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ACCURACY ANALYSIS OF WRIGHT'S CAPABILITY INDEX "CS" AND MODELLING NON-NORMAL DATA USING STATISTICAL SOFTWARE-A COMPARATIVE STUDY

Abstract: *Process Capability Indices (PCI) has been widely used as a means of summarizing process performance relative to set of specification limits. The proper use of process capability indices are based on some assumptions which may not be true always. Therefore, sometime whether the process capability indices can truly reflect the performance of a process is questionable. Most of PCIs, including Cp, Cpk, Cpm and Cpmk, neglect the changes in the shape of the distribution, which is an important indicator of problems in skewness-prone processes. Wright proposed a process capability index 'Cs' to detect shape changes in a process due to skewness by incorporating a penalty for skewness. In this paper, the effect of skewness on assessment of accuracy of Wright's capability index Cs is studied and comparison is made with Cp, Cpk, Cpm and Cpmk indices when the distribution of the quality characteristic (spring force) considered is skewed slightly. This paper also discusses how modelling the non normal data using statistical software and results were compared with other methods.*

Keywords: *Non- normal distribution, Process capability index, Skewness, Modeling non-normal data, normality*

1. Introduction

Process capability indices are widely used as a means to determine whether a process is capable of producing items on target and within a specific tolerance. There are number of process capability indices. The index Cp considers the overall process variability relative to the manufacturing tolerance, reflecting the product quality consistency. The index Cpk takes the process mean into

consideration but fail to distinguish between on-target processes from off-target processes. Chan *et al.*, (1988) have developed a more advanced process capability index, which has been referred to as Cpm. Pearn *et al.*, (1994) constructed another index Cpmk, designated by utilizing modifications of Cp that produced Cpk and Cpm (Gildeh and Asghari, 2011). These indices have been defined explicitly as follows:

$$Cp = \frac{USL - LSL}{6\sigma}, \quad (1)$$

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$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\}, \quad (2)$$

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(\mu-T)^2}}, \quad (3)$$

$$C_{pmk} = \min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\} \quad (4)$$

Where μ is the process mean, σ is the process standard deviation, T is the target value, USL and LSL are the upper and the lower specification limits respectively. It is essential that PCIs must be applied under the condition that the process is in statistical control. The estimation of PCI's introduced above is based on the assumption that the process is characterized by a normal process with symmetric tolerances. Most of PCI's, including C_p , C_{pk} , C_{pm} and C_{pmk} neglecting the changes in the shape of the distribution, which is an important indicator of problems in skewness-prone processes. When the distribution of a process quality characteristic is non-normal, which occurs frequently in practice, these process capability indices calculated using conventional method often lead to erroneous interpretation of the process capability.

The C_{pmk} index (Gildeh *et al.*, 2014; Sagbas *et al.*, 2014) provides an indication of increase in the process variation and the process departure from the target, but is not sensitive to the changes in the shape of the distribution, particularly its skewness. A modification of C_{pmk} proposed by Wright (1995) incorporates a skewness term in the denominator to reduce the index value when non-symmetry is present. Utilizing the third central moment μ_3 as a measure of skewness, Wright's index 'Cs' possesses the C_{pmk} characteristic of applicability to processes whose mean may not be centred between the specification limits and moreover may not be centred at the target. The index 'Cs' is defined as follows:

$$C_s = \frac{\min(USL-\mu), (\mu-LSL)}{3\sqrt{\sigma^2+(\mu-T)^2+\left|\frac{\mu_3}{\sigma^3}\right|}}, \quad (5)$$

OR

$$C_s = \frac{(d-|\mu-T|)/\sigma}{3\sqrt{\sigma^2+(\mu-T)^2+\left|\frac{\mu_3}{\sigma^3}\right|}} \quad (5)$$

Where, $d = (USL - LSL) / 2$ is the half-interval of the specification range. $\frac{\mu_3}{\sigma^3}$ is the skewness coefficient. The term μ_3 is divided by σ^3 to ensure that the skewness term is expressed in the same units as the other terms in the denominator and the absolute value of this term guarantees that a negative skewness will also incur a penalty. In the numerator, the well known identity $\min(x, y) = (x + y) / 2 - |x - y| / 2$, $x, y \in \mathbb{R}$ is utilized.

In practice, it is required to estimate the unknown process mean μ , the process standard deviation σ , and the third central moment μ_3 , to calculate Wright's C_s . Usually these are estimated from a sample of size n . A natural estimator of C_s is proposed by Wright (1995) to be:

$$C_s = \frac{d-|\bar{x}-T|}{3\sqrt{\frac{1}{n}\sum_{i=1}^n(x_i-T)^2 + \left|\frac{n^2 m_3}{(n-1)(n-2)} \times \left(\frac{n}{n-1} \times \frac{m_2}{c_4^2}\right)^{-\frac{1}{2}}\right|}} \quad (6)$$

$$\text{Where } m_2 = \sum_{i=1}^n (X_i - \bar{X})^2 / n, \quad (7)$$

$$m_3 = \sum_{i=1}^n (X_i - \bar{X})^3 / n \quad (8)$$

These are the sample central moments and

$$c_4 = \left[\frac{2}{n-1}\right]^{\frac{1}{2}} \times \Gamma(n/2) \Gamma\left[\frac{n-1}{2}\right]^{-1} \quad (9)$$

c_4 is the corrector for the bias.

Wright investigated this estimator C_s and studied its bias and variance using simulation for normal distributed processes. By evaluating the percentage bias in C_s and C_{pmk} , he has discovered that \hat{C}_s is more accurate estimator than C_{pmk} for small samples from off-target processes, when significant shifts in the mean have been occurred. For skewed distributions, Pearn and Chang (1997) examined the bias of \hat{C}_s ,

which turn out to be quite substantial. Chan and Kotz (1996) investigate the distributional properties of Wright's index and reach the conclusion that the asymptotic behaviour of the estimator \hat{C}_s is actually sensitive to skewness, and the contribution of the overall variability of the skewness correction factor is additive and non-interactive. These authors also substantiate Wright's conclusion that the asymptotic distribution of \hat{C}_s is normal when $\mu \neq T$ and $\mu_3 \neq 0$, which makes it a natural choice for process capability assessment for large sample sizes, regardless of the underlying distribution of the data. In general the C_s index provides a statistically sound approach to process capability analysis under the three conditions:

- a) $USL - \mu \neq \mu - LSL$ (i.e. $\mu \neq M = (USL + LSL) / 2$), b) $\mu \neq T$ and c) $\mu_3 \neq 0$

In the case study discussion, the process is assumed to be exactly centered ($\mu = T$) which implies that $C_s = C_{pk} = C_{pmk}$ for symmetric distributions.

2. Methodology

Methodology involves the following steps:

- Understanding the basic concepts of process capability and its measures.
- Process data collection.
- Calculate required statistics
- Validate the normality assumptions.
- Estimation of C_p , C_{pk} , C_{pm} and C_{pmk} indices.
- Estimation of Wright's Capability Index 'Cs'
- Modelling the non-normal data using statistical software (Wu *et al.*, 2007)
- Analysis and results of process capability study

3. Case study data

In order to discuss and assess the accuracy of Wright's method and to deal with skewness issues, the case study data consists of measurement of resistance for 125 coils. The measurement unit is ohm. The resistance of coils is one of the important quality characteristic to be controlled. Specification of the desired quality characteristic is 60 ± 12.50 ohm. Upper and Lower specification limits are 72.50 ohm and 47.50 ohm respectively. The specification mean is 60.00 ohm. Table 1 represents descriptive statistics for case study data and Table 2 presents measurements of resistance in ohm.

Table 1. Descriptive Statistics for case study data

Descriptive Statistics for case study data	
Mean: 60.657	Kurtosis : 1.63
Average of Average: 60.6568	Skewness : 0.81
Standard deviation : 4.067(over all)	Std Deviation(with in SG) = 3.889
Variance:16.540	Median :60.000
Range: 23.500	Minimum: 52.300,Maximum:75.800

Table 2. Measured values of Coil resistance in ohm (Rational groups)

Sl no	X1	X2	X3	X4	X5	X bar	Range
1	57.6	60.9	56.7	65.8	62.6	60.72	9.1
2	58.7	58.1	60.6	56.4	59.4	58.64	4.2
3	59.9	61.6	60.1	57.9	61.5	60.20	3.7
4	61.3	64.9	57.8	68.6	64.4	63.40	10.8
5	59.2	72.7	58.8	61.7	75.8	65.64	17.0
6	65.4	58.8	63.0	63.6	62.1	62.58	6.6
7	63.6	59.1	62.2	63.5	64.3	62.54	5.2
8	62.0	59.5	58.6	66.3	58.0	60.88	8.3
9	65.7	62.5	57.1	66.1	62.3	62.74	9.0
10	54.9	70.5	59.8	55.8	64.0	61.00	15.6
11	69.0	60.4	72.7	58.7	58.8	63.92	14.0
12	60.5	59.5	62.7	57.6	57.9	59.64	5.1
13	59.5	61.5	63.9	59.4	54.1	59.68	9.8
14	55.2	58.1	53.4	56.4	60.8	56.78	7.4
15	61.5	57.4	53.3	59.9	64.5	59.32	11.2
16	65.2	57.0	56.9	58.5	58.2	59.16	8.3
17	58.0	62.0	62.1	52.3	66.6	60.20	14.3
18	63.2	67.0	56.5	59.3	57.5	60.70	10.5
19	60.6	56.4	57.0	64.2	59.9	59.62	7.8
20	60.8	59.4	62.6	61.9	61.1	61.16	3.2
21	59.2	55.4	60.8	60.0	63.7	59.82	8.3
22	61.1	52.7	63.9	59.3	59.6	59.32	11.2
23	52.8	58.5	58.3	67.2	62.3	59.82	14.4
24	63.6	58.0	61.1	63.1	59.4	61.04	5.6
25	58.5	60.8	59.0	56.0	55.2	57.90	5.6

4. Process capability Analysis for the case study data

Process Capability Analysis was analyzed by many authors (Prabhuswamy and Nagesh, 2007; Wooluru *et al.*, 2014; Chang and Bai, 2002; according to Kotz and Johnson (1993), the following critical assumptions were made and checked before establishing the process capability indices:

- 1) The process must be in the state of statistical control.

- 2) The quality characteristic has a normal distribution.
- 3) In the case of two sided specifications, the process mean is centred between the lower and upper specification limits.
- 4) Observations must be random and independent to each other.

All the above assumptions will be verified as follows.

4.1. Construction of \bar{X} and R- Chart to assess the stability of the process

It has been observed from the Figure 1 that all the plotted sample range and mean values are fall within the control limits of both R-Chart as well as X-Bar chart but there is no random variation of sample mean on X-bar chart and indication of shift run has been noticed. Hence, it is concluded that the process is not under statistical control and not operating under the influence of only chance causes of variation.

Control limits for \bar{X} - Chart

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 60.6568 + [(0.577)(9.048)] = 66.02$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 60.6568 - [(0.577)(9.048)] = 55.30$$

Control limits for R-Chart

$$UCL = D_4 \bar{R} = 2.114 \times 9.048 = 19.65$$

$$LCL = D_3 \bar{R} = 0.00 \times 9.048 = 0.000$$

From Table 1, for n=5 $A_2 = 0.577$.

$$d_2 = 2.326. D_3 = 0.00. D_4 = 2.114$$

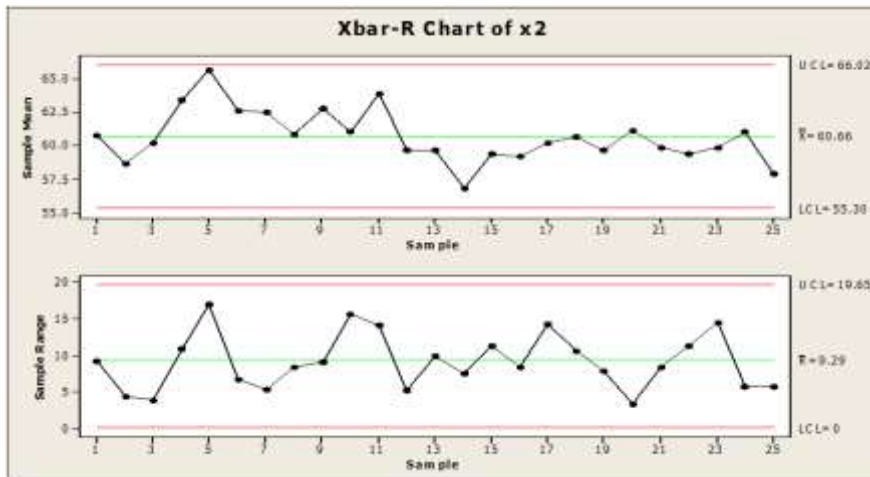


Figure 1. Control charts for case study data

It has been observed from the Figure 1 that all the plotted sample range and mean values are fall within the control limits of both R-Chart as well as X-Bar chart but there is no random variation of sample mean on X-bar chart and indication of shift run has been noticed. Hence, it is concluded that the process is not under statistical control and not operating under the influence of only chance causes of variation.

4.2. Histogram and Normal probability plot for validating the Normality assumption

Graphical methods including the histogram

and normal probability plot are used to check the normality of the data. Figure 2 display the histogram for the case study data and Figure 3 display the normal probability plot of 125 observations of case study data. The sample data appears to be non normal and positive skewed.

Test results of normal probability plot for the case study data from MINITAB -14 statistical software output shows that P-value: 0.010 is less than the significance level ($\alpha = 0.05$). This implies that the data was distributed non-normally. Thus, it is concluded that the sample data cannot be regarded as taken from the normal process.

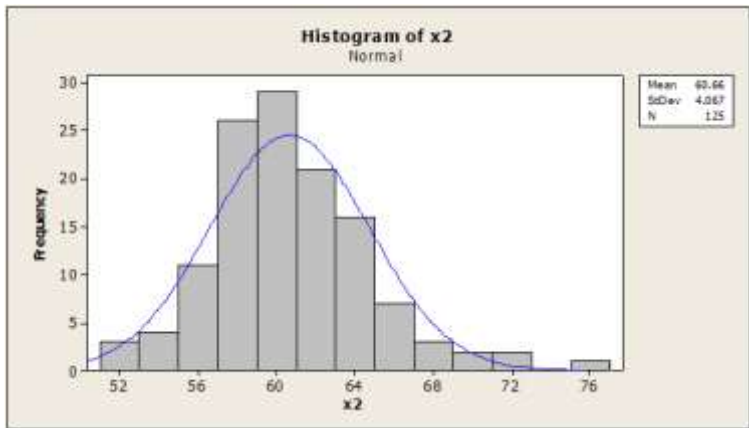


Figure 2. Histogram for capability case study data

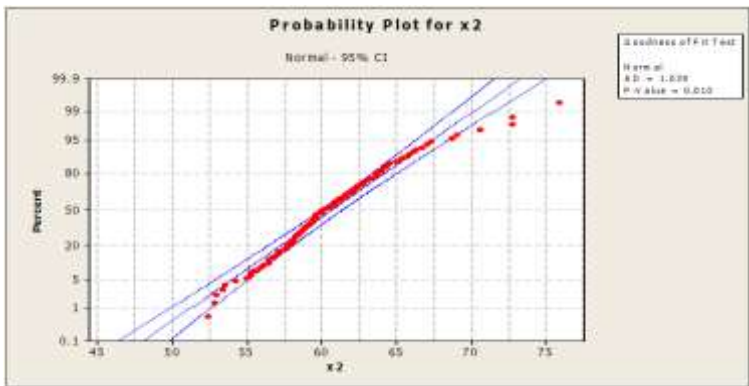


Figure 3. Normal Probability Plot for case study data

4.3. Construction of Run chart for checking the assumption of Randomness

where P - values for clustering, mixtures and oscillation and trend are greater than α value of 0.05.

Figure 4 shows Run chart for case study data

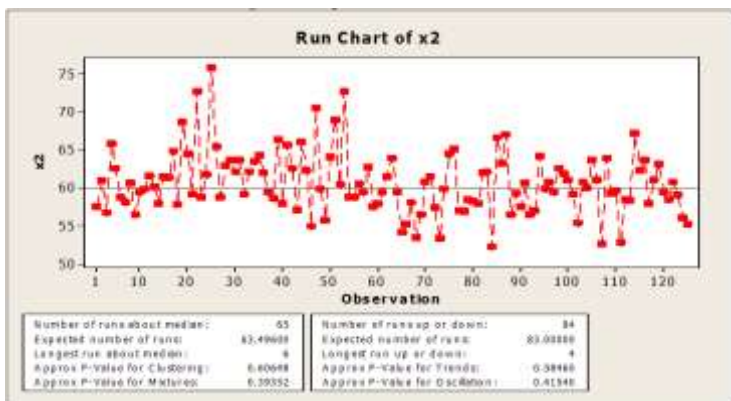


Figure 4. Run chart for case study data

The actual numbers of runs are close to the expected number of runs. Hence, it was concluded that the observations are random.

5. Quantification of Process Capability Indices

5.1. Process Capability Index - C_p

It simply relates the process capability to the specification range and it does not relate the location of the process with respect to the specifications. Values of C_p exceeding 1.33 indicate that the process is adequate to meet the specification. Values of C_p between 1.33 and 1.00 indicate that the process is adequate to meet specification but require close control. Values of C_p below 1.00 indicate the process is not capable of meeting specification. If the process is centred within the specification and is approximately “normal” then $C_p = 1.00$ results in a fraction nonconforming of 0.27%. It is also known as process potential.

The process standard deviation for the case study data (σ') = 3.8899

Process capability index:

$$C_p = \frac{(USL-LSL)}{6 \sigma'} = \frac{(72.50-47.50)}{(6 \times 3.8899)} = 1.07$$

Process capability (Overall):

$$Pp = \frac{(USL-LSL)}{6 \sigma'} = \frac{(72.50-47.50)}{(6 \times 4.067)} = 1.02$$

5.2. Process capability index - C_{pk}

(Second- generation capability index, developed from the original C_p). This process capability index considers process average and evaluates the process spread with respect to where the process is actually located. The magnitude of C_{pk} relative to C_p is a direct measurement of how off-centre the process is operating. It assumes process output is approximately normally distributed.

$$C_{pk} = \min \left\{ \frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma} \right\},$$

$$C_{pk} = \text{Min}(C_{pu}, C_{pl})$$

$$C_{pk} = \text{Min} \left(\frac{72.50-60.657}{3 \times 3.8899}, \frac{60.657-47.50}{3 \times 3.8899} \right),$$

$$C_{pk} = \text{Min}(C_{pu} = 1.0148, C_{pl} = 1.127)$$

Therefore,

$$C_{pk} = 1.01$$

$$P_{pk} = \min \left\{ \frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma} \right\},$$

$$P_{pk} = \text{Min}(C_{pu}, C_{pl})$$

$$P_{pk} = \text{Min} \left(\frac{72.50-60.657}{3 \times 4.067}, \frac{60.657-47.50}{3 \times 4.067} \right),$$

$$P_{pk} = \text{Min}(C_{pu} = 0.97, C_{pl} = 1.08)$$

Therefore,

$$P_{pk} = 0.97$$

5.3. Process capability index- C_{pm}

(Second- generation capability index, developed from the original C_p). It estimates process capability around the target T , it is always greater than zero and assumes process output is approximately normally distributed. It is also known as the Taguchi capability index, introduced in 1988. C_{pk} measures how well the process mean is centred within the specification limits, and what percentage of product will be within specification limits. Instead of focusing on specification limits, C_{pm} focuses on how well the process mean corresponds to the process target, which may or may not be midway between the specification limits. C_{pm} is motivated by Taguchi’s “Loss Function”.

$$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2 + (\mu-T)^2}}$$

USL and LSL are upper and lower specification limits, σ is process standard deviation

μ is process mean and T is target value.

$$C_{pm} = \frac{(72.5 - 47.50)}{6\sqrt{(3.8899)^2 + (60.657 - 60.00)^2}} = 1.05$$

$$P_{pm} = \frac{(72.5 - 47.50)}{6\sqrt{(4.067)^2 + (60.657 - 60.00)^2}} = 1.01$$

5.4. Process capability index - Cpk

C_{pmk} a third - generation capability index that incorporates the features of C_{pk} and C_{pm}. It estimates process capability around a target (T), and accounts for an off-centre process mean and assumes process output is approximately normally distributed. The process capability index - C_{pk} considers process average and evaluates half the process spread with respect to where the process average is actually located, though C_{pk} takes the process mean into consideration but it fails to differentiate an on-target process from off-target process. The way to address this difficulty is to use a process capability index C_{pm} that is better indicator of centring.

$$C_{pmk} = \min \left\{ \frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}$$

$$C_{pmk} = \min \left\{ \frac{72.50 - 60.657}{3\sqrt{(3.8899)^2 + (60.657 - 60.00)^2}}, \frac{60.657 - 47.50}{3\sqrt{(3.8899)^2 + (60.657 - 60.00)^2}} \right\}$$

$$C_{pmk} = \min \{1.00, 1.11\} = 1.00$$

$$P_{pmk} = \min \left\{ \frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}$$

$$P_{pmk} = \min \left\{ \frac{72.50 - 60.657}{3\sqrt{(4.067)^2 + (60.657 - 60.00)^2}}, \frac{60.657 - 47.50}{3\sqrt{(4.067)^2 + (60.657 - 60.00)^2}} \right\}$$

$$P_{pmk} = \min \{0.95, 1.06\} = 0.95$$

5.5. Process capability index - Cpk

The index C_{pmk} provides warnings about the increase of the process variation and the process departure from the target, but is sensitive to changes in the shape of the distribution, particularly its skewness. A modification of C_{pmk} proposed by Wright (1995) incorporates a skewness term in the denominator to reduce the index value when non-symmetry is present. Utilizing the third central moment μ_3 as a measure of skewness. The index C_s is defined as follows:

$$C_s = \frac{(d - |\mu - T|)}{3\sqrt{\sigma^2 + (\mu - T)^2 + \left|\frac{\mu_3}{\sigma}\right|}}$$

Where, $d = (USL - LSL)/2$ is the half-interval of the specification range. $\frac{\mu_3}{\sigma}$, is the skewness coefficient. μ_3 is the central moment as a measure of skewness.

$$\mu_3 = \sum_{i=1}^n \frac{(x_i - \bar{x})^3}{(n-3)} / S^3$$

where,

$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n-1)}}$$

$$C_s = \frac{(12.5 - |60.657 - 60.00|)}{3\sqrt{(4.067)^2 + (0.657)^2 + \left|\frac{0.81}{4.067}\right|}} = 0.95$$

6. Modelling Non-Normal data using Statistical software

Quality control engineers are frequently asked to evaluate process stability and capability for key quality characteristics that follow non-normal distributions. In the past, demonstrating process stability and capability require the assumption of normally distributed data. However, if data do not follow the normal distribution, the results generated under this assumption will be incorrect. Whether it is decided to transform data to follow the normal

distribution or identify an appropriate non-normal distribution model, statistical software's can be used. Identification of an appropriate non-normal distribution model is a good approach to find a non normal distribution that fits the data. Many non normal distribution can be used to model a response, but if an alternative to the normal distribution is going to be viable, the exponential, lognormal, and weibull distributions usually works well. Minitab statistical software can be used to verify the process stability and estimate process capability for non normal quality characteristics.

6.1. Modelling the data using MINITAB statistical software

This approach is to find a non-normal distribution that fits the case study data. Four non-normal distributions were used to model the process capability as shown in the Figure 5. Individual distribution identification was used to compare the fit of different distribution. In this case study, of the four shown, the logistic distribution provides the best fit as its p-value is highest among the four distributions. Logistic distribution was found to be appropriate for modelling the case study data and estimate the process capability.

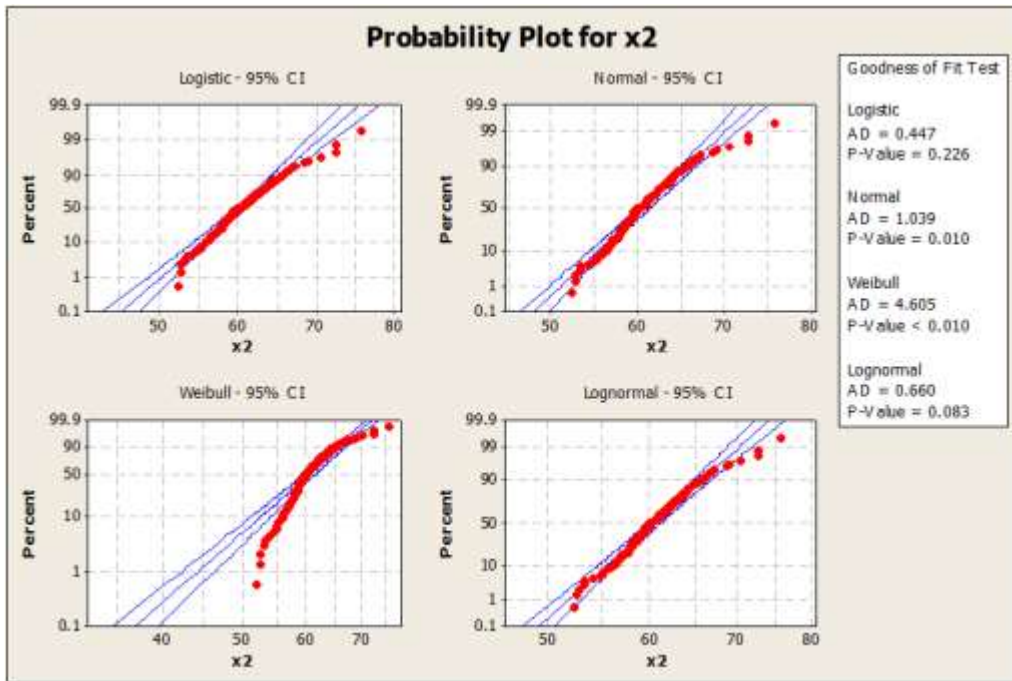


Figure 5. Probability plots for four non-normal distributions

6.2. Process Capability Analysis based on lognormal and logistic distribution model

Figure 6 shows Process Capability Analysis based on lognormal distribution model,

while Figure 7 shows Process Capability Analysis based on Logistic distribution model.

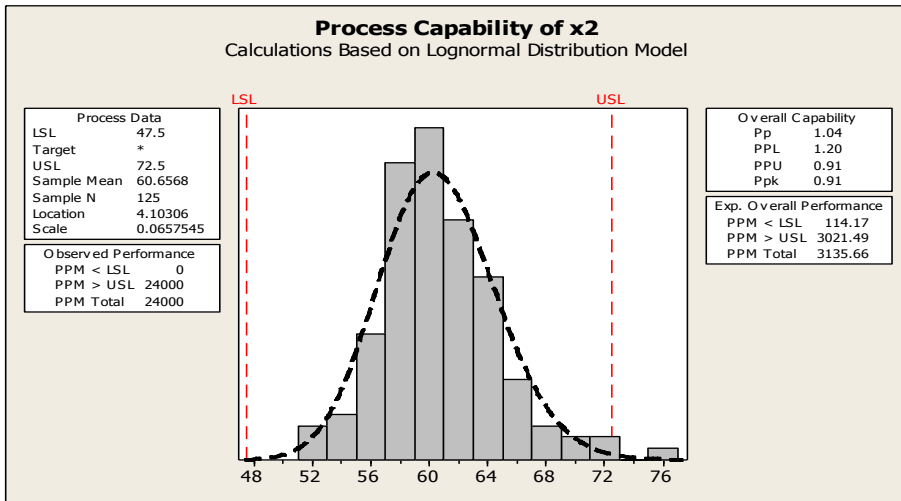


Figure 6. Process Capability Analysis based on lognormal distribution model using MINITAB software

The process capability, Ppk equal to approximately 0.91 with an expected overall performance of approximately 3135 PPM falling outside of the specification limit.

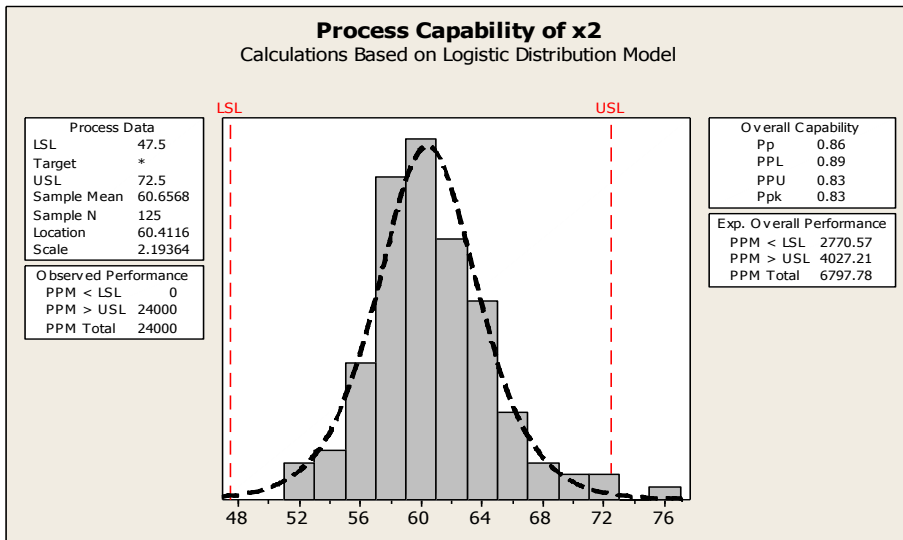


Figure 7. Process Capability Analysis based on Logistic distribution model using MINITAB software

The process capability, Ppk equal to 0.83 with an expected overall performance of approximately 6797 PPM falling outside of the specification limit.

7. Results and discussion

The case study results provide the information about the variation in resistance of the sample coils and estimated index values from conventional method that ignore

the critical assumptions. The analysis of results consists of computation of process capability indices for five different indices including Wright's process capability index

'Cs' and models from the non-normally distributed data using statistical software. The results are presented in the Table 4.

Table 4. Quantified values for Cp, Cpk, Cpm, Cpmk and Cs indices and Lognormal and Logistic Model

Sl.no.	Index	Index value	Index	Index Value	Lognormal Model	Logistic Model
1	Cp	1.07	Pp	1.02	1.04	0.86
2	Cpk	1.01	Ppk	0.97	0.91	0.83
3	Cpm	1.05	Ppm	1.01	-	-
4	Cpmk	1.00	Ppmk	0.95	-	-
5	Cs(with in)	0.89	Cs(Overall)	0.94	-	-

After computing the capability indices Cp, Cpk, Cpm, Cpmk and Cs, it has been observed that $C_p > C_{pk}$, $C_{pm} > C_{pmk}$, Cs is less than all estimated values. From development point of view, Cp value is greater than 1.0, it indicates that the process is capable of meeting the given specification limits (72.5 and 47.5), thus the process has the potential to meet specifications as long as the mean is centered.

The Cpk and Ppk measures process performance (1.01 & 0.97) and are lower than the Cp & Pp, it indicates that the process mean is not exactly equal to the mean of the specification limits and process mean has been drifted toward the upper specification limit and leads to rejections. The higher the index, the more closely the process is running to its specification and lower the defective parts per million. Cpm emphasizes on target deviation and as the process mean moves off of the target, it starts incurring greater penalty. The index Cpmk has increased sensitivity to departure from the process mean from the target value. It considers all the criteria that Cp, Cpk and Cpm. Wright's process capability index 'Cs' not only takes into account the process variation as well as the departure of the process mean from the target, but also the skewness of the distribution of the case study data. In this paper, comparative study has been made on the process capability indices when the distribution of the process characteristic i.e., coil resistance is skewed

and concluded that Wright's capability index gives accurate value than Cpmk and Ppmk.

When modeling the non normal data (Case study) using lognormal and logistic distribution, Logistic model provides good result compare to the lognormal model as well as Cs index.

8. Conclusions

Since the $C_{pk} < C_p$, the process is said to be off centered. This can be accepted as lower capability than the case that the process is centered. The reason is that the process is not operating at the midpoint of the specification limits.

$1 < C_{pk} < 1.33$ means that the process is barely capable. Automotive industry uses $C_{pk} = 1.33$ as a benchmark in assessing the capability of a process.

$C_{pmk} < C_{pk}$ means that the Cpmk index is sensitive to departure of the process mean from the desired target value.

$C_s < C_{pmk}$ indicates that the Wright's process capability index 'Cs' is still sensitive than Cpmk, as it not only takes into account the process variation as well as the departure of the process mean from the target, but also the skewness of the distribution of the resistance of the coil.

When compare the Cs index value with the results of modeling of non-normal data using lognormal and logistic distributions, it was

found to be less sensitive.

In this paper, comparison was made on the conventional capability indices when ignoring the critical assumptions with C_s index as well as modeling of non-normal data using appropriate distributions using MINITAB software. C_p , C_{pk} , C_{pm} , C_{pmk} ,

C_s and Lognormal distribution model under estimate the non-conforming coils. It is concluded that modeling technique is fast and accurate method for computation and analysis of process capability for non normal data.

References:

- Chan, L. K., Cheng, S. W., & Spring, F. A. (1988). A New measure of process capability: C_{pm} . *Journal of Quality Technology*, 20(3), 162–175.
- Chang, Y. S., & Bai, D.S. (2002). Process capability indices for skewed populations. *Quality and Reliability Engineering International*, 18(5), 383–393.
- Chen, H. F., & Kotz, S. (1996). An asymptotic distribution of Wright's process capability index sensitive to skewness. *Journal of Statistical Computation and Simulation*, 55, 147–158.
- Gildeh, B. S., & Asghari, S. (2011). A new method for constructing confidence interval for C_{pm} based Fuzzy data. *International Journal for Quality Research*, 5(2), 67–73.
- Gildeh, B. S., Iziy, A., & Ghasempour, B. (2014). Estimation of C_{pmk} processes capability index based on bootstrap method for Weibull distribution: A case study. *International journal for quality research*, 8(2), 255–264.
- Kotz, S., & Johnson, N. L. (1993). *Process capability indices*. Chapman and Hall, London, U.K.
- Pearn, W. L., & Kotz, S. (1994). Application of Clements method for calculating second and third generation process capability indices for non-normal pearsonian populations. *Quality Engineering*, 7(1), 139–145.
- Pearn, W. L., & Chang, C. S. (1997). The performance of the process capability index C_s on skewed distributions. *Communications in Statistics - Simulation and Computation*, 26, 1361–1377.
- Prabhuswamy, M. S., & Nagesh, P. (2007). Process capability Analysis made simple through graphical approach. *Kathmandu University Journal of science, Engineering and Technology*, 1(3).
- Sagbas, A., Carot, M. T. C., & Sanz, J. M. (2013). A new approach for measurement of the efficiency of C_{pm} and C_{pmk} Control charts. *International journal for quality research*, 7(4), 605–622.
- Wooluru, Y., Swamy, D. R., & Nagesh, P. (2014). The Process Capability Analysis-A Tool for Process Performance Measures and Metrics –A Case Study. *International Journal for Quality Research*, 8(3), 399–416.
- Wright, P. A. (1995). A process capability index sensitive to skewness. *Journal of Statistical Computation and Simulation*, 52, 195–203.
- Wu, C. W., Pearn, W. L., Chang, C. S., & Chen, H. C. (2007). Accuracy Analysis of the Percentile Method for Estimating Non Normal Manufacturing Quality. *Communication in statistics - Simulation and Computation*, 36, 657–697.

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