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Extended results on two domination number and chromatic number of a graph

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Abstract

A subset *S* of *V* is called a dominating set in *G* if every vertex in *V*-*S* is adjacent to at least one vertex in *S*. A Dominating set is said to be two dominating set if every vertex in *V*-*S* is adjacent to atleast two vertices in *S*. The minimum cardinality taken over all, the minimal two dominating set is called two domination number and is denoted by $\gamma_2(G)$. The minimum number of colors required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic number $\chi(G)$. In [6], it was proved that sum of two domination number and chromatic number is equals to $2n$-5 and $2n$-6. In this paper, we characterize all graphs whose sum of two domination number and chromatic number is $2n$-7.

Key words: Two Domination Number, Chromatic Number.

AMS Subject Classification (2010): 05C

1 Introduction

Let $G = (V, E)$ be a simple undirected graph. The degree of any vertex u in *G* is the number of edges incident with u and is denoted by $d(u)$. The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively. P_n denotes the path on *n* vertices. The vertex connectivity $\kappa(G)$ of a graph *G* is the minimum number of vertices whose removal results in a disconnected graph. A colouring of a graph is an assignment of colours to its vertices so that two adjacent vertices do not have the same colour. An n-colouring of a graph *G* uses *n* colours. The Chromatic Number χ is defined to be the minimum *n* for which *G* has an *n*-colouring. If $\chi(G) = k$ but $\chi(H) < k$, for every proper subgraph *H* of *G*, then *G* is *k*-critical.

A subset *S* of V is called a dominating set in *G* if every vertex in V-*S* is adjacent to atleast one vertex in *S*. The minimum cardinality taken over all dominating sets in *G* is called the domination number of *G* and is denoted by γ. A dominating set is said to be two dominating set if every vertex in V-*S* is adjacent to atleast two vertices in *S*. The minimum cardinality taken over all the minimal two dominating set is called two domination number and is denoted by $\gamma_2(G)$.

In [5], Harary and Haynes defined a subset *S* of V to be a double dominating set (DDS) of *G* if every vertex $v \in V$, $(N(v) \cap S) \geq 2$ .The double domination number $\gamma_2(G)$ is the minimum cardinality of a DDS.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [12], J. Paulraj Joseph and S. Arumugam proved that $\gamma + k \leq p$. In [13], J. Paulraj Joseph and S. Arumugam proved that $\gamma_c + \chi = p+1$. They also characterized the class of graphs for which the upper bound is attained. They also proved similar results for γ and γ_t. In [9], J. Paulraj Joseph and G. Mahadevan proved that $\gamma_{cc} + \chi \leq 2n-1$ and characterized the corresponding extremal graphs. In [8], J. Paulraj Joseph and G. Mahadevan proved that $\gamma_{pr} + \chi \leq 2n-1$ and characterized the corresponding extremal graphs. In [11], J. Paulraj Joseph and G. Mahadevan introduced the concept of complementary perfect domination number γ_{cp} and proved that $\gamma_{cp}+\chi \leq 2n-2$, and characterized the corresponding extremal graphs. They also obtained the similar results for the induced complementary perfect domination number and chromatic number of a graph. In this paper, we characterize all graphs whose sum of two domination number and chromatic number is $2n-7$. Terms not defined here, are used in the sense of Harary[1].

Notations. $K_n(P_m)$ denotes the graph obtained from K_n by attaching the end vertex of P_m to anyone vertices of K_n. $K_n(m_1,m_2,m_3,\ldots..m_k)$ denotes the graph obtained from K_n

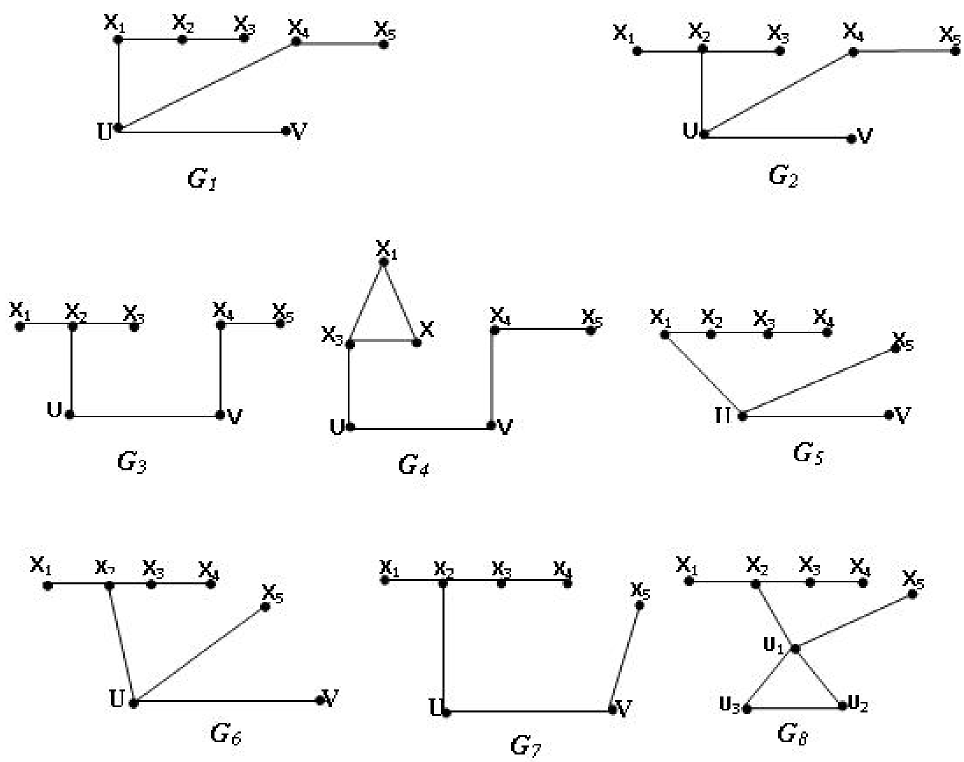
by attaching m_1 edges to any one vertex u_i of K_n, m_2 edges to any one vertex u_j for $i \neq j$ of K_n, m_3 edges to any one vertex u_k for $i \neq j \neq k$ of $K_n, \ldots,$ m_k edges to all the distinct vertices of K_n. $S(K_{1,m})$ is a graph obtained from $K_{1,m}$ by subdividing at one edge of $K_{1,m}$.

Theorem 1.1. *For any connected graph G,* $\gamma_2(G) \leq n$.

Theorem 1.2. *For any connected graph G,* $\chi(G) \leq \Delta(G) + 1$.

2 Main Results

Theorem 2.1. *For any connected graph G,* $\gamma_2(G) + \chi(G) = 2n - 7$ if and only if $G \cong S(K_{1,7})$, $K_4(5,0,0,0)$, $K_4(4,1,0,0)$, $K_4(3,2,0,0)$, $K_5(4,0,0,0,0)$, $K_5(3,1,0,0,0)$, $K_5(2,2,0,0,0)$, $K_3(P_5)$, $K_4(P_2.P_4,0,0)$, $K_3(P_3,P_3,0)$, $K_6(3,0,0,0,0,0)$, $K_6(2,1,0,0,0,0)$, $K_6(1,1,1,0,0,0)$, $K_5(P_4)$, $K_5(P_3,P_2,0,0,0)$, $K_5(P_3,P_2,0,0,0)$, $K_6(P_3)$, $K_7(2,0,0,0,0,0,0)$, $K_7(1,1,0,0,0,0,0)$, $K_8(1,0,0,0,0,0,0,0)$, K_9 *or any one of the graphs given in figure* 2.1.



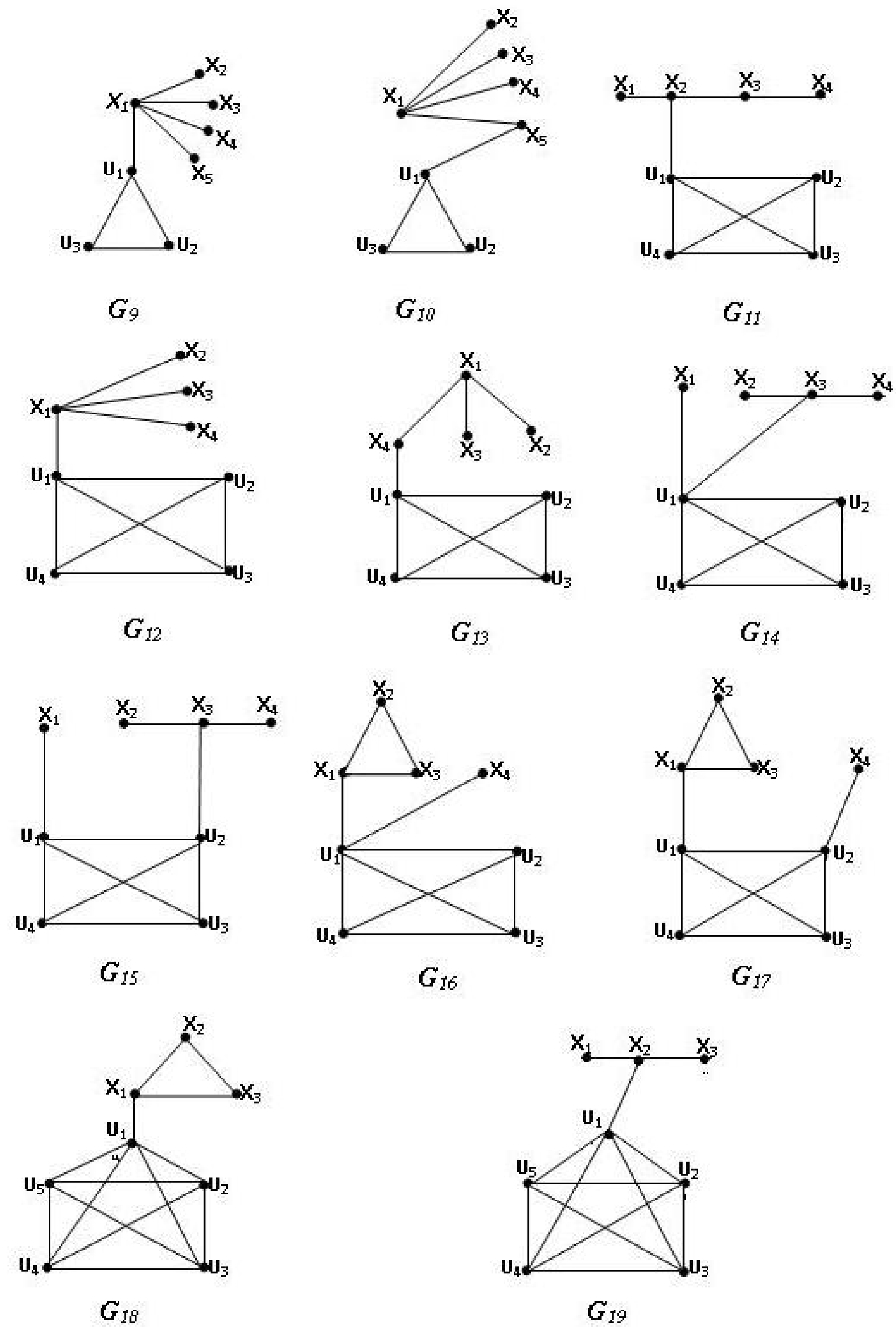


Figure 2.1.

Proof. If G is anyone of the graph given in the figure, then clearly $\gamma_2(G) + \chi(G) = 2n-7$. Conversely assume that $\gamma_2(G) + \chi(G) = 2n-7$. This is possible only if $\gamma_2 = n$, $\chi = n-7$ (*or*) $\gamma_2 = n-1$, $\chi = n-6$ (*or*) $\gamma_2 = n-2$, $\chi = n-5$ (*or*) $\gamma_2 = n-3$, $\chi = n-4$ (*or*) $\gamma_2 = n-4$, $\chi = n-3$ (*or*) $\gamma_2 = n-5$, $\chi = n-2$ (*or*) $\gamma_2 = n-6$, $\chi = n-1$(*or*) $\gamma_2 = n-7$, $\chi = n$.

Case (i) Let $\gamma_2(G) = n$ and $\chi(G) = n-7$. Since $\chi = n-7$, G contains a clique K on $n-7$ vertices or does not contains a clique K on $n-7$ vertices. Let $S = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \in V-S$. Let $<S> = K_7$, $\overline{K_7}$, P_7, $K_4 \cup K_3$, $K_2 \cup K_5$, $P_3 \cup K_4$, $P_4 \cup K_3$, $K_{1,6}$, $K_{2,5}$, $K_{3,4}$. *If* $<S> = K_7$, $\overline{K_7}$, P_7, $K_4 \cup K_3$, $K_2 \cup K_5$, $P_3 \cup K_4$, $P_4 \cup K_3$, $K_{1,6}$, $K_{2,5}$, $K_{3,4}$, then in all the cases no graph exists.

If G does not contain a clique K on $n-7$ vertices, then it can be verified that no new graph exists.

Case (ii) Let $\gamma_2(G) = n-1$ and $\chi(G) = n-6$. Since $\chi = n-6$, G contains a clique K on $n-6$ vertices. Let $S = \{x_1, x_2, x_3, x_4, x_5, x_6\} \in V-S$. *Let* $<S> = K_6$, $\overline{K_6}$, P_6, $K_3 \cup K_3$, $K_2 \cup K_4$, $P_3 \cup K_3$, $P_2 \cup K_4$, $K_{1,5}$, $K_{3,3}$, $K_{2,4}$. *If* $<S> = K_6$, $\overline{K_6}$, P_6, $K_3 \cup K_3$, $K_2 \cup K_4$, $P_3 \cup K_3$, $P_2 \cup K_4$, $K_{3,3}$, $K_{2,4}$, then in all the cases no graph exists.

Subcase (a) Let $<S> = K_{1,5}$. Let the root of $K_{1,5}$ be x_1. Let x_2, x_3, x_4, x_5, x_6 be adjacent to x_1. Since G is connected, there exists a vertex u_i in K_{n-6} which is adjacent to x_1 or any one of $\{x_2, x_3, x_4, x_5, x_6\}$. Then in both cases, $\{x_2, x_3, x_4, x_5, x_6, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n=8$. Hence $K=K_2 = uv$. If u is adjacent to x_1, then $G \cong S(K_{1,7})$. Let u be adjacent to any one of $\{x_2, x_3, x_4, x_5, x_6\}$, which is a contradiction no graph exists. If G does not contain a clique K on $n-6$ vertices, then it can be verified that no new graph exists.

Case (iii) Let $\gamma_2(G) = n-2$ and $\chi(G) = n-5$. Since $\chi = n-5$, G contains a clique K on $n-5$ vertices. *Let* $S = \{x_1, x_2, x_3, x_4, x_5\} \in V-S$. *Let* $<S> = K_5$, $\overline{K_5}$, P_5, $K_4 \cup K_1$, $P_3 \cup K_2$, $K_{1,4}$, $P_4 \cup K_1$, $K_{2,3}$, $K_3 \cup K_2$. *If* $<S>=K_5$, P_5, $K_4 \cup K_1$, $K_{2,3}$, then in all the cases no graph exists.

Subcase (a) Let $< S > = \overline{K_5}$, since G is connected. There exists a vertex u_i in K_{n-5} which is adjacent to all the vertices of S (or) four vertices of S (or) three vertices of S (or) two vertices of S (or) one vertex of S. Then in all the cases, $\{x_1, x_2, x_3, x_4, x_5, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 9$. Hence $K = K_4$. Let u_1, u_2, u_3, u_4 be the vertices of

K_4. If u_1 is adjacent to all the vertices of S and if $d(x_1) = d(x_2) = d(x_3) = d(x_4) = d(x_5) = 1$, then $G \cong K_4(5, 0, 0, 0)$. If u_1 is adjacent to four vertices of S and fifth one is adjacent to u_2, and if $d(x_1) = d(x_2) = d(x_3) = d(x_4) = d(x_5) = 1$, then $G \cong K_4(4,1,0,0)$. If u_1 is adjacent to three vertices of S and remaining two is adjacent to u_2, and if $d(x_1) = d(x_2) = d(x_3) = d(x_4) = d(x_5) = 1$, then $G \cong K_4(3, 2, 0, 0)$.

Subcase (b) Let $<S> = P_3 \cup K_2$. Let $P_3 = (x_1, x_2, x_3)$ and let x_4, x_5 be the vertices of K_2, since G is connected. There exists a vertex say $\underline{u_i}$ in K_{n-5} which is adjacent to x_1. Again since G is connected we consider the following two situations: (i) the vertex u_i is adjacent to x_1 (or equivalently x_3) or x_4. (ii)There exists a vertex u_i in K_{n-5} such that u_i is adjacent to x_2 and x_4 (or) the vertex u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_5. Then in all the cases, $\{x_1, x_3, x_5, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 7$. Hence $K = K_2$. Let u, v be the vertices of K_2. Let u be adjacent to x_1 (or equivalently x_3) and x_4 (or) let u be adjacent to x_2 and x_4 (or equivalently x_5) (or) let u be adjacent to x_2 and v is adjacent to x_4(or equivalently x_5) Then in all the cases, $G \cong G_1, G_2, G_3$. Let u be adjacent to x_1(or equivalently x_3) and v be adjacent to x_4 (or equivalently x_5), which is a contradiction. Hence no graph exists.

Subcase (c) Let $<S> = K_3 \cup K_2$, since G is connected. Let x_1, x_2, x_3 be the vertices of K_3 and x_4, x_5 be the vertices of K_2. There exists a vertex u_i in K_{n-5} is adjacent to x_1 and x_4 (or equivalently x_5) (or) u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_4 (or equivalently x_5). Then in both the cases, $\{x_1, x_2, x_5, u_i, u_j\}$ for $i \neq j$ is a γ_2 set of G, so that $n = 7$. Hence $K = K_2$. Let u, v be the vertices of K_2. Let u be adjacent to x_1 and x_4 (or equivalently x_5) then $G \cong G_4$. If u is adjacent to x_1 and v is adjacent to x_4 (or equivalently x_5) then $\gamma_2 = 4$, which is a contradiction. Hence no graph exists.

Subcase (d) Let $<S> = P_4 \cup K_1$, since G is connected. Let $P_4 = (x_1, x_2, x_3, x_4)$ and x_5 be the vertex of K_1. There exists a vertex u_i in K_{n-5} is adjacent to x_1(or equivalently x_4) and x_5 (or) If u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_5. In both the cases $\{x_2, x_4, x_5, u_i, u_j\}$ is a γ_2 set, so that $n = 7$. Hence $K = K_2$. Let u, v be the vertices of K_2. Let u be adjacent to x_1 (or equivalently x_4) and x_5, Hence $G \cong G_5$. Let u be adjacent to x_2 (or equivalently x_3) and x_5. Hence $G \cong G_6$. Let u be adjacent to x_2 (or equivalently x_3) and v be adjacent to x_5. Hence $G \cong G_7$. If u_i be adjacent to x_2 (or equivalently x_3) and x_5, then $\{x_1, x_3, x_4, x_5, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 8$. Hence $K = K_3$. Let u_1, u_2, u_3 be the vertices of K_3. If u_1 is adjacent to x_2 (or equivalently x_3) and x_5, then $G \cong G_8$.

Subcase (e) Let $<S> = K_{1,4}$. Since G is connected, the vertex x_1 be adjacent to x_2, x_3, x_4, x_5. There exists a vertex u_i in K_{n-5}, which is adjacent to x_1 or any one of $\{x_2, x_3, x_4, x_5\}$. Then $\{x_2, x_3, x_4, x_5, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n=8$. Hence $K=K_3$. If u_1 is adjacent to x_1, then $G \cong G_9$. If u is adjacent to x_5, then $G \cong G_{10}$.

For all the remaining cases, no new graph exists.

If G does not contain a clique K on $n-5$ vertices, then it can be verified that no new graph exists.

Case (iv) Let $\gamma_2(G) = n-3$ and $\chi(G) = n-4$. Since $\chi = n-4$, G contains a clique K on $n-4$ vertices or does not contain a clique K on $n-4$ vertices. Let $S = \{x_1, x_2, x_3, x_4\} \in V-S$. Let $<S> = K_4$, $\overline{K_4}$, P_4, $K_3 \cup K_1$, $K_{1,3}$, $K_2 \cup K_2$, $P_3 \cup K_1$.
If $<S> = K_4$, then no graph exists.

Subcase (a) Let $<S> = \overline{K_4}$. Since G is connected, one of the vertices of K_{n-4} say u_i is adjacent to all the vertices of S (or) three vertices of S (or) two vertices of S (or) one vertex of S. In all the cases, $\{x_1, x_2, x_3, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 9$. Hence $K = K_5$. Let u_1, u_2, u_3, u_4, u_5 be the vertices of K_5. If all the vertices of S are adjacent to u_1 and $d(x_1) = d(x_2) = d(x_3) = d(x_4) = d(x_5) = 1$, then $G \cong K_5(4, 0, 0, 0, 0)$. If three vertices of S are adjacent to u_1 and the fourth one is adjacent to u_2 and $d(x_1) = d(x_2) = d(x_3) = d(x_4) = d(x_5) = 1$, then $G \cong K_5(3, 1, 0, 0, 0)$. If two vertices of S are adjacent to u_1 and the remaining two vertices are adjacent to u_2 and $d(x_1) = d(x_2) = d(x_3) = d(x_4) = d(x_5) = 1$, then $G \cong K_5(2, 2, 0, 0, 0)$.

Subcase (b) Let $<S> = P_4 = (x_1, x_2, x_3, x_4)$. Since G is connected, there exists a vertex say u_i in K_{n-4} which is adjacent to x_1 (or equivalently x_4) (or) x_2 (or equivalently x_3). Let u_i be adjacent to x_1 then $\{x_2, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 6$. Hence $K = K_2 = uv$. If x_1 is adjacent to u, then $G \cong P_6$. Let u_i be adjacent to x_2 then $\{x_1, x_3, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 7$. Hence $K = K_3$. Let u_1, u_2, u_3 be the vertices of K_3. If x_1 of S is adjacent to u_1, then $G \cong K_3(P_5)$. If x_2 is adjacent to u_1, then no graph exists. Let u_i be adjacent to x_2 then $\{x_1, x_3, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 8$.

Hence $K = K_4$. Let u_1, u_2, u_3, u_4 be the vertices of K_4. If x_2 of S is adjacent to u_1, then $G \cong G_{11}$.

Subcase (c) Let $<S> = K_{1,3}$. Let the vertex x_1 be adjacent to x_2, x_3, x_4. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to x_1 or any one of (x_2, x_3, x_4). In all the cases, $\{x_2, x_3, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 8$. Hence $K=K_4$. Let u_1, u_2, u_3, u_4 be the vertices of K_4. If u_1 is adjacent to x_1, then $G \cong G_{12}$. If u_1 is adjacent to x_4, then $G \cong G_{13}$.

Subcase (d) Let $<S> = P_3 \cup K_1$. Let $P_3 = (x_2, x_3, x_4)$ since G is connected, there exists a vertex say u_i in K_{n-4} which is adjacent to x_1 .Again since G is connected we consider the following two situations: (i) The vertex u_i is adjacent to x_2 (or equivalently x_4) or x_3. (ii)There exists a vertex u_j for $i \neq j$ in K_{n-4} such that u_j is adjacent to x_2 (or equivalently x_4) or x_3. In all the cases, $\{x_1, x_2, x_4, u_i, u_j\}$ for $i \neq$j is a γ_2 set, so that $n = 8$. Hence $K= K_4$. Let u_1, u_2, u_3, u_4 be the vertices of K_4. Let u_1 be adjacent to x_1 and x_2 (or equivalently x_3) and let u_1 be adjacent to x_1 and u_2 be adjacent to x_2 (or equivalently x_3). In all the cases, $G \cong K_4 (P_2, P_4, 0, 0)$, G_{14}, G_{15}.

Subcase (e) Let $<S> = K_2 \cup K_2$. Let x_1x_2 and x_3x_4 be the edges in $<S>$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to x_1 and x_3 in S (or) u_i is adjacent to x_1 and u_j is adjacent to x_3 for $i \neq j$ in K_{n-4}. In both the cases, $\{x_2, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, hence $\gamma_2 = 4$, so that $n=7$. Hence $K = K_3$. Let u_1, u_2, u_3 be the vertices of K_3. If u_1 is adjacent to x_1 and x_3, then $G \cong K_3 (P_3, P_3, 0)$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_3, then $G \cong K_3 (P_3, P_3, 0)$.

Subcase (f) Let $<S> = K_3 \cup K_1$. Since G is connected, there exists a vertex u_i in K_{n-4} which is adjacent to x_1 and x_4 (or) u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_4. In both the cases, $\{x_2, x_3, x_4, u_i, u_j\}$ for $i \neq j$ is a γ_2 set of G, so that $\gamma_2 = 5$. Hence $n= 8$, since $\chi = n-4 =3$. Hence $K=K_4$. Let u_1, u_2, u_3, u_4 be the vertices of K_4. If u_1 is adjacent to x_1 and x_4, then $G \cong G_{16}$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_4, then $G \cong G_{17}$.

If G does not contain a clique K on $n-4$ vertices, then it can be verified that no new graph exists.

Case (v) Let $\gamma_2 = n\text{-}4$ and $\chi = n\text{-}3$. Since G is connected. Since $\chi = n\text{-}3$, G contains a clique K on $n\text{-}3$ vertices or does not contain a clique K on $n\text{-}3$ vertices. Let $S = \{x_1, x_2, x_3\} \in V\text{-}S$. Let $<S> = K_3, \overline{K_3}, P_3, K_2 \cup K_1$.

Subcase (a) Let $<S> = K_3$. Since G is connected, let x_1 be adjacent to u_i for some i in K_{n-3}. Then $\{x_2, x_3, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 8$. Hence $K = K_5$. Let u_1, u_2, u_3, u_4, u_5 be the vertices of K_5. If u_1 is adjacent to x_1, then $G \cong G_{18}$.

Subcase (b) Let $<S> = \overline{K_3}$. Since G is connected, one of the vertices of $K_{n-3,}$ say u_i is adjacent to all the vertices of S (or) two vertices of S (or) one vertex of S. In all the cases, $\{x_1, x_2, x_3, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n=9$. Hence $K=K_6$. Let $u_1, u_2, u_3, u_4, u_5, u_6$ be the vertices of $K_{6,}$ without loss of generality, u_1 is adjacent to all the vertices of S and $d(x_1) = d(x_2) = d(x_3) = 1$, hence $G \cong K_6\,(3, 0, 0, 0, 0, 0)$. If u_1 is adjacent to x_1, x_2 and u_2 is adjacent to x_3 and $d(x_1) = d(x_2) = d(x_3) = 1$, then $G \cong K_6\,(2, 1, 0, 0, 0, 0)$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_2 and u_3 is adjacent to x_3, then $G \cong K_6\,(1, 1, 1, 0, 0, 0)$.

Subcase (c) Let $<S> = P_3 = (x_1, x_2, x_3)$. Since G is connected, there exists a vertex u_i in K_{n-3} is adjacent to x_1 (or equivalently x_3) (or) u_i is adjacent to x_2. In both the cases, $\{x_1, x_3, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 8$. Hence $K = K_5$. Let u_1, u_2, u_3, u_4, u_5 be the vertices of $K_{5.}$ If u_1 is adjacent to x_1, then $G \cong K_5\,(P_4)$. If u_1 is adjacent to x_2, then $G \cong G_{19}$.

Subcase (d) Let $<S> = K_2 \cup K_1$. Let x_1x_2 be the edge in K_2. Since G is connected. There exists an u_i in K_{n-3} is adjacent to x_1 and x_3 (or) u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_3 . In both the cases, $\{x_2, x_3, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 8$ and hence $K=K_5$. Let u_1, u_2, u_3, u_4, u_5 be the vertices of K_5. If u_1 is adjacent to x_1 and x_3, then $G \cong K_5\,(P_3, P_2, 0, 0, 0)$. If u_1 is adjacent to x_1 and u_2 is adjacent to x_3, then $G \cong K_5\,(P_3, P_2, 0, 0, 0)$.

If G does not contain a clique K on $n\text{-}3$ vertices, then it can be verified that no new graph exists.

Case (vi) Let $\gamma_2 = n\text{-}5$ and $\chi = n\text{-}2$. Since $\chi = n\text{-}2$, G contains a clique K on $n\text{-}2$ vertices or does not contain a clique K on $n\text{-}2$ vertices. If G contains a clique K on $n\text{-}2$ vertices, then $S = \{x_1, x_2\} \in V\text{-}S$. Then $<S> = K_2$ or $\overline{K_2}$.

Subcase (a) Let <S> = K_2. Since G is connected, there exists a vertex u_i in K_{n-2} is adjacent to x_1. Then $\{x_2, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n=8$. Hence $K = K_6$. Let $u_1, u_2, u_3, u_4, u_5, u_6$ be the vertices of K_6. If u_1 is adjacent to x_1, then $G \cong K_6 (P_3)$.

Subcase (b) Let <S> = $\bar{K_2}$. Since G is connected, there exists a vertex u_i in K_{n-2} is adjacent to x_1 and x_2 (or) if u_i is adjacent to x_1 and u_j for $i \neq j$ is adjacent to x_2. In both the cases, $\{x_1, x_2, u_i, u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 9$. Hence $K = K_7$. Let $u_1, u_2, u_3, u_4, u_5, u_6, u_7$ be the vertices of K_7. If x_1 and x_2 be adjacent to u_1, then $G \cong K_7 (2,0,0,0,0,0,0)$. If x_1 is adjacent to u_1 and x_2 is adjacent to u_2, then $G \cong K_7(1,1,0,0,0,0,0)$.

If G does not contain a clique K on $n-2$ vertices, then it can be verified that no new graph exists.

Case (vii) Let $\gamma_2 = n-6$ and $\chi = n-1$. Since $\chi = n-1$, G contains a clique K on $n-1$ vertices or does not contain a clique K on $n-1$ vertices. If G contains a clique K on $n-1$ vertices, then there exists a vertex u_i in K_{n-1} adjacent to x. Hence $\{x,u_i,u_j\}$ for $i \neq j$ is a γ_2 set, so that $n = 9$. Hence $K = K_8$. Let $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$ be the vertices of K_8. If u_1 is adjacent to x, then $G \cong K_8 (1,0,0,0,0,0,0,0)$.

If G does not contain a clique K on $n-1$ vertices, then it can be verified that no new graph exists.

Case (viii) Let $\gamma_2 = n-7$ and $\chi = n$. Since $\chi = n$, we have $G = K_n$. But for K_n, $\gamma_2 = 2$, so that $n = 9$.

Hence $G \cong K_9$. ■

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