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Skolem difference mean labeling of *H*-graphs

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Abstract

A graph $G(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x\in V$ with distinct elements $f(x)$ from $1,2,3\ldots p+q$ in such a way that the edge $e=uv$ is labeled with $\frac{|f(u)-f(v)|}{2}$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edges get distinct labels from $1,2,3\ldots q$. A graph that admits skolem difference mean labeling is called skolem difference mean graph.

Key words: skolem difference mean labeling, skolem difference mean graphs.
AMS Subject Classification (2010): 05C78

1 Introduction

Throughout this paper, by a graph, we mean a finite, undirected, simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. A path on n vertices is denoted by P_n. The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1,v_2,\ldots v_n$ and $u_1,u_2,\ldots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey can be found in [2]. The concept of skolem mean labeling was introduced by T.Ramesh, A.Subramanian and V.Balaji [1].

In this paper, we define skolem difference mean labeling and show that the *H*-graphs are skolem difference mean.

2 Main Results

Definition 2.1. *A graph $G(V,E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,3\ldots p+q$ in such a way that the edge $e=uv$ is labeled with* $\frac{|f(u)-f(v)|}{2}$ *if $|f(u)-f(v)|$ is even and* $\frac{|f(u)-f(v)|+1}{2}$ *if $|f(u)-f(v)|$ is odd and the resulting edges get distinct labels from $1,2,3\ldots q$. A graph that admits skolem difference mean labeling is called skolem difference mean graph.*

The skolem difference mean labeling of C_4 is given in Figure 1.

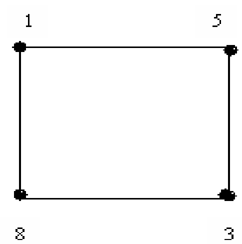


Figure 1: Skolem difference mean labeling of C_4

Theorem 2.2. *The H-graph G is a skolem difference mean graph.*

Proof. Let $v_1,v_2\ldots v_n$ and $u_1,u_2\ldots u_n$ be the vertices of the graph G.

Case (i) Let n be odd. We define a labeling f: $V(G) \rightarrow \{1,2,3,\ldots 4n\text{-}1\}$as follows:

$f(v_{2i+1}) = 2i+1; \quad 0 \le i < \frac{n+1}{2}$

$f(v_{2i}) = 4n+1-2i; \quad 1 \le i < \frac{n+1}{2}$

$f(u_{2i+1}) = 3n-2i; \quad 0 \le i < \frac{n+1}{2}$

$f(u_{2i}) = n+2i; \quad 1 \le i < \frac{n+1}{2}$

Case (ii) Let n be even. We define a labeling f: $V(G) \rightarrow \{1,2,3,\ldots 4n\text{-}1\}$as follows:

$f(v_{2i+1}) = 2i+1; \quad 0 \le i < \frac{n}{2}$

$f(v_{2i}) = 4n+1\text{-}2i; \quad 1 \le i \le \frac{n}{2}$

$f(u_{2i+1}) = n+1+2i; \quad 0 \le i < \frac{n}{2}$

$f(u_{2i}) = 3n+1\text{-}2i; \quad 1 \le i \le \frac{n}{2}$

In both the cases the induced edge labels are 1, 2…2n-1.Hence the theorem.

■

The skolem difference mean labeling of the H-graphs of P_3 and P_4 are given in Figure 2.

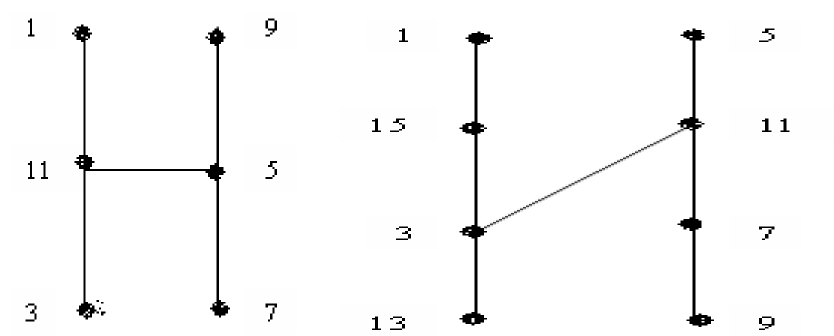


Figure 2: Skolem difference mean labeling of the H-graphs of P_3 and P_4

Theorem 2.3. *If a H-graph G is a skolem difference mean graph then G* $\odot S_1$ *is also a skolem difference mean graph.*

Proof. Let f be a skolem difference mean labeling of G with vertices $v_1, v_2 \ldots v_n$ and $u_1, u_2 \ldots u_n$. Let f^* be the induced edge labeling of G.

Let $v_1', v_2' \ldots v_n'$ and $u_1', u_2' \ldots u_n'$ be the corresponding new vertices in $G \odot S_1$.

Define a labeling g: $(G \odot S_1) \rightarrow \{1, 2, 3 \ldots 8n-1\}$ as follows:

Case (i) Let n be odd.

$g(v_{2i+1}) = f(v_{2i+1});$ $\quad 0 \le i < \frac{n+1}{2}$

$g(v_{2i}) = f(v_{2i}) + 4n;$ $\quad 1 \le i < \frac{n+1}{2}$

$g(u_{2i+1}) = f(u_{2i+1}) + 4n;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(u_{2i}) = f(u_{2i});$ $\quad 1 \le i < \frac{n+1}{2}$

$g(v_{2i+1}') = 4n - 2i;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(v_{2i}') = 4n + 2i;$ $\quad 1 \le i < \frac{n+1}{2}$

$g(u_{2i+1}') = g(v_{n-1}') + 2 + 2i;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(u_{2i}') = g(v_n') - 2i;$ $\quad 1 \le i < \frac{n+1}{2}$

For the vertex labeling g, the induced edge labeling g^* is defined by

$g^*(v_i v_{i+1}) = f^*(v_i v_{i+1}) + 2n;$ $\quad 1 \le i \le n-1$

$g^*(u_i u_{i+1}) = f^*(u_i u_{i+1}) + 2n;$ $\quad 1 \le i \le n-1$

$g^*(v_i v_i') = f(v_1) + 2n - i;$ $\quad 1 \le i \le n$

$g^*(u_i u_i') = f(u_1) - 2n - i + 1;$ $\quad 1 \le i \le n$

$$g^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right) = 3 f^*\left(v_{\frac{n+1}{2}} u_{\frac{n+1}{2}}\right)$$

Case (ii) Let n be even.

$g(v_{2i+1})=f(v_{2i+1});\qquad 0 \le i < \frac{n}{2}$

$g(v_{2i})=f(v_{2i})+4n;\qquad 1 \le i \le \frac{n}{2}$

$g(u_{2i+1})=f(u_{2i+1});\qquad 0 \le i < \frac{n}{2}$

$g(u_{2i})=f(u_{2i})+4n;\qquad 1 \le i \le \frac{n}{2}$

$g(v_{2i+1}')=4n-2i;\qquad 0 \le i < \frac{n}{2}$

$g(v_{2i}')=4n+2i;\qquad 1 \le i \le \frac{n}{2}$

$g(u_{2i+1}') =g(v_{n-1}')-2-2i;\ 0 \le i < \frac{n}{2}$

$g(u_{2i}') =g(v_n')+2i;\qquad 1 \le i \le \frac{n}{2}$

For the vertex labeling g, the induced edge labeling g* is defined by

$g^*(v_iv_{i+1})=f^*(v_iv_{i+1})+2n;\qquad 1 \le i \le n-1$

$g^*(u_iu_{i+1})=f^*(u_iu_{i+1})+2n;\qquad 1 \le i \le n-1$

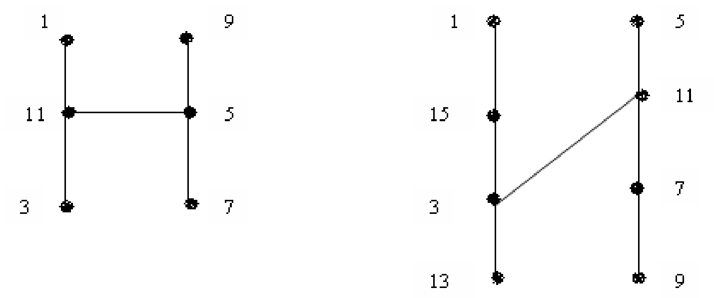
$g^*(v_iv_i')=f(v_1)+2n-i;\qquad 1 \le i \le n$

$g^*(u_iu_i')=f(u_1)-i;\qquad 1 \le i \le n$

$g^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}}) =3f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}})$

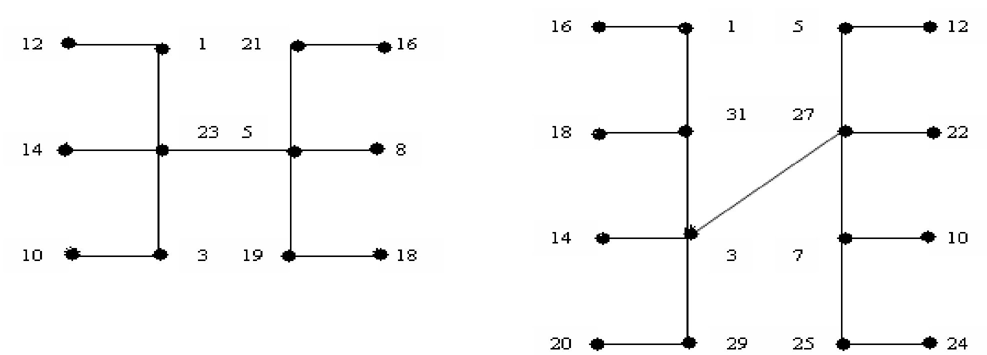
In both the cases it can be verified that $G \odot S_1$ is a skolem difference mean graph. ■

The skolem difference mean labelings of the two H-graphs G_1 and G_2 are given in Figure 3 and the skolem difference mean labelings of $G_1 \odot S_1$ and $G_2 \odot S_1$ are given in Figure 4.



G_1 G_2

Figure 3: Skolem difference mean labelings of the *H*-graphs G_1 and G_2



$G_1 \odot S_1$ $G_2 \odot S_1$

Figure 4: Skolem difference mean labelings of $G_1 \odot S_1$ and $G_2 \odot S_1$

Theorem 2.4. *If a H-graph G is a skolem difference mean graph then* $G \odot S_2$ *is also a skolem difference mean graph.*

Proof: Let f be a skolem difference mean labeling of G with vertices $v_1,v_2\ldots v_n$ and $u_1,u_2\ldots u_n$. Let f^* be the induced edge labeling of f.

Let $v_1',v_2'\ldots v_n'$ & $v_1'',v_2''\ldots v_n''$ and $u_1',u_2'\ldots u_n'$ & $u_1'',u_2''\ldots u_n''$ be the corresponding new vertices in $G \odot S_2$

Define a labeling g: $(G \odot S_2) \rightarrow \{1,2,3\ldots 12n\text{-}1\}$ as follows:

Case (i) Let n be odd.

$g(v_{2i+1})=f(v_{2i+1});$ $\quad 0 \le i < \frac{n+1}{2}$

$g(v_{2i}) = f(v_{2i})+8n;$ $\quad 1 \le i < \frac{n+1}{2}$

$g(u_{2i+1})=f(u_{2i+1})+8n;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(u_{2i}) = f(u_{2i});$ $\quad 1 \le i < \frac{n+1}{2}$

$g(v_{2i+1}')=2+10i;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(v_{2i+1}'')=4+10i;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(v_2')=g(v_2)-5$

$g(v_2'')=g(v_2)-7$

$g(v_4')= g(v_2')-11$

$g(v_4'')= g(v_2'')-11$

$g(v_{4+2i}')= g(v_4')-10i;$ $\quad 1 \le i < \frac{n+1}{2}$

$g(v_{4+2i}'')= g(v_4'')-10i;$ $\quad 1 \le i < \frac{n+1}{2}$

$g(u_{2i+1}')=g(v_{n-1}')-10-10i;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(u_{2i+1}'')=g(v_{n-1}'')-10-10i;$ $\quad 0 \le i < \frac{n+1}{2}$

$g(u_{2i}')=g(v_n')+10i;$ $\quad 1 \le i < \frac{n+1}{2}$

$g(u_{2i}'')=g(v_n'')+10i;$ $\quad 1 \le i < \frac{n+1}{2}$

For the vertex labeling g the induced edge labeling g^* is defined by

$g^*(v_iv_{i+1})=f^*(v_iv_{i+1})+4n;$ $\quad 1 \le i \le n-1$

$g^*(u_iu_{i+1})=f^*(u_iu_{i+1})+4n;$ $1 \le i \le n-1$

$g^*(v_iv_i')=2i-1;$ $1 \le i \le n$

$g^*(v_iv_i'')=2i;$ $1 \le i \le n$

$g^*(u_iu_i')=2n+2i-1;$ $1 \le i \le n$

$g^*(u_iu_i'')=2n+2i,$ $1 \le i \le n$

$g^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}})=5f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}})$

Case (ii) Let n be even.

$g(v_{2i+1})=f(v_{2i+1});$ $0 \le i < \frac{n}{2}$

$g(v_{2i}) \;=f(v_{2i})+8n;$ $1 \le i \le \frac{n}{2}$

$g(u_{2i+1})=f(u_{2i+1});$ $0 \le i < \frac{n}{2}$

$g(u_{2i}) \;=f(u_{2i})+8n;$ $1 \le i \le \frac{n}{2}$

$g(v_{2i+1}')=2+10i;$ $0 \le i < \frac{n}{2}$

$g(v_{2i+1}'')=4+10i;$ $0 \le i < \frac{n}{2}$

$g(v_2')=g(v_2)-5$

$g(v_2'')=g(v_2)-7$

$g(v_4')= g(v_2')-11$

$g(v_4'')= g(v_2'')-11$

$g(v_{4+2i}')= g(v_4')-10i;$ $1 \le i \le \frac{n}{2}$

$g(v_{4+2i}'')= g(v_4'')-10i;$ $1 \le i \le \frac{n}{2}$

$g(u_{2i+1}')=g(v_{n-1}')+10+10i;$ $0 \le i < \frac{n}{2}$

$g(u_{2i+1}'') = g(v_{n-1}'') + 10 + 10i; \quad 0 \le i < \frac{n}{2}$

$g(u_{2i}') = g(v_n') - 10i; \quad 1 \le i \le \frac{n}{2}$

$g(u_{2i}'') = g(v_n'') - 10i; \quad 1 \le i \le \frac{n}{2}$

For the vertex labeling g the induced edge labeling g^* is defined by

$g^*(v_i v_{i+1}) = f^*(v_i v_{i+1}) + 4n; \quad 1 \le i \le n-1$

$g^*(u_i u_{i+1}) = f^*(u_i u_{i+1}) + 4n; \quad 1 \le i \le n-1$

$g^*(v_i v_i') = 2i - 1; \quad 1 \le i \le n$

$g^*(v_i v_i'') = 2i; \quad 1 \le i \le n$

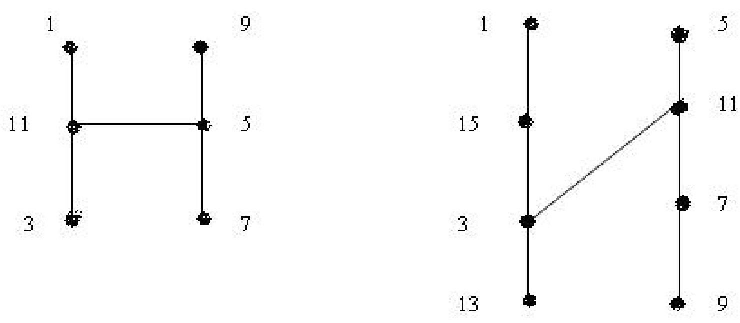
$g^*(u_i u_i') = 2n + 2i - 1; \quad 1 \le i \le n$

$g^*(u_i u_i'') = 2n + 2i; \quad 1 \le i \le n$

$g^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}}) = 5f^*(v_{\frac{n}{2}+1} u_{\frac{n}{2}})$

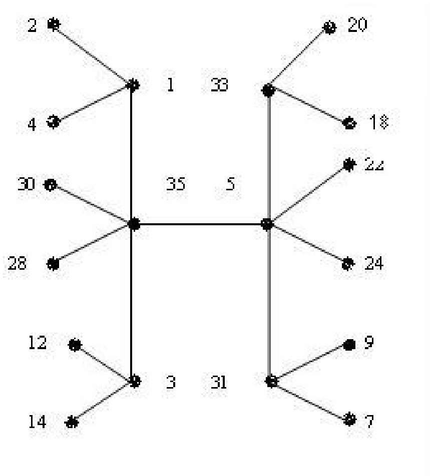
In both the cases it can be verified that $G \odot S_2$ is a skolem difference mean graph. ■

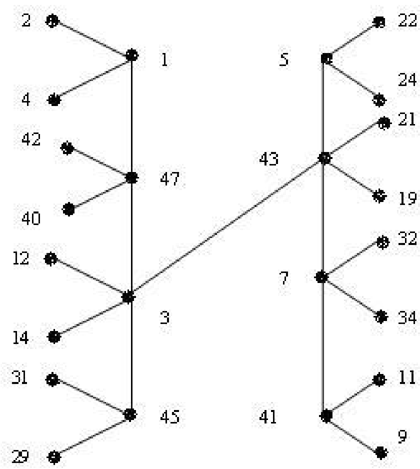
The skolem difference mean labelings of the H-graphs G_1 and G_2 are given in Figure 5 and the skolem difference mean labelings of $G_1 \odot S_2$ and $G_2 \odot S_2$ are given in Figure 6.



G_1 G_2

Figure 5: Skolem difference mean labelings of the H-graphs G_1 and G_2





$G_1 \odot S_2$ $G_2 \odot S_2$

Figure 6: Skolem difference mean labelings of $G_1 \odot S_2$ and $G_2 \odot S_2$

Theorem 2.5. *If G_1 and G_2 are two skolem difference mean H-graphs then $G_1 \cup G_2$ is also a skolem difference mean graph.*

Proof. Let $V(G_1)=\{v_i,u_i \ /1 \leq i \leq n\}$and $V(G_2)=\{s_i,t_i \ /1 \leq i \leq m\}$. Let f and g be a skolem difference mean labeling of G_1 and G_2 respectively.

Let f^* and g^* be the induced edge labeling of f and g respectively.

Define a labeling h: $V(G_1 \text{U} G_2) \rightarrow \{1,2,3\ldots 4n+4m-2\}$as follows:

$h(v_{2i+1})=f(v_{2i+1})$

$h(v_{2i})=f(v_{2i})+4m-2$

$h(u_{2i+1})=f(u_{2i+1})+4m-2$ when n is odd

$= f(u_{2i+1})$ when n is even

$h(u_{2i})=f(u_{2i})$ when n is odd

$= f(u_{2i}) +4m-2$ when n is even

$h(s_{2i+1})= g(s_{2i+1})+1$

$h(s_{2i})=g(s_{2i})+1$

$h(t_{2i+1})=g(t_{2i+1})+1$

$h(t_{2i})=g(t_{2i})+1$

For the vertex labeling h, the induced edge labeling h^* is defined as follows:

$h^*(v_iv_{i+1})=f^*(v_iv_{i+1})+2m-1;$ $\quad 1 \le i \le n-1$

$h^*(u_iu_{i+1})=f^*(u_iu_{i+1})+2m-1;$ $\quad 1 \le i \le n-1$

$h^*(s_is_{i+1})=g^*(s_is_{i+1})\ ;$ $\quad 1 \le i \le m-1$

$h^*(t_it_{i+1})=g^*(t_it_{i+1})\ ;$ $\quad 1 \le i \le m-1$

Case (i) When both n and m are odd

$$h^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}})=f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}})+2m-1$$

$$h^*(s_{\frac{m+1}{2}}t_{\frac{m+1}{2}})=g^*(s_{\frac{m+1}{2}}t_{\frac{m+1}{2}})$$

Case (ii) When n is odd and m is even

$$h^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}})=f^*(v_{\frac{n+1}{2}}u_{\frac{n+1}{2}})+2m-1$$

$$h^*(s_{\frac{m}{2}+1}t_{\frac{m}{2}})=g^*(s_{\frac{m}{2}+1}t_{\frac{m}{2}})$$

Case (iii) When n is even and m is odd

$$h^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}})=f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}})+2m-1$$

$$h^*(s_{\frac{m+1}{2}}t_{\frac{m+1}{2}})=g^*(s_{\frac{m+1}{2}}t_{\frac{m+1}{2}})$$

Case (iv) When both n and m are even

$$h^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}})=f^*(v_{\frac{n}{2}+1}u_{\frac{n}{2}})+2m-1$$

$h^*(s_{\frac{m}{2}+1}t_{\frac{m}{2}})=g^*(s_{\frac{m}{2}+1}t_{\frac{m}{2}})$.

It can be easily verified that g is a skolem difference mean labeling of $G_1 \cup G_2$. ■

For various skolem difference mean H-graphs G_1 and G_2, we find a skolem difference mean labeling for $G_1 \cup G_2$.

Illustration 2.6. The skolem difference mean labelings of G_1 (the H-graph of P_3), G_2 (the H-graph of P_5) and their union $G_1 \cup G_2$ are given in Figure 7.

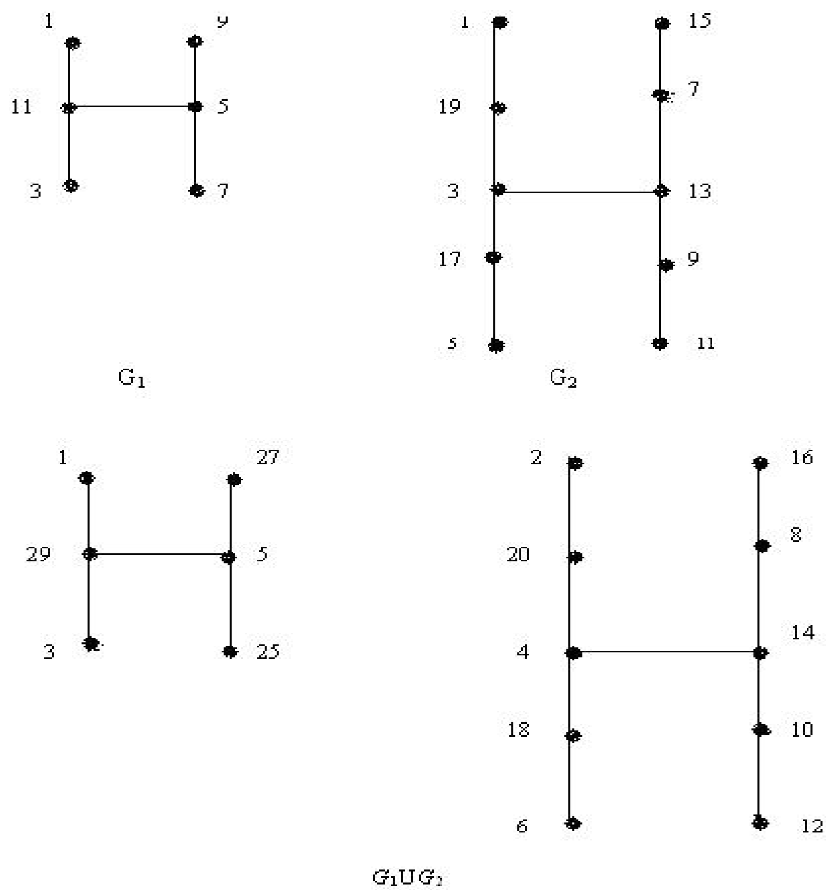


Figure 7: Skolem difference mean labelings of G_1, G_2 and $G_1 \cup G_2$

Illustration 2.7. The skolem difference mean labelings of G_1 (the H-graph of P_3), G_2 (the H-graph of P_4) and their union $G_1 \cup G_2$ are given in Figure 8.

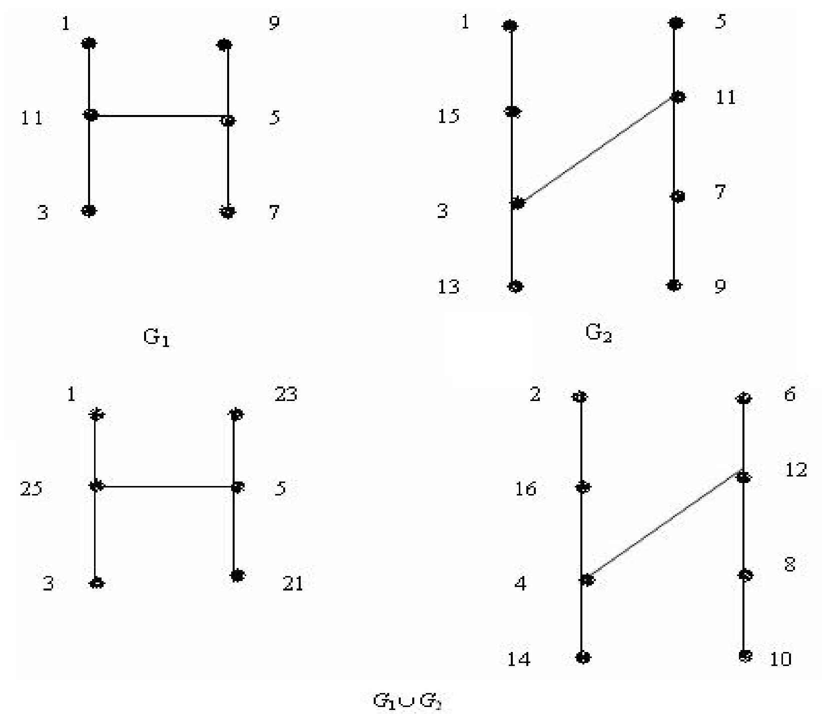
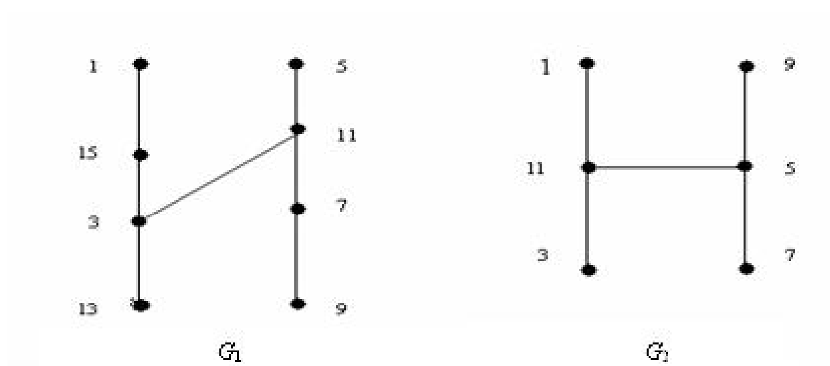


Figure 8: Skolem difference mean labelings of G_1, G_2 and $G_1 \cup G_2$.

Illustration 2.8. The skolem difference mean labelings of G_1 (the H-graph of P_4), G_2 (the H-graph of P_3) and their union $G_1 \cup G_2$ are given in Figure 9.



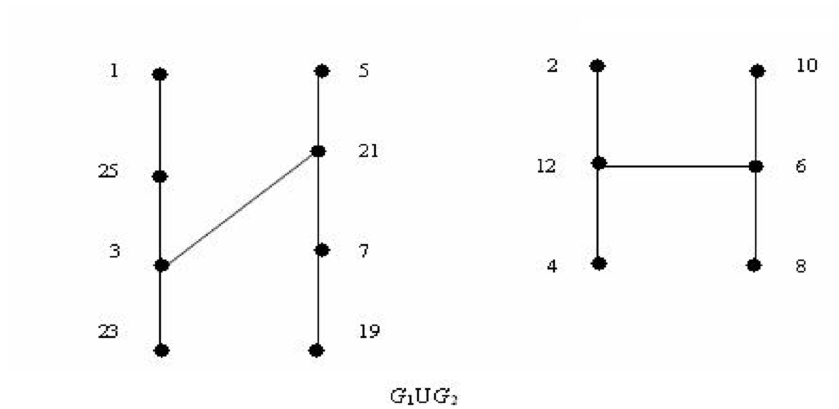


Figure 9: Skolem difference mean labelings of G_1, G_2 and $G_1 \cup G_2$

Illustration 2.9. The skolem difference mean labelings of G_1 (the H-graph of P_4), G_2 (the H-graph of P_6) and their union $G_1 \cup G_2$ are given in Figure 10.

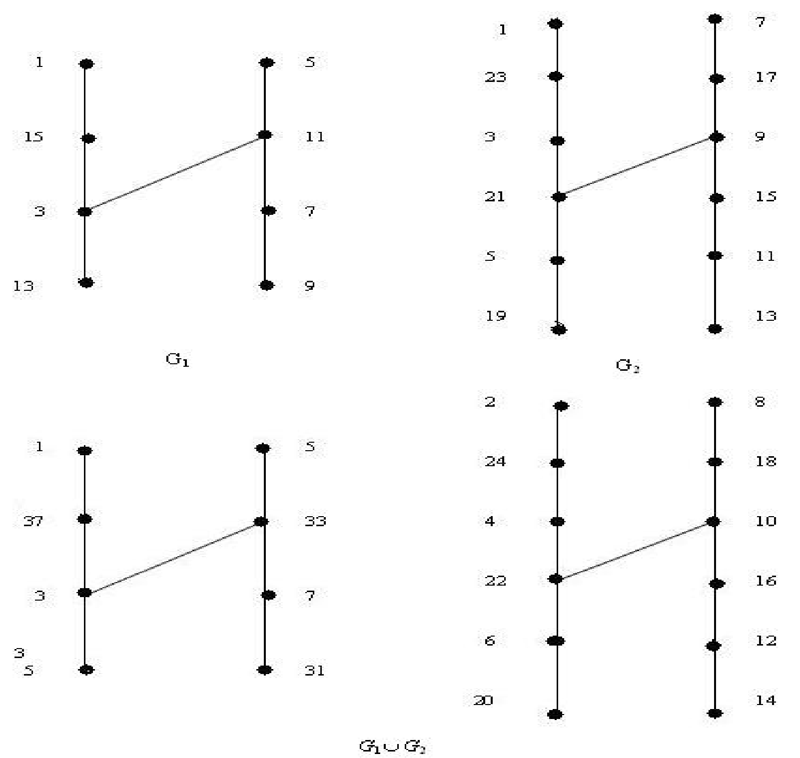


Figure 10: Skolem difference mean labelings of G_1, G_2 and $G_1 \cup G_2$

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