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Behavior of form factor models for pine boles using diameters at relative heights

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ABSTRACT

Background: Diameters taken at relative heights are good independent variables to be used in modeling the form factor; however they are oftentimes not easily accessible to be measured. **Objective:** The aim of this research was to evaluate the behavior of form factor models, using different combinations of the independent variables: diameter at 1.3 m, diameters at relative heights and bole height in 133 pine trees from forest stands in Brazil.**Methodology:**The incremental contribution of the variables: diameters at 10%, 30%, 50% and 70% of bole height were statistically evaluated. Form factor models were analyzed according to accuracy statistics and tests for autocorrelation and homogeneity of residues, besides residual dispersion. These four diameters were tested in four models, where variables doubly and triply combined was used, totalizing sixteen adjustments. Lastly, the bole volume was estimated using the form factor model adjusted in function of diameters taken at relative heights equated in function of diameter at breast height. **Results:** Form factor models presented different behaviors when the tested variables were changed. Tests for incremental contribution indicated significance at 95% or 99% probability level for all tested variables. In general, residual autocorrelation and heteroscedasticity behaved as a non-critical problem on the modeling, in spite of bias has been present in most models. Regarding to volumes calculated by the form factor models based on estimated diameters, the SEE reached value up to 13%, whereas the use of observed diameters reduced the SEE to 3%. **Conclusion:**The use of combined variables of diameters measured in relative heights with diameter at breast height reduces the standard error of estimate and to raise the coefficient of determination in the form factor models. However, such combinations can provoke heteroscedasticity and autocorrelation among residuals. The combined diameters used as independent variables have presented a strong tendency to provoke bias in the form factor model; therefore it is highly important evaluating them based on the behavior of their residual dispersion. The triply combined variables ($d_{rh1}d_{rh2}d^{-2}$, where rh is relative height) contribute more to reduce the errors and let the models to be less unbiased, when compared with the effect of the doubly combined variables ($d_{rh}d^{-2}$, where rh is relative height). Under the conditions in which an analysis regarding confidence intervals, tests t and F are not required, the form factor model 6.1 is the best option. It is possible to obtain volume in form factor models based on estimated diameters at relative heights, with an admissible error close to 5% on the estimated volume.

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INTRODUCTION

The knowledge of timber stock and growth in forest stands requires conduction of inventories, whose main goal is to obtain timber volume (Assmann, 1970). When regression models are used to solve volume estimation, the occurrence of errors is mostly due to the bole form variation. Husch *et al*. (1972) indicate the bole tapering as one of the main reasons of this, but such occurrence is also attributed to species variation, forest sites and age of the forest.

Conceptually, the tree boles fit into three geometric shapes which are described by truncated portions of a neiloid, a paraboloid and a cone, wherein in practice the tree boles are often referred as variations from conical up to cylindrical shapes (Gray, 1956). However, satisfactory solutions to explain the shape may not be obtained when it is considered only one aspect of the biological functions of the boles (Loetsch *et al*., 1973), what suggests us to search other factors linked to the tapering of the boles.

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Often methods based on diameter at breast height are used to estimate the form factor, even though there are other methods based on relative heights. The form factor's model, that are not specified with diameters at relative heights, presents as main independent variables: diameter at height breast, total height or combination of both, as is found in the following models presented by Loetsch *et al*. (1973): Often methods based on diameter at breast height are used to estimate the form factor, even though there are other methods based on relative heights. The form factor's model, that are not specified with diameters at relative heights, presents as main independent variables: diameter at height breast, total height or combination of both, as is found in the following models presented by Loetsch *et al*. (1973): Often methods based on diameter at breast height are used to estimate the form factor, even though there are other methods based on relative heights. The form factor's model, that are not specified with diameters at relative heights, presents as main independent variables: diameter at height breast, total height or combination of both, as is found in the following models presented by Loetsch *et al*. (1973):$\lambda = b_0 + b_1d + b_2d^2$, $\lambda = b_0 + b_1h + b_2hd^{-1}$ and $\lambda = b_0 + b_1h^{-1} + b_2d^{-2} + b_3d^{-2}h^{-1}$,where "$\lambda$" = artificial form factor; d = diameter at breast height; h = height of the bole; bi = parameters of the model.

Hohenadl (1936) was the pioneer to propose the form factor calculation based on one-tenth of bole height. After that, diameters at relative heights were proposed even as variables for form factor models, where some of them may be seen in Schneider and Schneider (2008). However, the difficulty encountered to measure diameters at relative heights makes this method less used. Measuring diameters along the stand tree requires more time of hand labor due to the demand either of using devices like a dendrometer or climbing the stand tree. These requirements might as consequence increase the forest inventories costs.

However, little is known regarding to how much of the dependent variable is explained when those variables are included in the form factor models. Considering that diameters along the bole can explain mightily its tapering, our hypothesis is that those variables should really improve the models, and such proof is quite challenging, mainly because when making form factor's modeling, no-biological aspects seldom are accessible, measured and used, according to what was presented by Loetsch *et al*. (1973).

The aim of this research was to find variables that can contribute to improve the accuracy of models of form factor and of the bole volume estimation. We analyzed the behavior of form factor models when diameters at relative heights were applied as doubly and triply combined variables. Models were evaluated by accuracy statistics and tests for homogeneity and autocorrelation of residues. Lastly, we analyzed the bole volume estimation when applying form factor models based on estimated diameters at relative heights.

MATERIAL AND METHODS

Database and purpose of research:

Database came from a forest inventory carried out in uneven-aged *Pinus taeda* L. stands located in Santa Catarina state, south Brazil, whose average coordinates are 51°34'56"W and 25°12'29"S. The forest stands have 4.0 up to 14.5 years old. Management was conducted applying one mixed thinning between 8 and 9 years old, when it was harvested 40% of total basal area.

The boles of 133 pine trees were cubed according to Hohenadl's method (1924), however further diameters were included in the measurement. The following diameters were measured at relative positions 0%, 1%, 2%, 3%, 4%, 5%, 10%, 15%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 83%, 85%, 90%, 93% e 95% of total bole height.

Volume of each section was calculated by Smalian's equation and the volume of the bole tip was calculated assuming it as geometric shape of a cone. The bole volume (1), in m³, was obtained adding up the volumes of the sections plus the bole's tip. Observing the cubed trees it was found a minimum bole volume equals to 0.01m³, an average volume equals to 0.42m³ and a maximum volume of 1.56m³. Models were used to estimate artificial form factor (λ) of the tree boles, whose calculation was made dividing the real tree volume by the volume of a cylinder considering diameter at breast height and total height of the bole as references, as specified in equation (2):

$$v = \frac{(g_{l_i} + g_{s_i})}{2} L_i + \frac{(g_{l_{i+1}} + g_{s_{i+1}})}{2} L_{i+1} + \ldots + \frac{(g_{l_n} + g_{s_n})}{2} L_n + \frac{g_{s_n}}{3} h_t \quad (1)$$

$$\lambda = \frac{v}{g_{1.3}h} \quad (2)$$

Where: g_{l_i} = cross-sectional area at large end of the bole part *i*, in m²; g_{s_i} = cross-sectional area at smaller end of the bole part *i*, in m²; $g_{1.3}$ = cross-sectional area at breast height, in m²; L_i = length of bole parts *i*, in m; h_t = tip height, in m; n = number of bole parts; h = total height of the bole, in m.

The purpose of this research was to evaluate the behavior of form factor models when diameters at different heights were used as combined variables. Form factor models were adjusted assuming some combinations of diameters measured at four different heights. Our objectives were completed presenting accuracy statistics of bole volume using estimated diameters at relative heights in the form factor model.

Form factor modeling and incremental contribution of variables:

Schneider and Schneider (2008) have presented some form factor models, from which four were selected and included diameters at relative heights in their specifications. We seek to answer how much it worth to take diameters at relative heights, knowing that they are variables little accessible and difficult to be localized the appropriate height where they should to be measured on stand trees.

Diameters at 10%, 30%, 50% and 70% of the bole height were analyzed by two combination types: the first test combining double variables (models from 3.1 to 4.4) and the second test combining triple variables (models from 5.1 to 6.4). The double combination was done with one diameter at relative height multiplied by the inverse of the squared diameter at breast height, while for the triple combination it is included one additional diameter measured at relative heights of the bole to the double combination. Then, four models with four combined variables were adjusted, which resulted in sixteen different models presented in Table 1.

Table 1: Models and variables tested to explain bole form factor of *Pinus taeda* L. in southern part of Brazil.

Variable tested	Number	Model	Relative heights	Variables to be tested
Doubly combined	3.1	$\lambda = b_0 + b_1 d_{0.1} d^{-2} + b_2 h d^{-2} + \varepsilon$	0.1	$d_{0.1}d^{-2}$
	3.2	$\lambda = b_0 + b_1 d_{0.3} d^{-2} + b_2 h d^{-2} + \varepsilon$	0.3	$d_{0.3}d^{-2}$
	3.3	$\lambda = b_0 + b_1 d_{0.5} d^{-2} + b_2 h d^{-2} + \varepsilon$	0.5	$d_{0.5}d^{-2}$
	3.4	$\lambda = b_0 + b_1 d_{0.7} d^{-2} + b_2 h d^{-2} + \varepsilon$	0.7	$d_{0.7}d^{-2}$
	4.1	$\lambda = b_0 + b_1 d_{0.1} d^{-2} + b_2 h d^{-2} + b_3 d^{-1} + \varepsilon$	0.1	$d_{0.1}d^{-2}$
	4.2	$\lambda = b_0 + b_1 d_{0.3} d^{-2} + b_2 h d^{-2} + b_3 d^{-1} + \varepsilon$	0.3	$d_{0.3}d^{-2}$
	4.3	$\lambda = b_0 + b_1 d_{0.5} d^{-2} + b_2 h d^{-2} + b_3 d^{-1} + \varepsilon$	0.5	$d_{0.5}d^{-2}$
	4.4	$\lambda = b_0 + b_1 d_{0.7} d^{-2} + b_2 h d^{-2} + b_3 d^{-1} + \varepsilon$	0.7	$d_{0.7}d^{-2}$
Triply combined	5.1	$\lambda = b_0 + b_1 d_{0.1} d_{0.3} d^{-2} + b_2 d_{0.5} d^{-2} + \varepsilon$	0.1, 0.3	$d_{0.1}d_{0.3}d^{-2}$
	5.2	$\lambda = b_0 + b_1 d_{0.1} d_{0.5} d^{-2} + b_2 d_{0.3} d^{-2} + \varepsilon$	0.1, 0.5	$d_{0.1}d_{0.5}d^{-2}$
	5.3	$\lambda = b_0 + b_1 d_{0.1} d_{0.7} d^{-2} + b_2 d_{0.5} d^{-2} + \varepsilon$	0.1, 0.7	$d_{0.1}d_{0.7}d^{-2}$
	5.4	$\lambda = b_0 + b_1 d_{0.3} d_{0.7} d^{-2} + b_2 d_{0.5} d^{-2} + \varepsilon$	0.3, 0.7	$d_{0.3}d_{0.7}d^{-2}$
	6.1	$\lambda = b_0 + b_1 d_{0.1} d_{0.3} d^{-2} + b_2 d_{0.5}^2 d^{-2} + b_3 h d^{-2} + \varepsilon$	0.1, 0.3	$d_{0.1}d_{0.3}d^{-2}$
	6.2	$\lambda = b_0 + b_1 d_{0.1} d_{0.5} d^{-2} + b_2 d_{0.3}^2 d^{-2} + b_3 h d^{-2} + \varepsilon$	0.1, 0.5	$d_{0.1}d_{0.5}d^{-2}$
	6.3	$\lambda = b_0 + b_1 d_{0.1} d_{0.7} d^{-2} + b_2 d_{0.5}^2 d^{-2} + b_3 h d^{-2} + \varepsilon$	0.1, 0.7	$d_{0.1}d_{0.7}d^{-2}$
	6.4	$\lambda = b_0 + b_1 d_{0.3} d_{0.7} d^{-2} + b_2 d_{0.5}^2 d^{-2} + b_3 h d^{-2} + \varepsilon$	0.3, 0.5	$d_{0.3}d_{0.7}d^{-2}$

λ = artificial form factor; d = diameter at breast height, in cm; h = bole height, in m; $d_{0.1}$, $d_{0.3}$, $d_{0.5}$, $d_{0.7}$ = diameter at 10%, 30%, 50% and 70% of the bole height, in cm, respectively; ε = random error.

To test if those variables contribute significantly to increase the expected sum of squares, the increase of their contributions were evaluated based on the F test (Gujarati, 2008), whose analysis of variance is effectuated as described in Table 2. The test consists in adjustment of the form factor models with and without those explanatory variables, in this case the diameters at relative heights.

Table 2: Analysis of variance for incremental contribution of the explanatory variables.

Source of variation	Degrees of freedom	Sum of squares (SS)	F test
ESS due to $x_2,\ldots,x_p$	p-1	$SS_1 = \hat{\beta}_{0,1}^2\ \Sigma x_1^2$	$F = \dfrac{SS_2/p-1}{SS_4/n-p-1}$
ESS due to the addition x_1	1	$SS_2 = SS_3 - SS_1$	
ESS due to $x_1,\ldots, x_p$	p	$SS_3 = \hat{\beta}_1 \Sigma y_i x_{1i} + \ldots + \hat{\beta}_p \Sigma y_i x_{pi}$	
RSS	n-p-1	$SS_4 = SS_5 - SS_3$	
Total	n-1	$SS_5 = \Sigma y_i^2$	

ESS = Expected Sum of Squares; RSS = Residual Sum of Squares; y and x = dependent and independent variables; β_i = parameters of the model; p = number of independent variables; n = number of observations.

Then, the test consists in evaluating if the addition of variables can contribute to increase the expected value of the Total Sum of Squares (TSS) of the model and consequently the goodness of fit. All tests were carried out at 95% probability level, assuming the initial hypothesis (H_0): The variable may not contribute to increase the ESS and the alternative hypothesis (Ha): The variable contributes to increase the ESS. Thus, H_0 is not rejected if the F test value is not significant at 95% probability level, otherwise H_0 is rejected.

Aiming to help understanding the behavior between the dependent and independent variables included in the form factor models, we estimated the Pearson's correlation coefficient ($\hat{\rho}$) whose formula is given by $\hat{\rho} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}}\sqrt{\sigma_{jj}}}$,, where$\sigma_{ij}$ = covariance between the variables i and j, $\sigma_{ii}$and σ_{jj} = variances of the variables i and j, respectively.

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Criteria to evaluate linear regression models:

It is desirable that the models can meet the main assumptions of linear regression, including absence of residual autocorrelation, homoscedasticity and appropriated specification of the model. Such assumptions must ensure efficient, consistent and unbiased estimators, consequently to estimate the dependent variable without tendency as well as generating the confidence interval, when it is appropriated. Thus, we evaluated possible residual autocorrelations by two ways, being the first visually by means of graphs of residual dispersion and the other by a statistical test proposed by Durbin and Watson (1951), here called DW test.

$$DW = \frac{\sum_{i=2}^{n}\left(\hat{\varepsilon}_i - \hat{\varepsilon}_{i-1}\right)^2}{\sum_{i=1}^{n}\hat{\varepsilon}_i^2} \quad i = 1, 2,..., n \qquad (7.1)$$

When fitting the equation (7.1), it is possible to obtain DW using Pearson’s correlation coefficient, as presented in equation (7.2):

$$DW = \frac{\sum_{i=2}^{n}\hat{\varepsilon}_i^2 + \sum_{i=2}^{n}\hat{\varepsilon}_{i-1}^2 - 2\sum_{i=2}^{n}\hat{\varepsilon}_i \hat{\varepsilon}_{i-1}}{\sum_{i=1}^{n}\hat{\varepsilon}_i^2}$$

$$DW = \frac{\sum_{i=2}^{n}\hat{\varepsilon}_i^2}{\sum_{i=1}^{n}\hat{\varepsilon}_i^2} + \frac{\sum_{i=2}^{n}\hat{\varepsilon}_{i-1}^2}{\sum_{i=1}^{n}\hat{\varepsilon}_i^2} - 2\frac{\sum_{i=2}^{n}\hat{\varepsilon}_i\hat{\varepsilon}_{i-1}}{\sum_{i=1}^{n}\hat{\varepsilon}_i^2} \quad \Rightarrow \quad DW \cong 2(1 - \hat{\rho}) \qquad (7.2)$$

Where: $\hat{\varepsilon}_i$ = Residues at *i* observation; $\hat{\rho}$ = Pearson’s correlation coefficient; n = number of observations.

The initial hypothesis (H_0) indicates absence of residual autocorrelation, against the alternative hypothesis (H_a) otherwise. To conclude about the residual autocorrelation by this test, Durbin-Watson's significance tables were necessary to frame DW statistical values in interval zones delimited by minimum and maximum tabled values. Under both hypotheses, while the test may indicate presence or absence of autocorrelation, DW values may fall in an indecision zone making it inconclusive (Gujarati, 2008).

The White's statistical test (1980) was used to evaluate residual homoscedasticity, which suggests fitting a regression model considering the square of the residuals in function of the used variables, combined or not, as it is presented in (8):

$$\hat{\varepsilon}_{in}^2 = \alpha_0 + \sum_{j=1}^{k}\sum_{k=j}^{k}\alpha_S x_{ij} x_{ik} + v \qquad i = 1,..., n;\ j = 1,\dots, k;\ s = 1,\dots, k(k+1)/2 \qquad (8)$$

Where: $\hat{\varepsilon}_i$ = Residuals of i^{th} observation; α_i = artificial coefficient; x = dependent variable; v = artificial residue; k = number of variables, n = number of observations.

The homoscedasticity can be confirmed when $nR^2 < \chi^2_{DF}$, whereas heteroscedasticity is detected otherwise, where n = number of observations; R^2 = coefficient of determination; χ^2 = chi-square and DF = degrees of freedom equals to the number of parameters of the artificial model minus 1.

As criteria to select models, besides Durbin-Watson’s test and White’s test, models also were evaluated according to accuracy statistics as the standard error of estimate in percentage (SEE %), presented in equation (9) and adjusted coefficient of determination $\bar{R}^2$, presented in equation (10). The best model was selected based on the lowest SEE (%) and the higher $\bar{R}^2$. Finally, graphics of residuals or estimated random errors ($\hat{\varepsilon}$) were analyzed, of which the models without tendencies of residuals in function the dependent variable were selected. Equation (11) was used to compute estimated random errors in percentage.

$$SEE\ (\%) = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n - p}}\ \frac{100}{\bar{y}} \qquad (9)$$

$$\bar{R}^2 = \left[1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}\right] \cdot \left[\frac{n - 1}{n - p}\right] \qquad (10)$$

$$\hat{\varepsilon}\ (\%) = \frac{(y_i - \hat{y}_i)}{y_i} 100 \qquad (11)$$

Where: y_i and $\hat{y}_i$ = observed and estimated variables at i^{th} observation; $\bar{y}$ = average of observed variable; n = number of observations; p = number of coefficients excluding the intercept.

Modeling diameters along the bole:

Considering what was treated about obtaining the diameters at relative heights, the same 133 cubed boles of pine trees were used to adjust the models aiming to estimate diameters at relative heights in function to diameters at breast height (Dacosta, 2008), using the equation as it is presented in (12):

$$d_{rh} = b_0 + b_1 d_{1.3} \qquad (12)$$

Where: d_{rh} = diameter at relative height; $d_{1.3}$ = diameter at breast height; b_i = regression coefficients.

Using the Model (12) those four diameters at relative heights were estimated and tested on the data set aiming to readjust the selected form factor model, but now using the observed and/or estimated diameters at relative heights. This allows us to evaluate the behavior of volume estimation using the selected form factor model based on observed and estimated diameters at relative heights. Thus it was aimed to select the most economical and accurate way to estimate the bole volume of those pine trees.

RESULTS AND DISCUSSIONS

Initially the form factor models (3.1 to 6.4) were adjusted by ordinary least square method and we evaluated the significance of their coefficients by t test (Table 3). From the total of 56 parameters, 48 (86%) of them presented significance at 99% probability level by t test, what shows a strong relation between dependent and independent variables. Coefficients highly significant also were observed by Dacosta (2008) when adjusting form factor models in boles of *Pinus taeda* L., in forest stands located in the province of Corrientes, Argentina.

Table 3: Linear regression coefficients from form factor models for boles of *Pinus taeda* L. in south Brazil.

Variable tested	Model number	Coefficients of equations			
		b_0	b_1	b_2	b_3
Doubly combined	3.1	0.4314**	0.6515*	0.5041^{ns}	-
	3.2	0.4211**	1.5984**	-0.9129*	-
	3.3	0.4098**	3.1124**	-1.4347**	-
	3.4	0.4119**	4.8136**	-1.2846**	-
	4.1	0.4878**	6.6112**	1.0913**	-8.0710**
	4.2	0.4498**	8.8917**	0.3045^{ns}	-7.3571**
	4.3	0.4232**	6.1198**	-0.3465^{ns}	-2.8358**
	4.4	0.4169**	5.0416**	-0.9186*	-0.4316^{ns}
Triply combined	5.1	0.0927**	0.4506**	-0.0646^{ns}	-
	5.2	0.1601**	0.4515**	0.4505**	-
	5.3	0.2616**	0.4391**	0.9912**	-
	5.4	0.2792**	0.4427**	1.2608**	-
	6.1	0.0695**	0.3521**	0.2413**	0.1984**
	6.2	0.1004**	0.2778**	0.2583**	0.3421**
	6.3	0.2603**	0.2211**	0.2369**	0.6006**
	6.4	0.2772**	0.1909**	0.2452**	0.6825**

* and **: Significant at 95% and 99% of probability level; ns: not significant.

The t test indicated only three coefficients (5%) as significant at 95% probability level. Finally, five coefficients (9%) were not significant at 95% probability level, attributed to the variables hd^{-2} in the models 3.1, 4.2, 4.3, the variable d^{-1} in the model 4.4 and the variable $d_{0.5}d^{-2}$ in the model 5.1. Such results can be better understood by analyzing the Pearson's correlation coefficients between the form factor ("λ") and the tested variables in the models, as presented in Table 4.

Table 4: Pearson's correlation coefficients for variables employed in the form factor models.

Variables	λ	$d_{0.1}d^{-2}$	$d_{0.3}d^{-2}$	$d_{0.5}d^{-2}$	$d_{0.7}d^{-2}$	$d_{0.1}d_{0.3}d^{-2}$	$d_{0.1}d_{0.5}d^{-2}$	$d_{0.1}d_{0.7}d^{-2}$	$d_{0.3}d_{0.7}d^{-2}$	$d^2_{0.5}d^{-2}$	$d^2_{0.3}d^{-2}$	hd^{-2}
$d_{0.1}d^{-2}$	*0,34**											
$d_{0.3}d^{-2}$	*0,39**	$0,99^{**}$										
$d_{0.5}d^{-2}$	*0,46**	$0,96^{**}$	$0,97^{**}$									
$d_{0.7}d^{-2}$	*0,49**	$0,87^{**}$	$0,89^{**}$	$0,94^{**}$								
$d_{0.1}d_{0.3}d^{-2}$	*0,85**	$0,54^{**}$	$0,56^{**}$	$0,56^{**}$	$0,49^{**}$							
$d_{0.1}d_{0.5}d^{-2}$	*0,85**	$0,15^{ns}$	$0,18^{*}$	$0,32^{**}$	$0,31^{**}$	$0,66^{**}$						
$d_{0.1}d_{0.7}d^{-2}$	*0,67**	$-0,06^{ns}$	$-0,01^{ns}$	$0,11^{ns}$	$0,34^{**}$	$0,32^{**}$	$0,65^{**}$					
$d_{0.3}d_{0.7}d^{-2}$	*0,63***	$-0,24^{**}$	$-0,16^{ns}$	$-0,05^{ns}$	$0,17^{ns}$	$0,25^{**}$	$0,60^{**}$	$0,95^{**}$				
$d^2_{0.5}d^{-2}$	*0,64***	$-0,25^{**}$	$-0,19^{*}$	$-0,03^{ns}$	$0,03^{ns}$	$0,28^{**}$	$0,86^{**}$	$0,72^{**}$	$0,79^{**}$			
$d^2_{0.3}d^{-2}$	*0,83***	$0,11^{ns}$	$0,21^{*}$	$0,21^{*}$	$0,23^{**}$	$0,79^{**}$	$0,68^{**}$	$0,52^{**}$	$0,62^{**}$	$0,59^{**}$		
hd^{-2}	*0,28***	$0,92^{**}$	$0,92^{**}$	$0,92^{**}$	$0,89^{**}$	$0,37^{**}$	$0,12^{ns}$	$0,07^{ns}$	$-0,08^{ns}$	$-0,16^{ns}$	$0,07^{ns}$	
d^{-1}	*0,29**	$0,99^{**}$	$0,99^{**}$	$0,96^{**}$	$0,87^{**}$	$0,48^{**}$	$0,09^{ns}$	$-0,09^{ns}$	$-0,25^{**}$	$-0,23^{*}$	$0,07^{ns}$	0,93

* and **: significant at 95% and 99% of probability; ns: not significant at 95% probability ; Cells painted in dark gray show correlations larger than 70% and painted in light gray show correlations varying from 50% up to 70%.

Some variables presented weak correlation with form factor (λ), whose two lowest correlations occurred to hd^{-2} and d^{-1}, being both variables indicated as not significant at 95% probability by the*t* test. Regarding to the variable $d_{0.5}d^{-2}$, also has not showed significant when it was included in the model 5.1, however presented significance when it was included in the models 5.3 and 5.4. Such findings allow us to conclude that the variable $d_{0.5}d^{-2}$ contributed to increase the expected sum of squares of models that included$_{0.1}d_{0.7}d^{-2}$ or $d_{0.3}d_{0.7}d^{-2}$ as variables, but do not contribute significantly when the form factor model has included the variable $d_{0.1}d_{0.3}d^{-2}$. Indeed, these results make sense to what was observed in Table 4, since the correlations between $d_{0.1}d_{0.7}d^{-2}$ and

$d_{0.3}d_{0.7}d^{-2}$ with λ were lower when compared to correlation between $d_{0.1}d_{0.3}d^{-2}$ and λ, indicating that this last variable triply combined was enough to explain the form factor through model 5.1.

Other particular situation observed in Table 4 refers to the variables$d_{0.3}d^{-2}$ and$d_{0.5}d^{-2}$, whensubstituting the diameter measured on a relative height by its squared form, its correlation became stronger between $d^2_{0.3}d^{-2}$ and $d^2_{0.5}d^{-2}$ with the form factor.

Regarding to the incremental contribution of the triply and doubly combined variables, the F test indicated significance at the 95% or 99% probability in all models, what lead us to keep all the tested variables in their correspondent models (Table 5).

Table 5: Incremental contribution for variables tested in the form factor models for boles of *Pinus taeda* L. in south Brazil.

Model	Variable tested	F test	Conclusion
3.1	$d_{0.1}d^{-2}$	5.37*	Keep $d_{0.1}d^{-2}$ in the model
3.2	$d_{0.3}d^{-2}$	17.86**	Keep $d_{0.3}d^{-2}$ in the model
3.3	$d_{0.5}d^{-2}$	43.13**	Keep $d_{0.5}d^{-2}$ in the model
3.4	$d_{0.7}d^{-2}$	54.76**	Keep $d_{0.7}d^{-2}$ in the model
4.1	$d_{0.1}d^{-2}$	60.30**	Keep $d_{0.1}d^{-2}$ in the model
4.2	$d_{0.3}d^{-2}$	190.11**	Keep $d_{0.3}d^{-2}$ in the model
4.3	$d_{0.5}d^{-2}$	109.78**	Keep $d_{0.5}d^{-2}$ in the model
4.4	$d_{0.7}d^{-2}$	56.53**	Keep $d_{0.7}d^{-2}$ in the model
5.1	$d_{0.1}d_{0.3}d^{-2}$	229.99**	Keep $d_{0.1}d_{0.3}d^{-2}$ in the model
5.2	$d_{0.1}d_{0.5}d^{-2}$	351.03**	Keep $d_{0.1}d_{0.5}d^{-2}$ in the model
5.3	$d_{0.1}d_{0.7}d^{-2}$	132.58**	Keep $d_{0.1}d_{0.7}d^{-2}$ in the model
5.4	$d_{0.3}d_{0.7}d^{-2}$	153.24**	Keep $d_{0.3}d_{0.7}d^{-2}$ in the model
6.1	$d_{0.1}d_{0.3}d^{-2}$	447.92**	Keep $d_{0.1}d_{0.3}d^{-2}$ in the model
6.2	$d_{0.1}d_{0.5}d^{-2}$	137.81**	Keep $d_{0.1}d_{0.5}d^{-2}$ in the model
6.3	$d_{0.1}d_{0.7}d^{-2}$	15.42**	Keep $d_{0.1}d_{0.7}d^{-2}$ in the model
6.4	$d_{0.3}d_{0.7}d^{-2}$	9.81**	Keep $d_{0.3}d_{0.7}d^{-2}$ in the model

d = diameter at breast height; $d_{0.1}$, $d_{0.3}$, $d_{0.5}$, $d_{0.7}$ = diameter at 10%, 30%, 50% and 70% of bole height, respectively; * and **: significant at 95% and 99% of probability level; ns: not significant at 95% probability level.

By means of the F test values showed in the Table 5 it was observed the occurrence of most cases of incremental contributions significant at 99% probability, compared to the statistical significance, at 95% probability, which occurred only on model 3.1, indicating a strong contribution of the tested variables to increase the expected value of the sum of squares of the model. Regarding to doubly combined variables of the model 3, it was observed a clear tendency to increase the contribution when the heights are raised from 10% to 70%, whereas the opposite occurred for those triply combined variables of the model 6, in which the inclusion of diameters at larger heights provoked a decreasing contribution (F test).

Such results imply that when the diameters at relative heights are combined only with d^{-2}, there is a trend in increasing the coefficient of determination (R^2) proportionally to the height where the diameter was measured, while if it is included one more diameter measured at a relative height in the combined variables, such relation becomes inversely proportional, i.e., the inclusion of diameters measured at higher heights tends to decrease the R^2 value.

The strongest incremental contribution was observed in the model 6.1, whose specification had included $d^{0.1}d^{0.3}d^{-2}$ as tested variable. Curiously, that the same variable presented a weakest incremental contribution when tested in the model 5.1, whose difference between both models is the presence of hd^{-2} as independent variable, what strengthens the previously reported information on distinct behaviors on models when using the same variable. In addition, it is suggested to test models in which height is included as independent variable, even if diameters at relative heights have being used in the model.

Table 6 presented the accuracy statistics: standard error of estimate in percentage (SEE %), adjusted coefficient of determination in percentage ($\bar{R}^2$ %), Durbin-Watson's (DW) test for residual autocorrelation analysis and finally the White's test for residual heteroscedasticity analysis. Results obtained for all models are presented in the Table 6. SEE varied from 3.31% up to 9.98% and $\bar{R}^2$ varied from 10.66% up to 90.16%.

Table 6: Statistical tests for form factor models for boles of *Pinus taeda* L. in south Brazil.

Variable tested	Model	SEE (%)	$\bar{R}^2$(%)	DW test	White test	Conclusion about residues
Doubly combined	3.1	9.98	10.66	2.25^{ns}	9.49*	No autocorrelation but heteroscedasticity
	3.2	9.57	17.92	2.23^{ns}	8.95^{ns}	No autocorrelation and homoscedasticity
	3.3	9.45	19.83	2.35*	7.33^{ns}	Autocorrelation but homoscedasticity

	3.4	8.56	34.31	2.36*	5.42^{ns}	Autocorrelation but homoscedasticity
	4.1	8.45	36.01	1.95^{ns}	49.24**	No autocorrelation but heteroscedasticity
	4.2	6.51	62.04	2.00^{ns}	11.30^{ns}	No autocorrelation and homoscedasticity
	4.3	7.52	49.27	2.35*	43.92**	Autocorrelation and heteroscedasticity
	4.4	8.53	34.71	2.37*	12.93^{ns}	Autocorrelation but homoscedasticity
Triply combined	5.1	5.70	70.83	2.28^{ns}	0.34^{ns}	No autocorrelation and homoscedasticity
	5.2	5.09	76.74	2.15^{ns}	91.74**	No autocorrelation but heteroscedasticity
	5.3	6.68	60.01	2.31^{iz}	16.74*	Heteroscedasticity
	5.4	6.43	62.92	2.14^{ns}	23.90**	No autocorrelation but heteroscedasticity
	6.1	3.31	90.16	1.93^{ns}	35.37**	No autocorrelation but heteroscedasticity
	6.2	3.73	87.52	1.86^{ns}	24.35**	No autocorrelation but heteroscedasticity
	6.3	6.62	60.68	2.12^{ns}	33.45**	No autocorrelation but heteroscedasticity
	6.4	6.75	59.09	2.05^{ns}	33.21**	No autocorrelation but heteroscedasticity

SEE: standard error of estimate; $\overline{R}^2$: adjusted coefficient of determination; DW: Durbin-Watson; * and **: significant at 95% and 99% of probability level; ns: not significant at 95% probability level; iz: indecision zone.

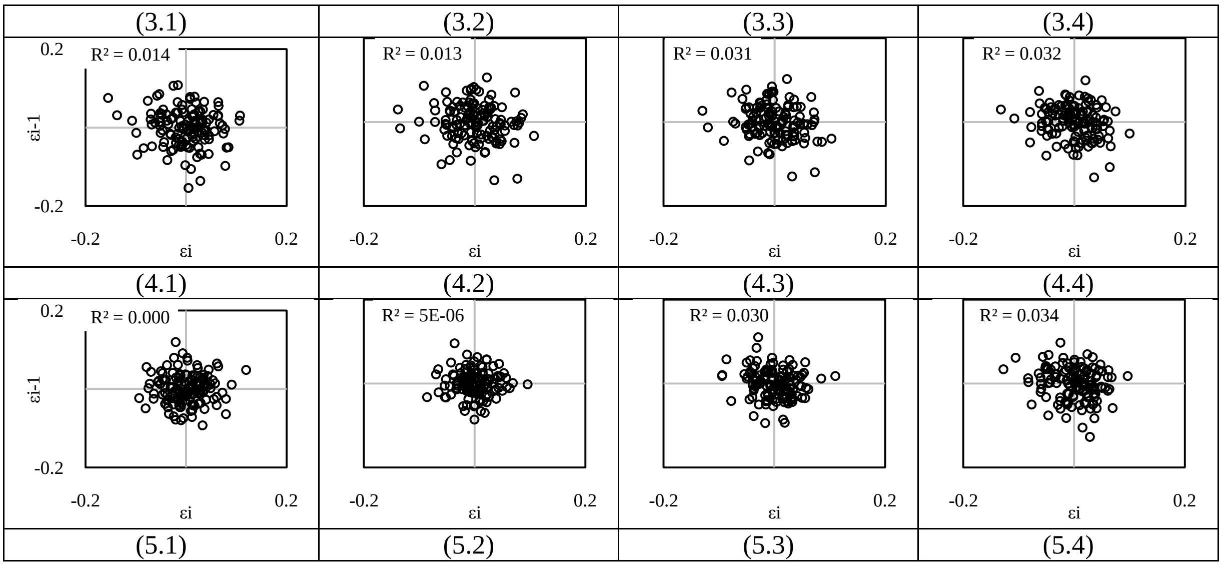
Considering only SEE (%) and $\overline{R}^2$(%), it was observed, in general, an improvement of both statistics in the models 3 to 6. Approximately, values of SEE (%) have reduced up to three times in the models 3 to 6, reaching the lowest error of 3%. Testing the same models proposed by us, Drescher *et al*. (2001) have obtained very similar errors when adjusting form factor in boles of *Pinus elliottii* Engelm. in forest stands in Rio Grande do Sul state, Brazil. Drescher *et al*. (2010) have not obtained likewise good results, when applying the same models in boles of *Tectona grandis* L. f. in young forest stands in Mato Grosso state, Brazil.

In relation to $\overline{R}^2$(%), its value has increased up to nine times more, reaching a ceiling of 90%. Such results showed that there is an expressive trend to reduce errors when more complex models are used instead the simpler ones. In other words, the triple combinations were more efficient than double combinations. Drescher *et al*. (2001) and Drescher *et al*. (2010) also obtained similar results for the coefficient of determination.

Accuracy statistics of models 3 improved when the position of the diameters at relative heights was changed and this issue was already discussed above. However, no trend was observed when it was changed the positions of the diameters in the model 4. For the models 5 and 6, in which triply combined variables were tested, those that contributed sometimes more and sometimes less to improve accuracy statistics were different among the models. However, there were invariably improvements from models 3 toward models 4 and from the 5th toward the 6th models, due to inclusion of the variables d^{-1} and hd^{-2} in the models 4 and 6, respectively.

Analyzing the Durbin-Watson's test for residual autocorrelation, four of the sixteen models have presented autocorrelation of residuals at the 95% probability level. In the other models, Durbin-Watson's tests have presented not significant autocorrelated residuals at the same probability level, except for model 5.3 whose DW calculated value was framed in the indecision zone, preventing us to conclude decisively about it.

In order to evaluate visually the dispersion between nearest residues (relationship between ε_i and ε_{i-1}), whose correlations were used in Durbin-Watson test calculation, the residues of the all sixteen models were graphically plotted and are presented in the Figure 1. Furthermore, simple linear models were adjusted and their coefficients of determination (R²) are presented in Figure 1, where the number of models is shown in parentheses.



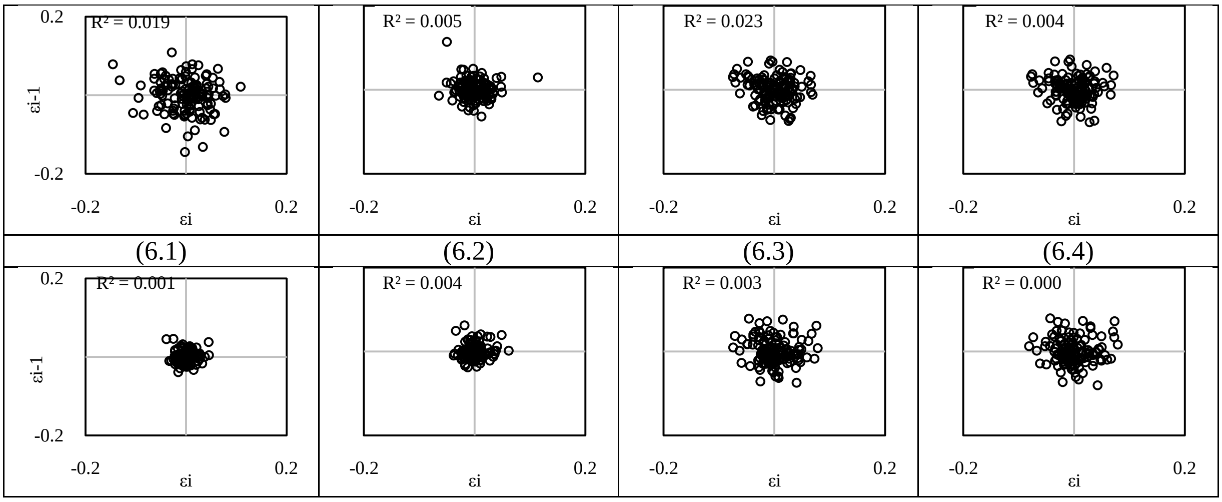


Fig. 1: Dispersion of residuals from form factor models adjusted in boles of *Pinus taeda* L. in south Brazil.

Through the Figure 1, we can note that, in general, there is a low correlation and absence of tendency in the residuals, even in the models (3.3, 3.4, 4.3 and 4.4), whose residuals have showed autocorrelations at 95% probability by Durbin-Watson's test. The residual dispersions tend to concentrate in the center of the graphic from the models 3 to the models 6, confirming what it was mentioned before about reduction of SEE.

White's test indicated eight models of the sixteen with heteroscedasticity of residuals at the 99% probability, two models with heteroscedasticity of residuals at 95% probability and six models not significant for heteroscedasticity at this same probability level (Table 6).

Curiously, the combination of variables tested in models 5.1 to 6.4 provided satisfactory results with respect to autocorrelation (Table 6), however they presented undesirable results if the residual heteroscedasticity is considered. Three of these models (5.2, 6.1 and 6.2) showed better residual dispersion but with heteroscedasticity.

Considering the models 3.1 to 4.4, in which doubly combined variables were used, a better scenario was observed, because in most cases the tests have not indicated significance at 95% probability level, highlighting models 3.2 and 4.2 without autocorrelated residuals and homoscedasticity.

These two models have in common the doubly combined variable $d^{0.3}d^{-2}$ and the difference between them is the variable inverse of the diameter at breast height (d^{-1}), which was included only in the model 4.2. Though specifications of these two models are well similar, it is relevant to emphasize that the inclusion of the third variable d^{-1} contributed to reduce the SSE in almost 32%. In this sense, it is important to test different specifications even when diameters at relative heights are included in the form factor models.

A brief discussion about heteroscedasticity and autocorrelation:

Maddala and Lahiri (2009) have explained that the problem of heteroscedasticity may generate errors linked to tests which are based on variance of the coefficients of the linear model, since the coefficients $\hat{b}_i$ in a model with heteroscedasticity of residuals are inefficient and do not provide, for example, proper confidence intervals. When the error variance σ_i^2 of a model with heteroscedasticity, as presented in (13), a transformation of variables using the generalized least square method provides the Best Linear Unbiased Estimators (BLUE). Gujarati (2008) proposed this transformation waiting the variables by σ_i as described in Model (14.1) and (14.2):

$$\frac{y_i}{\sigma_i} = b_0\frac{x_{0_i}}{\sigma_i} + b_1\frac{x_i}{\sigma_i} + \frac{\varepsilon_i}{\sigma_i} \qquad (14.1)$$

Biased estimators b_i become BLUE, where they are represented by b_i^* in Model (14.2):

$$y_i^* = b_0^*x_{0_i}^* + b_1^*x_i^* + \varepsilon_i^* \qquad (14.2)$$

When the linear regression is performed without considering the evaluation of the heteroscedasticity, the key problem highlighted by Gujarati (2008) refers to super estimation of variance of b1 ($\sigma_{b_1}^2$) in the Model (13). As commented before, confidence intervals have become unreliable, due to the consequences provoked by the variance problem. Furthermore, higher values of variances can change the results of the t and F tests for such estimators, since both also are dependent on the estimated variance.

Knowing that the overestimated variances tend to reduce the calculated t value resulting in coefficients not significant in the models, it is not appropriated judging the heteroscedasticity as a very serious problem, considering that only three coefficients were not significant at 95% probability (Table 3), what indicate values of$\sigma_{b_1}^2$ not so expressive. In this case, in spite of the detected heteroscedasticity and attested by White's test

(Table 6) for the majority of the tested models, these results are sufficiently adequate, but other significance tests would be recommended for detection of heteroscedasticity when the presentation of confidence intervals are relevant.

Regarding the autocorrelation between residuals, the models have presented almost no problems for the evaluation of heteroscedasticity. The models had not presented residual autocorrelation indicated by Durbin-Watson's test at 99% probability and, in four cases only, the test indicated autocorrelation at 95% probability.

Low autocorrelation between residuals can be seen in Figure 1, in which the residuals have not presented tendency to autocorrelations. Gujarati (2008) explains that the problem of residual autocorrelation generates exactly the same consequences encountered in the case of heteroscedasticity. Analogously the b_1 estimator of the model (13) resulted in super estimation of variance $\sigma^2_{b_1}$. Thus, even the models with heteroscedasticity of residuals or with autocorrelation between them have presented unreliable variance of coefficients. They have presented unbiased coefficients whose sample average is the closest to the population average.

Considerations and recommendations to select form factor models:

After performed all statistical analysis and discussed the problem of residual heteroscedasticity and autocorrelation, the final consideration requires clarifying how the residuals are distributed. Knowing its importance as criterion to select models, disturbances in the linear model has been a subject well studied by several researchers dealing with linear regression field, mainly to demonstrate the complications caused by residuals which appear to be unbiased, according to Teil (1965), Dubbelman (1978) and Cook and Weisberg (1982).

In this sense, it was considered essential that models shall be evaluated according to the behavior of residual distributions regarding the observed artificial form factors (Figure 2). In most cases, residues will distribute themselves within a percentage interval of ± 30%.

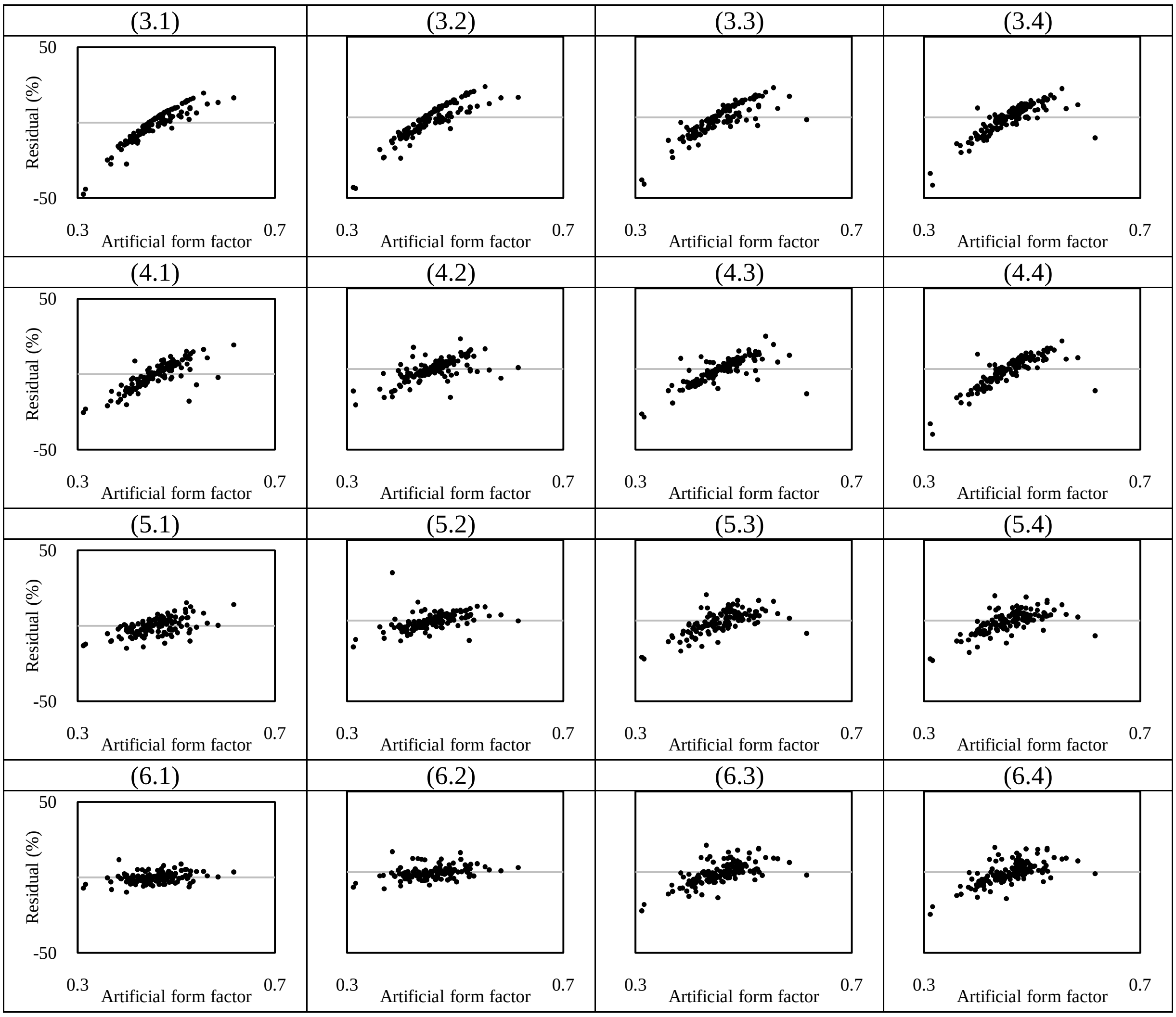


Fig. 2: Scatter plot of residuals in relation to the observed form factor in boles of *Pinus taeda* L. in south Brazil.

Analyzing all graphics, it is noted that some models presented bias problems, mainly in the models 3.1 up to 5.1. In other circumstances the bias problems were not present. In models 6.1 and 6.2 the best residual dispersion was observed, where no tendency occurred along the x axis (observed artificial form factor).

Model 3.1 was the only one adjusted without employing diameters at relative heights as independent variable. Due to this, it was noted a different pattern of residual dispersion, when compared to the other models. Though no tendency was noted for the residual dispersion in model 3.1, its distribution was not as efficient as was observed on the models 6.1 and 6.2.

Finally on the last considerations, in order to recommend the best form factor model after everything that was discussed, we reinforce about both important problems: residual heteroscedasticity and autocorrelations, since they should not bother us when $\sigma^2_{\hat{b}_i}$ is an estimator of no interest to the researcher.

Under such conditions in which an analysis regarding confidence intervals, tests t and F are not required, we recommend the form factor model numbered 6.1, since it presented the lowest SEE, the best residual distribution and no autocorrelation among residuals, beyond the significance of coefficients and the evaluation of heteroscedasticity that did not revealed be a serious problem.

Application of the methodology aiming to estimate bole volume:

According to what was discussed before, the form factor models are used in forest inventories to obtain bole volume. When the bole volume is estimated by the form factor model , as in the Model presented in (15), the estimated residues ($\hat{"\varepsilon"}$) are the same because the form factor is obtained directly from the volume. Thus, the graph of the model 6.1, presented in the Figure 2, suggests no bias occurs in the form factor function. However it does not imply that the statistical estimators of the observed volumes should be unbiased.

$$\hat{v} = \hat{\lambda} g_{1.3} h, \text{ where in this case } \hat{\lambda} = (\hat{b}_0 + \hat{b}_1 d_{0.1} d_{0.3} \hat{d}^{-2} + \hat{b}_2 d_{0.5}^2 d^{-2} + \hat{b}_3 h d^{-2}) \quad (15)$$

Where: $\hat{\lambda}$ = estimated form factor; $g_{1.3}$ = cross-sectional area at breast height, in m² and h = total height of the bole, in m; d = diameter at breast height, in cm; $d_{0.1}$, $d_{0.3}$, $d_{0.5}$= diameters at 10%, 30%, 50% of the bole height, in cm, respectively; $\hat{b}_i$= parameters of the model.

As proposed in the methodology, we evaluated the behavior of these volume estimations making a combination of observed and estimated diameters at relative heights by using a simple linear model in function of diameter at breast height. In the Table 7 are presented accuracy statistics for the Model (15), in which eight possibilities of observed and estimated variables were combined.

The aim with such procedure was to detect how much the resulting accuracy from the volume model could be worse when it were used estimated diameters at relative heights. Furthermore, we sought to know how much the combined variables could result in smaller errors in the volume estimation. The coefficients used to estimate the volumes in the Model (15) were taken from the model 6.1 (Table 3).

Table 7: Accuracy statistics for volume estimation using form factor models based on observed and estimated diameters at relative heights, besides the coefficients from diametric models.

Possibilities	Accuracy statistics	
	SEE (%)	R² (%)
$d_{0.1}$, $d_{0.3}$ and $d_{0.5}$ estimated	12.98	98.36
$d_{0.1}$ and $d_{0.3}$ estimated, $d_{0.5}$ observed	7.40	99.47
$d_{0.1}$ and $d_{0.5}$ estimated, $d_{0.3}$ observed	5.15	99.74
$d_{0.3}$ and $d_{0.5}$ estimated, $d_{0.1}$ observed	8.28	99.33
$d_{0.1}$ estimated, $d_{0.3}$ and $d_{0.5}$ observed	4.29	99.82
$d_{0.3}$ estimated, $d_{0.1}$ and $d_{0.5}$ observed	6.06	99.64
$d_{0.5}$ estimated, $d_{0.1}$ and $d_{0.3}$ observed	4.59	99.80
$d_{0.1}$, $d_{0.3}$ and $d_{0.5}$ observed	*3.03*	*99.91*
Simple linear diametric model based on diameter at breast height	Coefficient	
	Intercept	Slope
($d_{0.1}$) – diameter at 10% of bole height	1.6678	0.9035
($d_{0.3}$) – diameter at 30% of bole height	-0.1427	0.8249
($d_{0.5}$) – diameter at 50% of bole height	-1.4389	0.6941

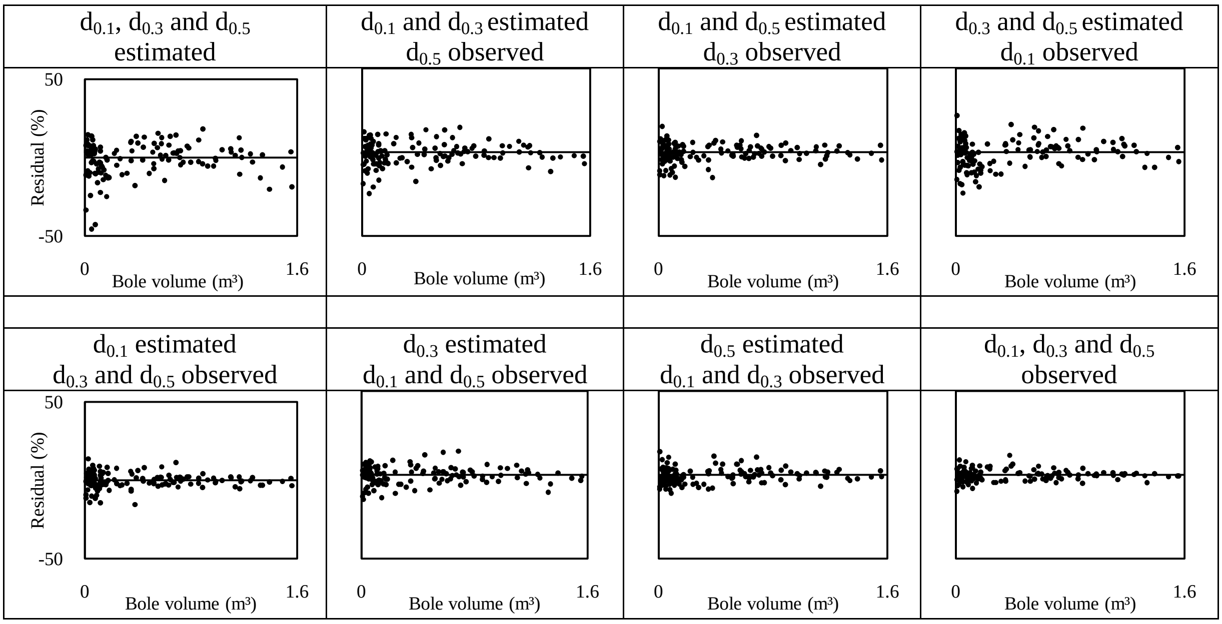
SEE: standard error of estimate; R^2: coefficient of determination.

It was expected that the form factor models based on estimated diameters would result in the worst volume estimation. Indeed, these can be observed by comparing the case in which the three relative diameters were estimated with the other combinations when only the relative observed diameters were used. Confronting these two possibilities, the SEE (%) values have reduced from approximately 13% to 3%.

Of course if it is measured these three diameters at relative heights, the obtained errors on volume estimate will be smaller than if only two of them are measured. In this last situation, admitting a maximum error of 5%, in the Table 7 it is clear that the better to be done is: estimating $d_{0.1}$, and measure $d_{0.3}$ and $d_{0.5}$, or estimating $d_{0.5}$ and to measure $d_{0.1}$ and $d_{0.3}$. Considering the difficulty to measure diameters at higher heights, the last recommendation is better. However, if it is measured only one relative diameter and the others are estimated, the error of 5% will be exceeded. As can be observed in Table 7, in this situation the better to be done is to estimate $d_{0.1}$ and $d_{0.5}$ and measure $d_{0.3}$, since this combination provides the smallest error of all proposed situations.

However, except the situation in which it was tested the models to estimate the three diameters, the possibilities presented in the Table 7 have resulted in better accuracy statistics when comparing to some works done on methods for bole volume estimation, as presented by Araújo *et al.* (2012), Li and Weiskittel (2010), Ozdemir (2008), Diamantopoulou (2006), Machado *et al.* (2005) and Eerikäinen (2001).

Figure 3 illustrates the residuals in function of observed bole volume, in m³, for the eight combined possibilities as presented in the Table 7.



According to Figure 3, the fact of the diameters at relative heights be obtained by a linear regression model has not provoked biased residuals, what allows us to confirm that the modeling is proper to be used in volume estimations. However, there was tendency of increasing the scatter of the points, when diameters were estimated and not measured.

Conclusions:

The use of combined variables of diameters measured in relative heights with diameter at breast height contributes to reduce standard error of estimate and to raise the coefficient of determination in the form factor models. However, such combinations can provoke heteroscedasticity in the model and autocorrelation among residuals.

The combined diameters used as independent variables have a strong tendency to provoke bias in the form factor model; therefore it is highly important evaluating them based on the behavior of their residual dispersion. The triply combined variables ($d_{rh1}d_{rh2}d^{-2}$, where rh is relative height) contribute more to reduce the errors and let the models to be less unbiased, when compared with the effect of the doubly combined variables ($d_{rh}d^{-2}$, where rh is relative height).

Under the conditions in which an analysis regarding confidence intervals, tests t and F are not required, the form factor model 6.1 is the best option to be selected, since it presented the lowest SEE, the best residual distribution and no autocorrelation among residuals, beyond the significance of coefficients and the evaluation of heteroscedasticity that did not revealed be a serious problem.

It is possible to obtain volume in form factor models based on estimated diameters at relative heights, with an admissible error close to 5% on the estimated volume.

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