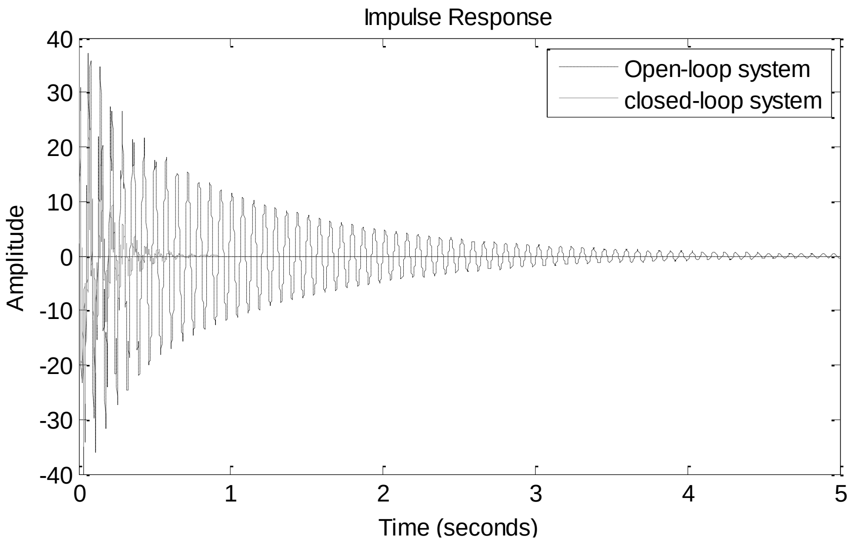

Fig. 6 Impulse response of the open-loop and closed-loop system

Actuator voltage of this controller during the impulse response is plotted in Fig. 7.

As one can see, the maximum amplitude of the actuator voltage is under about 20 Volts.

In addition, comparison of frequency responses of closed-loop system and open-loop system is shown in Fig. 8 which shows that the amplitude is reduced in the nominal model natural frequencies.

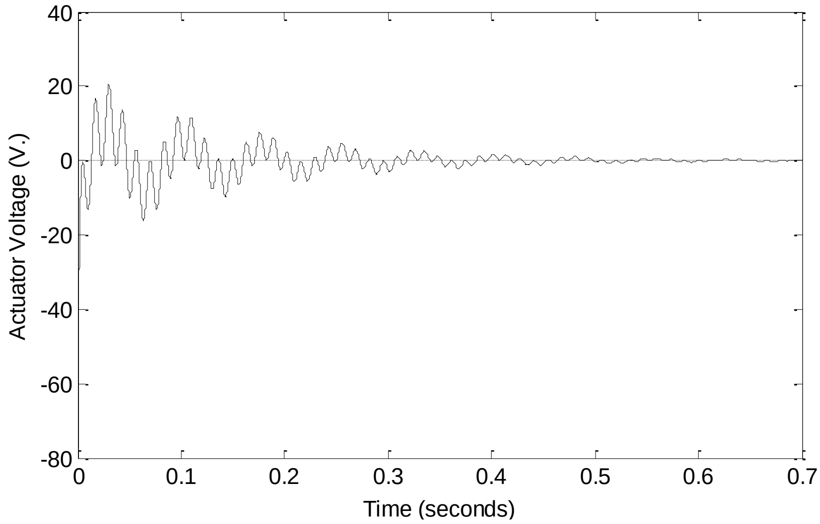

Fig. 7 Input control for impulse response of the closed-loop system

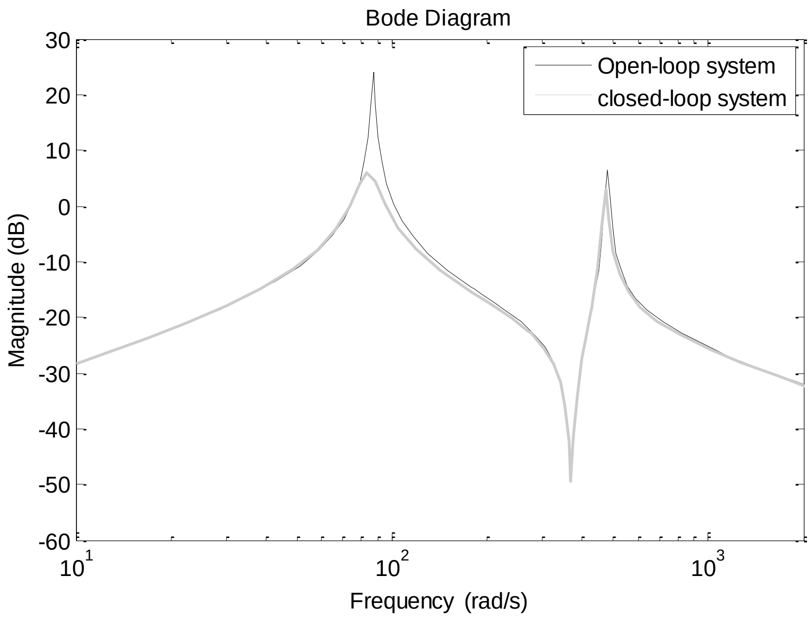

Fig. 8 Bode diagram of closed-loop system and open-loop system

For investigation of the robust performance of the uncertain closed-loop system with the designed controller by structured singular value analysis Fig. 9 is obtained. This plot shows upper/lower bounds of uncertain closed-loop structured singular values in frequency domain.

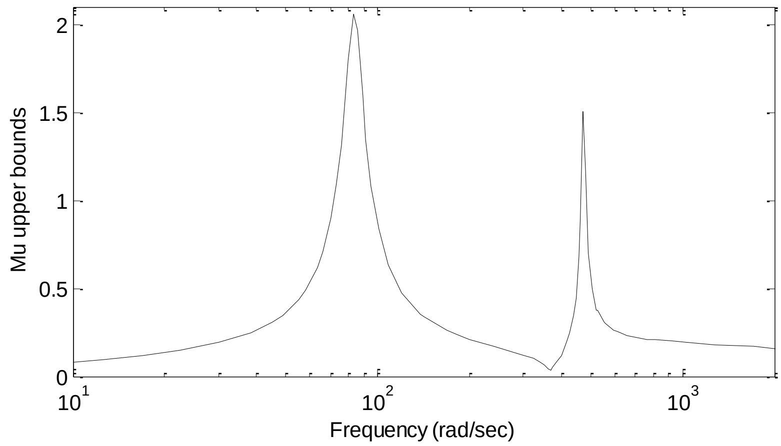

Fig. 9 μ bounds of uncertain closed-loop system

The performance margin is the reciprocal of the structured singular value and if the magnitude of the structured singular value were under unit, in entire frequency range, the system would have robust performance. Therefore, upper bounds from structured singular value become lower bounds on the performance margin and critical frequency associated with the upper bound of the structured singular value, here is $\omega_{critical} = 87$rad/sec. In addition, the system can tolerate up to 557% of the modeled uncertainty without losing desired performance.

Through the experimental implementation of the control law on the smart structure the possibility of the successful vibration control performance is evaluated on full order system. The vibration amplitude suppression will be demonstrated under the harmonic excitation of the piezo-beam through the control channel and the results obtained using hardware in loop system with dSPACE RTI platform. Experimental excitation is considered to be harmonic $F(t) = A\sin(2\pi f_j t)$, with f_j being the first bending resonant frequency of the clamped piezo-beam. The closed-loop system is implemented on the real time data acquisition platform of the dSPACE with sampling frequency of 10 kHz. The predefined task of the controller is to guarantee the robust stability and performance in conjugation with real time vibration amplitude suppression in frequency ranges close to resonance eigenvalues. Therefore, investigations are carried out in time domain by means of the experimental setup shown in Fig. 10.

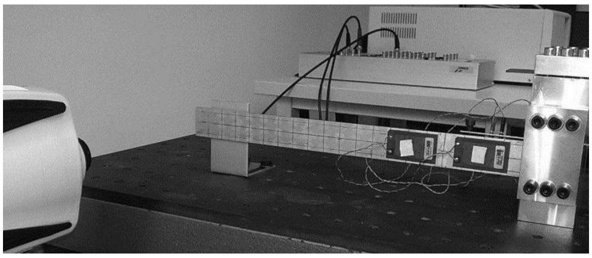

Fig. 10 Experimental rig of the closed-loop system

For the analysis in time domain the sinusoidal excitation signal is generated in Simulink and lead out through the dSPACE DAC. The frequency of the excitation is adjusted experimentally to reach the highest vibration amplitude representing the actual eigenfrequency. The response of the system for controlled and uncontrolled case is shown in Fig. 11 based on the measurement signal generated by Doppler vibro-meter.

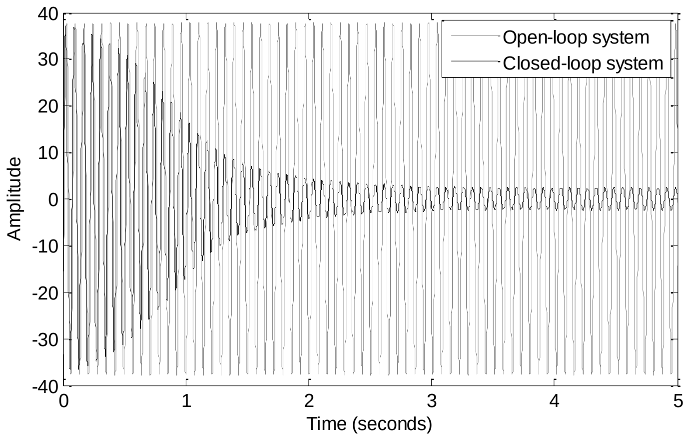

Fig. 11 Experimental comparison of velocity response

This diagram shows the velocity magnitudes of the beam measured by dSPACE ADC board. In addition the corresponding control effort generated for piezo-actuator patches by the dSPACE DAC board is shown in Fig. 12.

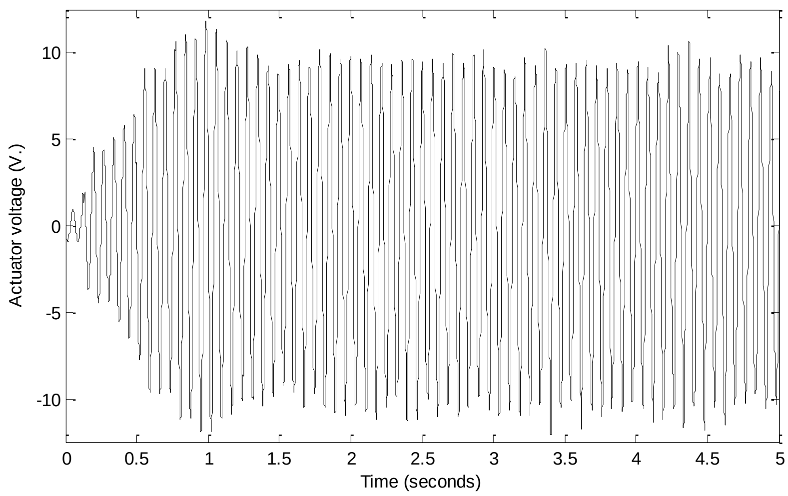

Fig. 11 Control effort of the piezo-patch actuator

The experimental results show the obvious performance of the robust control system in attenuating the vibration amplitude.

6. CONCLUSION

Vibration control of a clamped-free beam with piezoelectric actuator and vibrometer has been achieved by using a multi-objective robust output feedback control strategy with regional pole placement constraints in an LMI framework, based on H_2 / H_∞ weighting objective functions. The robustness of the closed loop smart beam with respect to external input disturbance increased to 557% of the modeled uncertainty. The regional pole placement constraints guaranteed the improvement of the transient response of the closed-loop system and the optimality of the control effort is achieved by satisfying the appropriate H_2 LMI based performance index. All these constraints are presented in a LMI formulation, which is solvable in the MATLAB environment. Finally, the performance of the approach is proven to be effective and robust on the experimental set up where the higher order modes take effect in the dynamics of the smart beam.

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ROBUSTNO H_2/H_∞ UPRAVLJANJE VIBRACIJAMA U CILJU PRIGUŠENJA AKTIVNE KONZOLE

Ograničena robustnost u odnosu na neprecizno modeliranje dinamike i spoljašnjih poremećaja mehaničkih sistema značajno utiče na njihove performanse i optimalna svojstva aktivnih struktura. U ovom radu prikazujemo na koji način se ovaj nedostatak može prevazići u cilju postizanja željenih performansi aktivnih struktura. U tom cilju predložen je robustni upravljački zakon za redukciju vibracija aktivne konzole sa piezoelektričnim aktuatorom i laserskim senzorom u LMI okruženju, koji se uspešno može primenjivati u prisustvu eksternih ulaza sa težinskim funkcijama,a koji obezbeđuje željeno podešavanje polova i robustne performanse u isto vreme. Model homogene konzole dobijen je primenom modalne analize metodom konačnih elemenata. Zatim je modalni sistem redukovanog reda posmatran kao nominalni model, dok su modeli višeg reda su razmatrani kao mulitiplikativna nesigurnost. Potom je na osnovu proširenog modela, koji sačinjava nominalni model sa pratećom neizvesnošću, projektovan robustni kontroler sa lokalnim podešavanjem polova, rešavanjem konveksnog optimizacionog problema. Na kraju je simulacijom i eksperimantalno analizirana robustnost nizvesnog modela zatvorenog kola, kao i uticaj težinskog indeksa performansi na izlaz sistema.

Ključne reči: *piezoelektrični materijali, redukcija vibracija, robustno upravljanje, aktivna konozla, metod konačnih elemenata.*