Australian Journal of Basic and Applied Sciences, 9(2) February 2015, Pages: 277-291

AENSI Journals

Australian Journal of Basic and Applied Sciences

(ISSN 1991-8178)

Journal home page: www.ajbasweb.com

Reliability Based Design of Pressure Vessels Containing Axial Through Crack Using Probabilistic Fracture Mechanics

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ARTICLE INFO

Article history:
Received 25 October 2014
Received in revised form 26 November 2014
Accepted 29 December 2014
Available online 15 January 2015

Keywords:
Probabilistic fracture mechanics; reliability, safety factor, axial through cracked pressure vessel; failure pressure; modified two parameter fracture criterion, operating pressure.

ABSTRACT

Background: Deterministic Fracture Mechanics analysis does not address the uncertainties involved in material properties, loads, location and size of the flaws etc. However, in real-life situations such uncertainties can affect significantly the conclusions drawn out of a deterministic analysis. The principles of Probabilistic Fracture Mechanics (PFM) are used to ascertain the effects of such uncertainties. PFM is becoming increasingly popular for realistic evaluation of fracture response and predicting the reliability of cracked structures. **Objective:** A probabilistic failure assessment methodology has been proposed in this study to compute the reliability-based safety factor for the prediction of safe operating pressure for the pressure vessels. Modified two parameter fracture criterion has been used to predict the failure of cracked pressure vessels. In this study, the scatter in the crack geometry, fracture parameter, material properties and geometric parameters of the pressure vessel are considered. Monte- Carlo simulation method is used to perform the PFM analysis. **Results:** Based on the study carried out using PFM, the upper and lower limits of failure pressure of pressure vessel at various confidence levels are proposed. Using Stress-Strength interference theory, reliability based safety factor has also been proposed. **Conclusion:** The safe operating pressure for pressure vessels containing axial through crack is computed for the specified reliability levels. The proposed work will help the design engineer to decide the operating pressure of the cracked structure for the specified safety and reliability.

To Cite This Article: A. Krishnaveni, D. Jebakani, K. Jeyakumar, T. Christopher, Reliability Based Design of Pressure Vessels Containing Axial Through Crack Using Probabilistic Fracture Mechanics.***Aust. J. Basic & Appl. Sci.,*** 9(2)*: 277-291, 2015*

INTRODUCTION

The mechanical integrity of pressure vessel is of great importance for both economical and safety reasons. In practice, this integrity is assured by careful attention to all aspects of design, manufacturing, installation and operation of the pressure vessel. Cracks are inherent in many components owing to the process by which they are manufactured or fabricated. The maximum pressure withstanding capacity of a cracked pressure vessel is determined by applying the principles of fracture mechanics. The crack under severe load can become unstable and thereby causing the failure of pressure vessel at cracked sections. Therefore the cracks are an important consideration for safety analysis (Rohit Rastogi *et al*., 2002). However many of the input parameters are statistically distributed. The traditional approach of safety assessment and design lies in a deterministic model. The probabilistic approach is obviously the best choice in practical applications when sufficient information on the distribution of the random variables is known (Lin *et al*., 2004).

Probabilistic Fracture Mechanics (PFM) is a rapidly developing field with numerous applications in science and engineering which blends fracture mechanics and probability theory (Rahman and Kim, 2001). PFM provides a more rational way of describing the actual behavior and reliability of structures than the traditional deterministic models (Rahman and Kim, 2001). Now-a-days many researchers are exploring applicability of the PFM approach and few of them are listed here. Rahman (1997) developed a PFM model for analyzing circumferential through-walled-cracked pipes subject to bending loads. New equations were developed to represent the functions. Both analytical and simulation methods were formulated to determine the probabilistic characteristics of J integral. Jian Ping Zhao *et al*. (1997) introduced a new concept of probabilistic failure assessment diagram for defect assessment. Genki Yagawa *et al*. (1997) established standard PFM procedures for evaluating failure probabilities of nuclear pressure vessel and pipes. Sensitivity analyses are performed to

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quantify the effects of uncertainty of flow stress, fracture toughness, fatigue crack growth rate and Copper content on failure probabilities.

Lei and Xiamen (1997) presented a probabilistic analysis of fatigue crack growth, fatigue life and reliability of elastic structural components on the basis of fracture mechanics and the theory of random process. Jin Xing (1997) developed a PFM assessment method based on the R6 procedure for the integrity of the structures. Sharif Rahman (2001) conducted studies on probabilistic analysis based on J integral estimation model that provides accurate estimates of failure probability and also shown that the uncertainty in the crack with a large coefficient of variation (COV) has a significant effect on the probability of failure. Rahman and Kim (2001) developed a probabilistic methodology for Elasto-Plastic fracture mechanics analysis of non-linear cracked structures, which is capable of predicting accurate deterministic and probabilistic characteristics of J integral. Xhou Xun and Yu Xiao-li (2006) made a reliability evaluation for diesel engine crank shaft based on 2D stress strength interference model, in which multi axial loading fatigue criteria has been employed. Rohit Rastogi*et al*. (2002) presented the determination of failure probabilities of a straight pipe with a through wall crack, in the circumferential direction subjected to in plane bending moment, using the R6 method. The probability of failure is studied by constructing Failure Assessment Diagram (FAD) using all the three options of R6 method for the variation in the applied loads.

Tronskar *et al*. (2003) studied the influence on the failure probability of modified R6 FAD and the influence of constraint correction on the combined fatigue and fracture failure probability for the vessels subjected to wave loading. The uncertainties in various internal operating loadings and external forces, including earthquake and wind, flaw sizes, material fracture toughness and flow stress are considered. Lin*et al*. (2004) proposed a probabilistic assessment methodology for in-service nuclear piping containing defects and also developed software for failure assessment based on R6 method. Albert Bagaviev and Artur Ulbrich (2004) outlined the theoretical background of life assessment with the help of PFM and have demonstrated its application for typically heavy loaded components of large steam turbines. Prem Navinand Ramachandramoorthy (2005) carried out a probabilistic study on through wall circumferentially cracked pipe, to obtain a reliableestimate of maximum moment capacity. Also, a reduction factor has been proposed for net section plastic collapse moment to predict the maximum moment capacity. Cristina Gentilini*et al*. (2005) presented a simple and reliable method for the probabilistic characterization of the linear elastic response of cracked truss and frame structures with uncertain damage.

Past literatures are focused the importance of PFM in the evaluation of the reliability of cracked structures. The failure pressure estimation of cylindrical pressure vessel having axial through crack using PFM approach is scarcely reported by the researchers. The objective of this paper is to predict reliability based safety factor for various cylindrical pressure vessels containing axial through crack subjected to internal pressure. PFM analysis is carried out using the experimental literature test data from the references (Anderson and Sullivan, 1966; Peter and Kuhn, 1957; Calfo, 1968). The uncertainty with respect to material properties viz. ultimate tensile strength and yield strength, geometric parameters viz. thickness and diameter and the crack geometry viz. crack length and crack depth and fracture parameter (K_F) has been considered. It is observed from the literature that in most of the PFM study, failure of the cracked structure is assessed by J_{1c} and K_{1c} based failure criterion (Krishnaveni and Christopher, 2011) and R6 method. In this work, the modified two parameter fracture criterion (Christopher *et al*., 2005) is used to perform the probabilistic fracture mechanics analysis. The operating pressure for various cylindrical pressure vessels is obtained using Stress-Strength interference theory for the specified reliability.

Nomenclatures:

Symbol	**-**	**Item description**
$2c$	-	Crack length (mm)
a	-	Crack depth (mm)
c	-	Half the crack length (mm)
k	-	Level of confidence (%)
K	-	Stress intensity factor
L	-	Strength random variable (MPa)
M, φ	-	Correction factor
N	-	Safety factor
r	-	Inner radius of cylinder (mm)
R	-	Reliability of the component (%)
S	-	Stress random variable (MPa)
t	-	Thickness of the cylinder (mm)
V	-	Coefficient of variation of random variable (%)
X	-	Random vector
y	-	Margin of safety
Y	-	Random variable

Z	-	Standard normal random variable
D_o	-	Outer diameter of cylinder (mm)
$E(p_{bf})$	-	Expected mean value of random variable
fx_i	-	Density function of random variable
K_{max}	-	Stress intensity factor at failure (MPa√m)
p_b	-	Failure pressure of unflawed cylindrical vessel (MPa)
p_f	-	Probability of failure (%)
p_{bf}	-	Failure pressure of flawed cylindrical vessel (MPa)
$Var(p_{bf})$	-	Variance of random variable
V_L	-	Coefficient of variation of predicted operating pressure (%)
V_S	-	Coefficient of variation of predicted failure pressure (%)
Z_0	-	Lower limit of integral Equation(5)
σ	-	Standard deviation
σ_f	-	Net section failure stress (MPa)
σ_u	-	Hoop stress at the burst level of the unflawed cylindrical shell (MPa)
σ_{ult}	-	Ultimate tensile strength (MPa)
σ_{ys}	-	Yield strength or 0.2 % proof stress (MPa)
μ	-	Mean of random variable
μ_S	-	Mean of predicted failure pressure (MPa)
μ_L	-	Mean of predicted operating pressure (MPa)
Φ	-	Cumulative distribution function

Probabilistic Failure Assessment Methodology:

In this study, a probabilistic failure assessment methodology (Krishnaveni *et al.*, 2014) has been proposed to compute the reliability based safety factor for the prediction of safe operating pressure of the pressure vessels containing axial through cracks. The flow chart (See Figure 1) depicts the procedure of proposed probabilistic failure assessment methodology. This methodology consists of three phases viz. goodness of fit test, probabilistic fracture analysis and computation of safety and reliability.

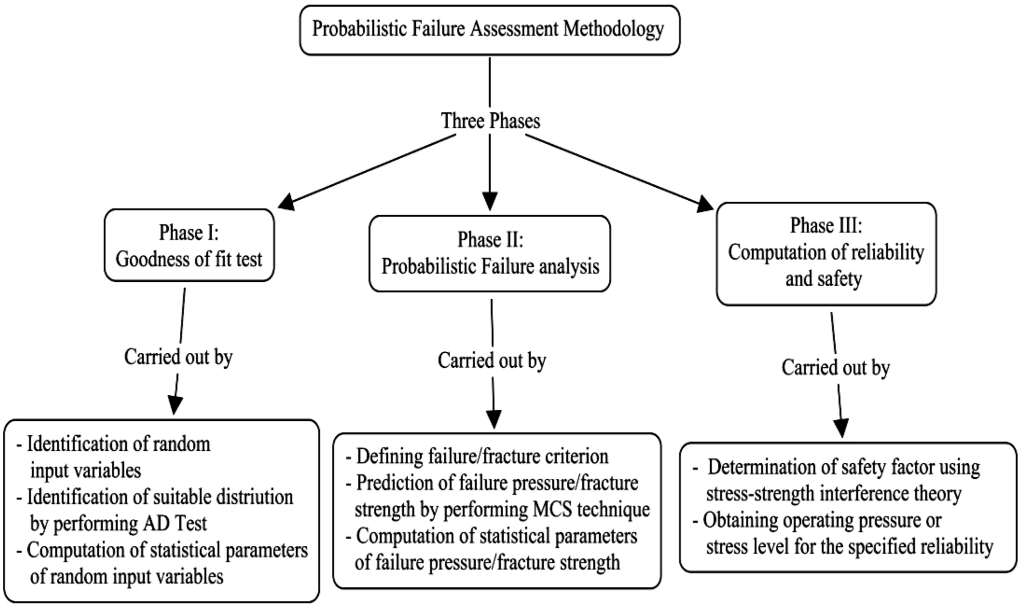

Fig. 1: Proposed Probabilistic Failure Assessment Methodology.

Modified Two Parameter Fracture Criterion:

The significant parameters affecting the size of a critical crack in a structure are the applied stress levels, the fracture toughness of the material, the location of the crack and its orientation. Because the stress intensity factor (K)is a function of load, geometry and crack size. It will be more useful to have a relationship between the stress intensity factor at failure (K_{max}) and the net section failure stress (σ_f) from the fracture data of cracked specimens for the estimation/prediction of the fracture strength/failure pressure to any cracked configuration. The relationship between K_{max} and σ_f can be of the form shown in Equation (1) See Refs. Christopher *et al*. (2005a) and Potti *et al*. (2000) for more details.

$$K_{max} = K_F \left\{ 1 - m\left(\frac{\sigma_f}{\sigma_u}\right) - (1-m)\left(\frac{\sigma_f}{\sigma_u}\right)^p \right\} \quad (1)$$

where, σ_u is the nominal stress required to produce a fully plastic region (or hinge) on the net section. For cylindrical pressure vessels, σ_u is the hoop stress at the burst pressure level of the unflawed thin cylindrical shell. K_F, *m*and*p*are the three fracture parameters to be determined from the fracture data. Figure 1 shows a cylindrical vessel containing an axial through crack. Stress intensity factor expressions for these cracked configurations are available based on finite element solutions (Newman, 1976; Newman and Raju, 1979). Using the value of average net section failure stress (σ_f) in the stress intensity factor expression, the stress intensity factor at failure (K_{max}) for the cracked configuration can be obtained. For the cracked configurations in Figure 2, expressions for K_{max} and σ_f are shown in Equation (2) and Equation (3).

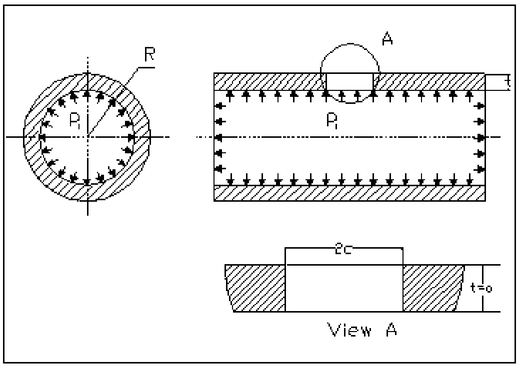

Fig. 2: Closed end cylindrical pressure vessel with an axial through crack under internal pressure.

$$K_{max} = \sigma_f (\pi a)^{1/2} \frac{M}{\varphi} \quad (2)$$

$$\sigma_f = \frac{p_{bf} r}{t} \quad (3)$$

Where,

$$M = M_e f_s \quad (4)$$

$$M_e = M_1 + \left(\varphi\sqrt{\frac{c}{a}} - M_1\right)\left(\frac{a}{t}\right)^q \quad (5)$$

$$M_1 = 1.13 - 0.1\left(\frac{a}{c}\right) \quad for\ a \le c \quad (6)$$

$$M_1 = \left\{1 + 0.03\left(\frac{c}{a}\right)\right\}\left(\frac{c}{a}\right)^{1/2} \quad for\ a > c \quad (7)$$

$$\varphi^2 = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad for\ a \le c \quad (8)$$

$$\varphi^2 = 1 + 1.464\left(\frac{c}{a}\right)^{1.65} \quad for\ a > c \quad (9)$$

$$f_s = \left(1 + 0.52\lambda_s + 1.29\lambda_s^2 - 0.074\lambda_s^3\right)^{\frac{1}{2}} \quad for\ 0 \le \lambda_s \le 10 \quad (10)$$

$$\lambda_s = \left[\frac{c}{\sqrt{Rt}}\right]\left(\frac{a}{t}\right) \quad (11)$$

$$\sigma_u = \frac{p_b R}{t} \quad (12)$$

$$q = 2 + 8\left(\frac{a}{c}\right)^3 \quad (13)$$

where M and φ are correction factors relevant to the geometry chosen, r is the radius of the cylindrical vessel, t is the thickness, p_b is the general failure or bursting pressure of an unflawed cylindrical vessel, p_{bf}isthe failure pressure of a cylindrical vessel with an axial surface crack. A comparative study Christopher *et al*. (2002) on failure pressure estimation of unflawed cylindrical vessels indicates the validity of the Faupel's formula for steel vessels shown in Equation (14).

$$p_b = \frac{2}{\sqrt{3}}\sigma_{ys}\left[2 - \frac{\sigma_{ys}}{\sigma_{ult}}\right] ln\left[1 + \frac{t}{r}\right] \quad (14)$$

where σ_{ys} is the 0.2% proof stress or the yield stress of the material. The non-linear equation for determining the net section failure stress (σ_f) for a specified crack size is obtained using Equation (1) and Equation (2) and shown in Equation (15).

$$(1-m)\left(\frac{\sigma_f}{\sigma_u}\right)^p + \left\{m + \sigma_u(\pi a)^{\frac{1}{2}}\frac{M}{\varphi K_F}\right\}\left(\frac{\sigma_f}{\sigma_u}\right) - 1 = 0 \quad (15)$$

Using the Newton–Raphson iterative method, the non-linear Equation (23) is solved for $\sigma_f$and failure pressure of cylindrical vessel having an axial through-thickness crack is obtained from Equation (3). In this present work, modified two parameter fracture criterion is used to predict the probabilistic failure pressure of the cracked pressure vessel.

Statistical Parameters of Random Input Variables:

The pressure vessel containing axial through crack subjected to internal pressure as shown in Figure 2 is considered for the study. The literature test data for the pressure vessel (Anderson and Sullivan 1966; Peter and Kuhn 1957; Calfo, 1968) viz. ELI Titanium alloy, Aluminum alloy and AISI stainless steel cylinders as shown in Tables (1-5) have been used to perform probabilistic fracture analysis with different materials under cryogenic temperature. In this analysis uncertainty with respect to crack geometry, strength properties, fracture parameter and geometric parameters are taken into account.

The failure pressure estimates values for various Cylindrical Pressure Vessels (CPV) are presented in Tables (1-5) which are considered as mean and their coefficient of variations are taken from the literature (Sang-Min Lee *et al*., 2006). In the fracture analysis, K_F is one of the important and changeable fracture parameters. The variation in K_F is quantified by constructing FAD using Equation (1). In order to accommodate all the literature test data, Failure Assessment Line (FAL) are constructed on either side of already drawn FAD. The K_F value available in Tables (1-5) is assumed as mean of K_F. Figure 3 shows that the FAL corresponds to mean, upper and lower K_F values for 5Al-2.5Sn-Ti cylinders at 20 K along with literature test data. The higher and lower K_F values are equated to upper and lower limit of 3σ limits. The standard deviation and COV for K_F is computed and is represented in Table 6. The uncertainty (i.e. COV) in K_F is obtained as 14%. The random variables are assumed to follow normal and log-normal distributions. The fracture parameters m and q in the MTPFC (Equation (1)) are assumed to be deterministic. The statistical properties of all random input variables are summarized in Table 6.

Table 1: Literature Test data of ELI Titanium Alloy 5Al-2.5Sn-Ti Cylinders at 20K (D_o=152.4mm, t=0.51 mm, σ_{ys}=1525 MPa, σ_{ult}=1675 MPa) at 20°K, K_F=228.2 MPa√m, m=1.0.

Cylinder No.	Crack length (2c) mm	Failure Pressure, p_{bf} (MPa)		Relative error (%)
		Test (Anderson and Sullivan, 1966)	Deterministic Analysis (Christopher *et al*., 2002 b)	
1	2.4	7.88	7.64	3.0
2	3.9	7.4	6.82	7.9
3	4.8	7.13	6.44	-5.0
4	7.0	5.58	5.63	-0.9
5	7.1	5.25	5.61	-6.8
6	12.2	3.49	4.23	-3.0
7	13.3	3.89	4.01	-21.1
8	19.4	2.83	3.00	1.9
9	20.4	2.92	2.87	14.5
10	23.9	2.90	2.48	-6.0
11	25.0	2.37	2.37	-0.1
12	39.6	1.73	1.47	23.8
13	40.6	1.87	1.43	15.2

Table 2: Literature Test data of Aluminium Alloy 2024-T3Al Cylinders (D_o=182.9 mm, σ_{ys}=250 MPa, σ_{ult}=450 MPa) t=0.30 mm (K_F=124 MPa√m, m=1.0).

Cylinder No.	Crack length (2c) mm	Failure Pressure, p_{bf} (MPa)		Relative error (%)
		Test (Peter and Kuhn, 1957)	Deterministic Analysis (Christopher *et al*., 2002 b)	
1	6.1	0.92	1.010	-10.0
2	6.1	0.95	1.010	-6.5
3	12.7	0.70	0.763	-9.0
4	12.7	0.61	0.763	-25.1
5	24.4	0.46	0.488	-7.1
6	24.4	0.47	0.488	-3.8
7	24.4	0.48	0.488	-1.6
8	47.5	0.25	0.258	-3.2
9	97.0	0.11	0.126	-14.8
10	24.9	0.53	0.479	9.5
11	50.3	0.29	0.243	16.2
12	102.1	0.13	0.121	6.9
13	24.4	0.52	0.488	6.2
14	48.5	0.23	0.252	-9.8
15	97.3	0.12	0.126	-5.0

Table 3: Literature Test Data of Aluminium Alloy 2024-T6Al Cylinders (D_o = 142.2 mm, σ_{ys} = 560 MPa, σ_{ult} = 680 MPa, t= 1.52 mm; at 20K (K_F = 68.9 MPa√m, *m* = 0.601, p = 20.4).

Cylinder No.	Crack length (2c) mm	Failure Pressure, p_{bf} (MPa)		Relative error (%)
		Literature test (Anderson and Sullivan, 1966)	Deterministic Analysis (Christopher *et al*., 2002 b)	
1	2.6	12.15	11.72	3.6
2	6.4	9.37	8.65	7.7
3	12.7	5.85	6.02	-2.9
4	19.1	4.73	4.48	5.3
5	25.4	3.10	3.48	-12.1
6	31.8	2.93	2.79	4.9
7	44.5	1.95	1.93	1.0
8	50.8	1.76	1.65	7.1

Table 4: Literature Test Data of ELI Titanium Alloy 5Al-2.5Sn-Ti Cylinders at 78K (D_o= 52.4 mm, t=0.51 mm, σ_{ys}=1200 MPa, σ_{ult}=1400 MPa) at 78 K: K_F=274.8 MPa√m, m=0.763,p=29.5.

Cylinder No.	Crack length (2c) mm	Failure Pressure, p_{bf} (MPa)		Relative error (%)
		Literature test (Anderson and Sullivan, 1966)	Deterministic Analysis (Christopher *et al*., 2002 b)	
1	3.2	8.75	8.11	7.3
2	6.1	7.58	6.80	10.3
3	5.8	7.20	6.92	3.9
4	11.2	5.31	5.18	2.5
5	11.5	4.83	5.09	-5.4
6	18.6	3.90	3.63	6.8
7	19.4	3.43	3.51	-2.3
8	24.0	3.30	2.91	12.0
9	25.2	3.04	2.78	8.5
10	37.6	2.03	1.85	9.1
11	37.1	1.65	1.87	-13.5

Table 5: Literature Test Data of AISI 301 Stainless Steel cylinders, 152 mm Diameter Tanks (σ_{ys}=1830 MPa, σ_{ult}=2220 MPa, t=0.6 mm)(Non-stress relieved) at 20 K: K_F=201.2 MPa√m, m=0.614, p=20.9.

Cylinder No.	Crack length (2c) mm	Failure Pressure, p_{bf} (MPa)		Relative Error (%)
		Test (Calfo, 1968)	Deterministic Analysis (Christopher *et al*., 2002 b)	
1	4.3	10.32	9.95	3.6
2	4.8	9.79	9.52	2.8
3	10.6	6.33	6.06	4.3
4	10.8	6.52	6.35	2.6
5	17.9	3.90	4.02	-3.0
6	24.4	3.03	2.89	4.7
7	37.5	1.97	1.80	8.5
8	37.6	1.65	1.80	-9.4

Table 6: Statistical Properties of Random Input Variables for the Flawed CPV.

Flawed cylindrical pressure vessels	Diameter (D) mm	Thickness (t) mm	Crack length (2c) mm	Ultimate strength (σ_{ult}) MPa	Yield strength (σ_{ys}) MPa	Fracture parameter (K_F) MPa√m
Mean (μ)						
5Al-2.5Sn-Ti cylinders At 20 K	152.4	0.51	2.0 – 50[b]	1675	1525	228.2
2024-T3Al cylinders	182.9	0.3	5.0 – 120[b]	450	250	124
2024-T6Al cylinders	142.2	1.52	2.0 – 50[b]	680	560	68.9
5Al-2.5Sn-Ti cylinders At 78 K	152.4	0.51	2.0 – 40[b]	1400	1200	274.8
AISI 301 stainless steel	152	0.6	2.0 – 50[b]	2220	1830	201.2
COV [a]	2%	2%	10%	7%	7%	14%
Probability distribution	Normal	Normal	Log normal	Normal	Normal	Normal
Reference	(Sang-Min Lee *et al*., 2006)					_[c]

[a]COV =standard deviation /mean; [b]assumed; [c]statistically obtained

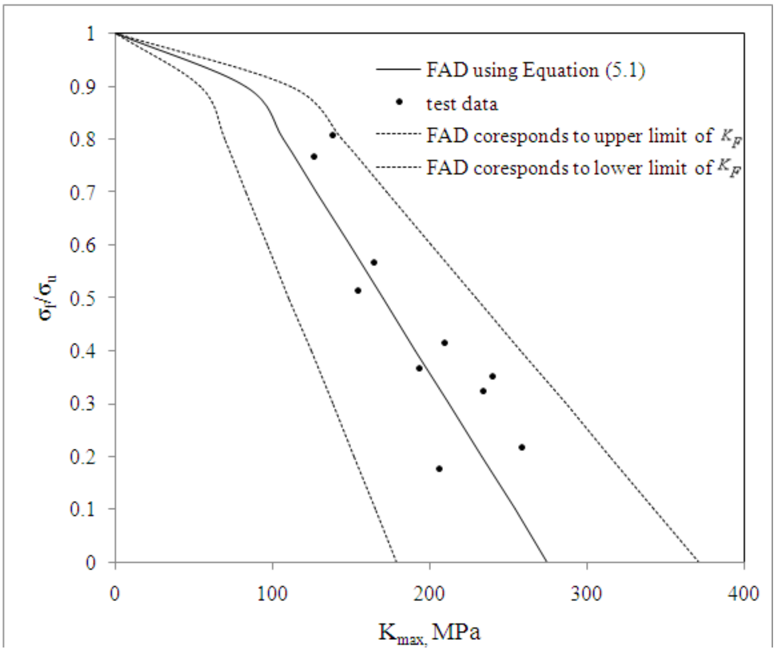

Fig. 3: FAD for Mean, Upper and Lower K_F values for 5Al-2.5Sn-Ti cylinders at 20 K.

Probabilistic Failure Pressure Prediction Using Monte -Carlo Simulation (MCS):

The probabilistic failure analysis is carried out using the proposed methodology explained in Figure 1. The failure pressure is predicted for five different materials under cryogenic temperature using MCS. For performing the MCS the mean, COV and probability distribution of random variables are required which are taken from Table 6. In this study, 1000 simulation trials are used to perform the analysis. 1000 data for each random variables (σ_{ys}, σ_{ult}, K_F, D_o, a, t and 2c) are generated by a code written using MATLAB software and the a sample of 10 simulated data for ELI Titanium alloy 5Al-2.5Sn-Ti at 20K is shown in Table 7.

Table 7: Simulated Sample Data of ELI Titanium Alloy (5Al-2.5Sn-Ti) at 20K using MCS Method.

Sample No.	Diameter, D_o (mm)	Thickness, t (mm)	Ultimate Strength, σ_{ult} (MPa)	Yield Strength, σ_{ys} (MPa)	Fracture Parameter, K_F (MPa√m)	Crack Length, 2c (mm)
1	146.8	0.511	1758.8	1367.8	227.9	4.8
2	154.6	0.501	1559.4	1533.6	214.4	5.0
3	152.9	0.482	1632.8	1618.3	165.9	5.6
4	156.2	0.509	1697.7	1337.6	232.8	4.8
5	156.9	0.503	1714.2	1522.9	244.6	5.0
6	151.2	0.522	1585.4	1445.8	282.3	5.0
7	146.2	0.515	1563.6	1582.4	186.2	4.8
8	156.3	0.506	1551.8	1543.3	235.3	5.6
9	154.3	0.508	1654.2	1454.8	188.7	4.2
10	155.9	0.524	1455.8	1357.8	231.6	5.2

For performing the Monte Carlo simulation the mean, COV and type of probability distribution are required. All these values are taken from Table 6. As mentioned in (Genki Yagawa*et al*., 1997)the simulation trial should be greater than 700 to obtain satisfactory result. In this study, 1000 simulation trials are carried out. The Monte Carlo based probabilistic fracture analysis requires sampling of parameters using random numbers. The following procedure is adopted for generating random variables of 1000 trials. Terminologies used in the simulation procedure are given below.
μ= vector of dimension n containing mean values of input parameters
X= vector of dimension n containing uniform random variables in the range [0, 1]
Z= vector of dimension n obtained as standard Gaussian vector corresponding to random vector, X
Y= vector of dimension n containing parameter's value, to be used for Monte Carlo analysis for a given Monte Carlo trial, the vector $Y$is determined as:

i. Generate vector X using a uniform random number generator. One random number for each parameter.
ii. Generate vector Zby mapping all elements of X.

$$Z = \phi^{-1}(X)$$

Where Φ is the cumulative distribution function of a standard Gaussian random variable. Y is obtained from the following relation

$$Y = X + Z$$

In case of lognormal distribution, to get the actual parameter, antilog of vector Yshould be taken.

Using the above procedure, 1000 trials data for each random variables (σ_{ys}, σ_{ult},K_F, D_o, a, t and $2c$) are generated by a code written using MATLAB software and the sample of 10 simulated data is given in Table 7. Using the simulated data shown in Table 3, the failure stress (σ_f) of the cracked pressure vessel is estimated through Equation (15) by a code written in MATLAB software. Since the Equation (15) is a non-linear one, the Newton-Raphson iterative method is implemented for finding failure stress (σ_f).

The failure pressure (p_{bf}) is computed with σ_f value using Equation (11) as shown in Table 8. The statistical properties like mean, COV and standard deviation of failure pressure (p_{bf}) are computed using 1000 simulated data in Minitab software. The range of probabilistic failure pressure at various confidence levels like 1σ, 2σ and 3σ are computed using the Equation (16) for different crack lengths and presented in Tables (9-13). Confidence bounds give an indication of how much the predicted response is expected to fluctuate. Confidence interval is based on normal approximation for a given level of confidence (k) is given as

$$Confidence\ \ interval = E\left(p_{bf}\right) \pm k\sqrt{Var\left(p_{bf}\right)} \tag{16}$$

where $E(p_{bf})$ is the expected mean and $Var(p_{bf})$ is the variance of the predicted failure pressure.

Table 8: Predicted Failure Pressure of ELI Titanium Alloy (5Al-2.5Sn-Ti) Cylinders at 20K Corresponding To Simulated Sample Data.

Sample No.	Crack Length, 2c (mm)	Probabilistic failure Pressure, p_{bf} (MPa)
1	4.8	7.18
2	5.0	7.18
3	5.6	5.10
4	4.8	6.69
5	5.0	6.79
6	5.0	7.47
7	4.8	6.36
8	5.6	7.15
9	4.2	6.43
10	5.2	6.29

Table 9: Predicted Failure Pressure Range for 5Al-2.5Sn-Ti Cylinders at 20K at Various Confidence Levels.

Crack Length (2c) mm	Predicted failure pressure, P_{bf} (MPa)			Probabilistic failure pressure, (p_{bf})range at various confidence levels (MPa)		
	Mean (μ_L)	COV_L (%)	Standard deviation (σ_L)	84.13 % (1σ)	97.72 % (2σ)	99.86 % (3σ)
2	8.46	7.37	0.62	7.83 - 9.08	7.15 – 9.69	6.59 – 10.32
5	6.72	8.80	0.59	6.13 – 7.32	5.54 – 7.91	4.96 – 8.47
10	4.97	11.31	0.56	4.41 – 5.53	3.85 – 6.07	3.28 – 6.63
15	3.79	13.28	0.50	3.28 – 4.29	2.78 – 4.79	2.28 – 5.29
20	2.99	14.33	0.43	2.56 - 3.41	2.13 – 3.84	1.70 – 4.27
25	2.42	15.52	0.38	2.05 – 2.80	1.67 – 3.17	1.29 – 3.55
30	2.01	16.30	0.33	1.68 – 2.34	1.35 – 2.66	1.03 – 2.99
35	1.72	17.04	0.29	1.43 – 2.01	1.13 – 2.31	0.84 – 2.60
40	1.48	17.31	0.26	1.22 – 1.73	0.97 – 1.99	0.71 – 2.24
45	1.30	17.84	0.23	1.07 – 1.53	0.83 – 1.76	0.60 – 1.99
50	1.15	18.42	0.21	0.94 – 1.36	0.73 – 1.58	0.52 – 1.79

Table 10: Predicted Failure Pressure Range for2024-T3Al Cylinders at Various Confidence Levels.

Crack Length (2*c*) mm	Predicted failure pressure, P_{bf} (MPa)			Probabilistic failure pressure (p_{bf})range at various confidence levels (MPa)		
	Mean (μ)	COV (%)	Standard deviation (σ)	84.13 % (1σ)	97.72 % (2σ)	99.86 % (3σ)
5	0.91	6.5	0.06	0.85-0.97	0.79- 1.03	0.73- 1.09
10	0.81	8.2	0.067	0.74-0.88	0.68-0.94	0.61-1.01
15	0.66	10.0	0.06	0.59-0.73	0.53-0.79	0.48-0.86
20	0.54	11.9	0.065	0.48-0.61	0.42-0.68	0.35-0.74
30	0.39	14	0.056	0.34-0.45	0.29-0.51	0.23-0.56
40	0.3	15.4	0.046	0.26-0.35	0.21-0.40	0.16-0.44
50	0.24	16.1	0.039	0.22-0.28	0.16-0.32	0.12-0.36
60	0.20	16.7	0.036	0.16-0.23	0.12-0.27	0.09-0.31
70	0.17	16.3	0.026	0.14-0.19	0.11-0.22	0.09-0.24
80	0.15	16.1	0.023	0.12-0.17	0.10-0.19	0.08-0.21
90	0.14	15.7	0.02	0.12-0.16	0.10-0.18	0.08-0.20
100	0.12	15.6	0.019	0.10-0.13	0.08-0.15	0.06-0.17
110	0.113	16.1	0.02	0.09-0.13	0.07-0.15	0.05-0.17
120	0.107	17.0	0.019	0.08-0.12	0.06-0.14	0.05-0.16

Table 11: Predicted Probabilistic Failure Pressure Range for 2024-T6Al Cylinders at Various Confidence Levels.

Crack Length (2c) mm	Predicted failure pressure, P_{bf} (MPa)			Probabilistic failure pressure (p_{bf})range at various confidence levels (MPa)		
	Mean (μ)	COV (%)	Standard deviation (σ)	84.13 % (1σ)	97.72 % (2σ)	99.86 % (3σ)
2	11.81	7.80	0.920	10.90-12.73	9.90-13.7	9.05-14.6
5	9.100	9.85	0.890	8.21-9.99	7.32-10.8	6.43-11.7
10	6.750	11.90	0.800	5.95-7.55	5.14-8.36	4.34-9.16
15	5.290	13.30	0.700	4.59-5.90	3.88-6.70	3.18-7.40
20	4.280	14.20	0.600	3.67-4.89	3.07-5. 50	2.43-6.11
25	3.530	15.50	0.549	2.98-4.08	2.43-4.63	1.82-5.18
30	2.972	15.80	0.469	2.52-3.42	2.03-3.91	1.56-4.38
35	2.547	17.00	0.430	2.11-2.97	1.68-3.41	1.25-3.84
40	2.200	16.80	0.367	1.83-2.57	1.46-2.94	1.11-3.31
45	1.940	17.20	0.330	1.60-2.27	1.27-2.61	0.92-2.94
50	1.720	17.60	0.304	1.42-2.02	1.11-2.33	0.81-2.63

Table 12: Predicted Probabilistic Failure Pressure Range for 2024-T3Al Cylinders at Various Confidence Levels.

Crack Length (2c) mm	Predicted failure pressure, P_{bf} (MPa)			Probabilistic failure pressure (p_{bf})range at various confidence levels (MPa)		
	Mean (μ)	COV (%)	Standard deviation (σ)	84.13 % (1σ)	97.72 % (2σ)	99.86 % (3σ)
2	9.35	6.5	0.612	8.73-9.96	8.12-10.57	7.5-11.18
5	7.64	8.2	0.628	7.01-8.277	6.38-8.90	5.76-9.53
10	5.72	10.7	0.61	5.11-6.33	4.49-6.95	3.88-7.56
15	4.41	12.7	0.56	3.85-4.97	3.29-5.53	2.73-6.09
20	3.5	14.46	0.50	3.01-4.02	2.50-4.53	2.00-5.09
25	2.86	14.92	0.43	2.42-3.27	1.99-3.74	1.57-4.18
30	2.38	16.44	0.38	2.00-2.71	1.62-3.15	1.23-3.53
35	2.04	16.62	0.33	1.70-2.37	1.35-2.71	1.03-3.04
40	1.76	17.91	0.32	1.46-2.05	1.62-2.35	0.86-2.70

Table 13: Predicted Probabilistic Failure Pressure Range for AISI 301 Stainless Steel Cylinders at Various Confidence Levels.

Crack Length (2c) mm	Predicted failure pressure, P_{bf} (MPa)			Probabilistic failure pressure (p_{bf})range at various confidence levels (MPa)		
	Mean (μ)	COV (%)	Standard deviation (σ)	84.13 % (1σ)	97.72 % (2σ)	99.86 % (3σ)
2	14.4	8.9	1.28	13.14-15.68	11.86-16.96	10.58-18.2
5	10.3	11.3	1.16	9.14-11.41	7.97-12.64	6.82-13.7
10	6.90	13.6	0.93	5.96-7.84	5.04-8.78	4.09-9.72
15	4.97	16.0	0.79	4.17-5.77	3.39-6.57	2.57-7.37
20	3.79	16.27	0.61	3.17-4.41	2.56-5.03	1.96-5.64
25	3.03	16.85	0.52	2.51-3.54	2.00-4.05	1.49-4.56
30	2.43	17.6	0.42	2.00-2.86	1.57-3.29	1.17-3.72
35	2.05	18.0	0.37	1.68-2.42	1.30-2.79	0.93-3.16
40	1.75	18.4	0.32	1.42-2.07	1.10-2.39	0.78-2.71
45	1.50	18.7	0.28	1.22-1.78	0.94-2.06	0.65-2.34
50	1.34	18.6	0.24	1.09-1.59	0.86-1.84	0.62-2.06

Computation of Safety Factor and Reliability:

Stress - Strength Interference Theory:

The reliability of existing structure or design of new structure or system with certain desired reliability can be determined using stress strength interference theory. With this knowledge, it is possible to increase the

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control on the safety factors that have an important effect on the product quality, and relax over stringent control on other factors that are not critical to the product quality (Zhou Xun and Yu Xiao-li, 2006). The computation of the reliability of the component requires the knowledge of the random nature of strength (*S*) and the stress (*L*). If the probability density function of *S* and *L* are known then, the reliability of the component can be evaluated by constructing integral equations as shown in Equation (17). In certain cases, *S* and *L* follow normal, lognormal, exponential or Weibull distributions, and the integral equations can be reduced to simple form. Otherwise the reliability of the component can be found only by evaluating the integral numerically. The reliability (*R*) of a component is given by

$$R = P(S > L) = P(S - L > 0) = \int f_{S.L}(S,L)\,ds\,dL \quad (17)$$

The probability of failure (P_f) can be expressed as shown in Equation (18)

$$P_f = 1 - R = 1 - P(S \geq L) \quad (18)$$

If S and L are random variables, the safety margin y is also a random variable and given as in Equation (19).

$$y = S - L \quad (19)$$

then the reliability of the component is represented as in Equation (20)

$$R = p(y \geq 0) \quad (20)$$

When strength and stress have normal density functions, y is also normally distributed and the reliability (R) is represented17 in the Equation (21).

$$R = \frac{1}{2\pi} \int_{-\frac{(\mu_S - \mu_L)}{\sqrt{\sigma_S^2 + \sigma_L^2}}}^{\infty} exp\left(-\frac{z^2}{2}\right) dz \quad (21)$$

where z is the standard normal random variable, μ_S is the mean value of the strength, μ_L is the mean value of the stress, and σ_S and σ_L represents the standard deviations of strength and stress, respectively. The negative sign portion of the lower limit of the integral in Equation (21) is denoted by Z_0. Therefore

$$Z_0 = \frac{(\mu_S - \mu_L)}{\sqrt{\sigma_S^2 + \sigma_L^2}} \quad (22)$$

The factor of safety *N* is defined as $N = \frac{\mu_S}{\mu_L}$ and the coefficients of variation of the strength and stress is denoted by V_S and V_L respectively, then $V_S = \frac{\sigma_S}{\mu_S}$ and $V_L = \frac{\sigma_L}{\mu_L}$

The Equation (22) is rewritten in terms of safety factor (Kapur and Lamberson, 1977) *N* as presented in Equation (23) and Equation (24).in terms of *N*, V_S and V_L

$$Z_0 = \frac{N - 1}{\sqrt{\left(\frac{\sigma_S}{\mu_L}\right)^2 + \left(\frac{\sigma_L}{\mu_L}\right)^2}} \quad (23)$$

$$Z_0 = \frac{N - 1}{\sqrt{V_S^2 N^2 + V_L^2}} \quad (24)$$

In this section, a safety factor is computed using stress-strength interference theory by considering the experimental failure pressure as strength (S) variable and the predicted failure pressures as stress (L) variable. If the S and L are independent, then the interference area between the PDF of S and L gives a measure of the probability of failure (Rao 1996). The mean and standard deviation of strength and stress random variables are taken from the Tables (9-13) which is obtained from probabilistic failure analysis.

Safety Factor Computation:

For instance, the pressure vessel made of 5Al-2.5Sn-Ti alloy at 20K containing an axial through crack of length 2c=3.9 mm is considered for safety factor computation. The mean and standard deviation of stress and strength variables are taken from Table 14. The safety factor is computed using Equation (24) for the specified reliability. Similarly, the safety factor is computed for other materials and is shown in Table 15.

Operating Pressure Computation:

The safe operating pressure of the vessel is determined using Equation (25) as given below.

Operating pressure $\sigma_{op} = \mu_L/N$ (25)

where μ_L is the predicted failure pressure mean and N is safety factor for the specified reliability. The operating pressure is computed for five different cylindrical pressure vessels and is summarized in Table 15.

Table 14: Predicted Safety Factor for Flawed Pressure Vessels Containing Axial Crack.

Material	Crack length (2c) mm	Failure Pressure, P_{bf} (MPa)			
		Test		Predicted	
		Mean (μ_S)	Standard deviation (Lin *et al* 2004) (σ_S)	Mean (μ_L)	Standard deviation (σ_L)
5Al-2.5Sn-Ti at 20K	3.9	7.40	0.740	7.305	0.602
2024-T3Al	6.1	0.92	0.092	0.913	0.067
2024-T6Al	2.6	12.15	1.215	11.379	1.046
5Al-2.5Sn-Ti at 78K	3.2	8.75	0.875	8.625	0.626
AISI 301 stainless steel	4.3	10.32	1.032	10.992	1.221

Table 15: Predicted Safety Factor and Operating Pressure of Cylinders at a Particular Crack Length for the Specified Reliability.

Cylinders	Crack length (2c) mm	Safety factor (N)				Operating pressure (MPa)			
		90%	95%	99%	99.99%	90%	95%	99%	99.99%
5Al-2.5Sn-Ti at 20K	2.4	1.185	1.246	1.372	1.704	6.17	5.86	5.32	4.29
2024-T3Al	7.1	1.177	1.236	1.359	1.681	0.78	0.74	0.67	0.54
2024-T6Al	2.6	1.188	1.250	1.379	1.714	9.58	9.10	8.25	6.64
5Al-2.5Sn-Ti at 78K	3.2	1.177	1.236	1.363	1.681	7.33	6.98	6.33	5.13
AISI 301 stainless steel	4.3	1.211	1.271	1.419	1.781	9.08	8.65	7.75	6.17

RESULTS AND DISCUSSION

In this chapter, the PFM analysis is carried out for the pressure vessels containing crack and the following observations made during the analysis are summarized below.

i. In this study, the failure pressure of the cylindrical pressure vessel having axial through crack is predicted using probabilistic fracture mechanics approach. With the use of statistical properties of predicted failure pressure, the probabilistic failure pressure range has been determined and is presented in Tables (9-13).

ii. The predicted failure pressure range at 3σ level and deterministic failure pressure along with test data against various crack length is drawn as shown in Figure 4 for 5Al-2.5Sn-Ti Cylinders at 20K. Out of thirteen test data considered shown in Tables (1-5), seven of them fall below the deterministic prediction (53.8%) and three of them in line with prediction (23%), then the remaining three values fall above the deterministic prediction (23%). Whereas in the case of probabilistic failure prediction, all the simulated test data fall within upper and lower limits of failure pressure shown in Figure 4 and thereby eliminating the negative error.

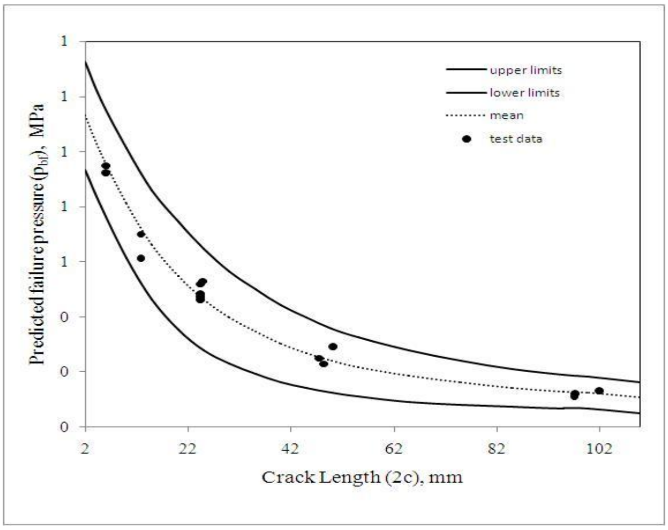

Fig. 4: Comparison of Deterministic and Probabilistic Failure Pressure Limits at 3σ Along With Literature Experimental Test Data for 5Al-2.5Sn-Ti Cylinders at 20K

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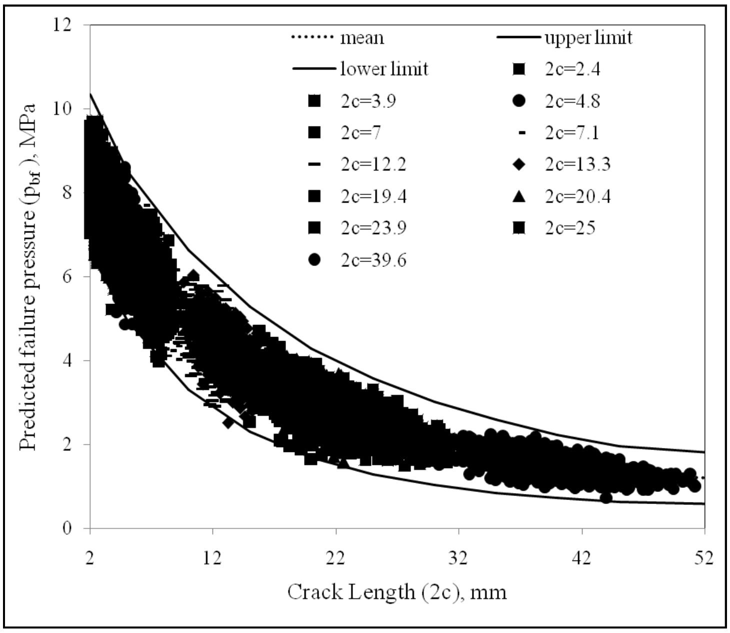

Fig. 5: Simulated test data of ELI Titanium alloy 5Al-2.5Sn-Ti at 20 K along with Probabilistic Failure Pressure Limits at 3σ Confidence Level.

The predicted probabilistic failure pressure range at 3σ is shown in Figure 5 for 5Al-2.5Sn-Ti alloy along with deterministic failure pressure for various crack length. From the Figure 5 it is clear that 1000 simulated test data generated for all the materials fall within the predicted probabilistic failure pressure range at 3σ for various crack length. Thus ensuring that probabilistic approach predicts correctly under all circumstances than the deterministic approach.

iii. From Figure 6, it is clear that distribution of predicted failure pressure through proposed methodology has good agreement with the distribution of literature test failure pressure

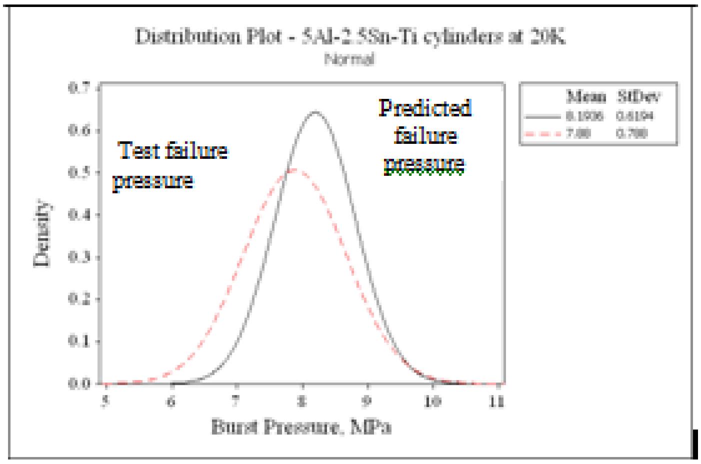

Fig. 6: Stress-Strength interference graph of 5Al-2.5Sn-Ti cylinders at 20K.

iv. From the results of probabilistic analysis, reliability-based safety factor is suggested for the specified reliability using stress-strength interference theory . The safe operating pressure is obtained by shifting the mean of the distribution of predicted failure pressure for the specified reliability. The stress – strength interference graph for 5AL-2.5 Sn-Ti cylinders at 20 K for the reliability of 90% and 99.99% respectively is shown in Figures 7 and 8. It is observed from Figures 7 and 8 that interference area is more for 90% reliability and less for reliability 99.99%. Based on this approach, a design engineer can design the system for the specified reliability and safety.

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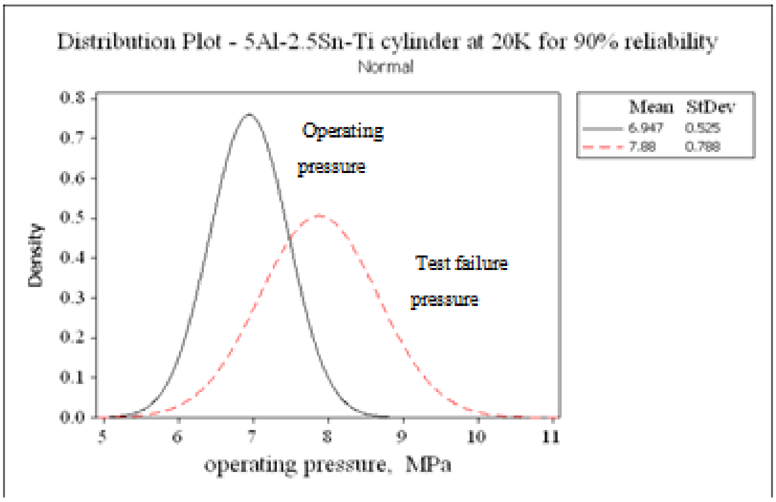

Fig. 7: Stress- Strength Interference Graph of 5Al-2.5Sn-Ti Cylinders at 20 K for 90 % Reliability.

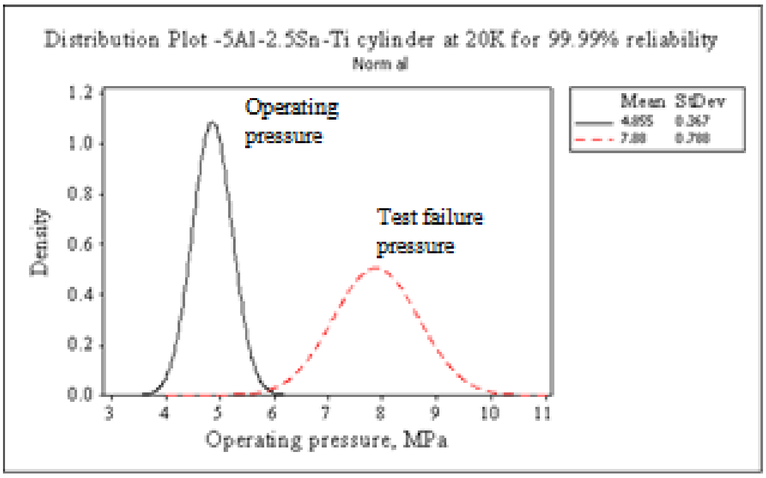

Fig. 8: Stress- Strength Interference Graph of 5Al-2.5 Sn-Ti Cylinders at 20 K for 99.99 % Reliability.

v. Figure 9 shows the effect of crack length on safety factor for 5Al-2.5Sn-T6 titanium alloy cylinder at 20K. It is inferred from Figure 9 that as the crack length increases, the safety factor is also increases for the specified reliability.

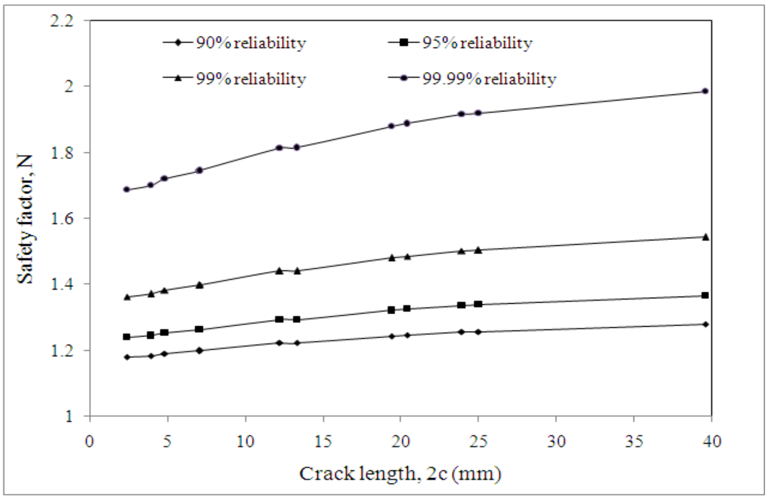

Fig. 9: Effect of Crack Length on Safety Factor for 5Al-2.5Sn-Ti Cylinders at 20 K for The Specified Reliability Level.

Conclusion:

In this study, PFM analysis of pressure vessels containing axial through crack under cryogenic temperatures is performed using the proposed probabilistic failure assessment methodology. The Monte Carlo simulation technique is adopted for performing probabilistic fracture analysis using MATLAB software. As a result, reliability-based operating pressure is suggested for five different cylinders having axial through crack subjected to internal pressure. The proposed work will help the design engineer to decide the operating pressure of the cracked structure for the specified safety and reliability. In future, this procedure can be extended to the pressure vessel containing circumferential through crack, combination of axial and circumferential and pressure vessels made of composite materials.

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