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(k, d)-even mean labeling of $P_m \odot nK_1$

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Abstract

In this paper, we introduce the concept of (k, d)-even mean labeling and investigate (k, d)-even mean labeling of $P_m \odot nK_1$.

Key words: k-even mean labeling, k-even mean graph, (k, d)-even mean labeling, (k, d)-even mean graph

AMS Subject Classification(2010): 05C78

1 Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [9]. The symbols $V(G)$ and $E(G)$ denote the vertex set and edge set of a graph. Labeled graphs serve as useful models for a broad range of applications [1–4].

Many studies in graph labeling refer to Rosa's research in 1967 [11]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). Graph labeling was first introduced in the late 1960's.

The concept of mean labeling was introduced in [12] and that of odd mean labeling in [10]. k-odd mean labeling and (k, d)-odd mean labeling are introduced and discussed in [5–7]. k-even mean labeling was introduced in [8]. In this paper, we introduce the concept of (k, d)-even mean labeling and investigate the (k, d)-even mean labeling of $P_m \odot nK_1$.

Throughout this paper, k and d denote any positive integer ≥ 1. For brevity, we use (k, d)-EML for (k, d)-even mean labeling.

2 Main Results

Definition 2.1. *A (p, q) graph G is said to have a (k, d)-even mean labeling if there exists an injection*

$$f : V \to \{0, 1, 2, \ldots, 2k + 2(q-1)d\}$$

such that the induced map f^* defined on E by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

is a bijection from E to $\{2k, 2k + 2d, 2k + 4d, \ldots, 2k + 2(q-1)d\}$.

A graph that admits a (k, d)-even mean labeling is called a (k, d)-even mean graph.

Theorem 2.2. *$Pm \odot nK_1$ ($m \geq 3$, $n \geq 2$) is a (k, d)-even mean graph if*

(i) m is even and for any k and d

(ii) m is odd, n is odd and for any k and d

(iii) m is odd, n is even and for any $k \geq d$.

Proof. Let $V(P_m \odot nK_1) = \{u_i,\ 1 \leq i \leq m\} \cup \{u_{ij},\ 1 \leq i \leq m,\ 1 \leq j \leq n\}$ and $E(P_m \odot nK_1\} = \{e_i, 1 \leq i \leq m-1\} \cup \{e_{ij},\ 1 \leq i \leq m, 1 \leq j \leq n\}$ (see Fig.1)

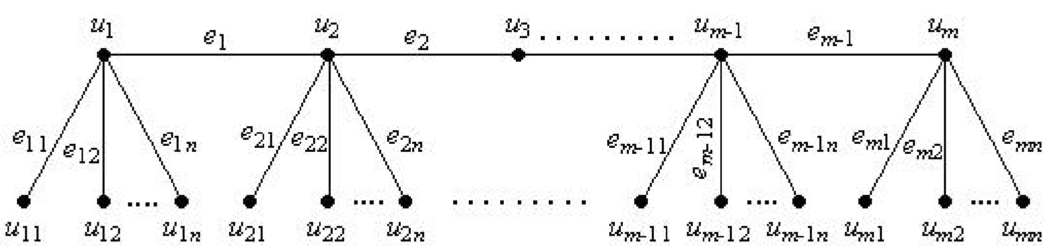

Figure 1: Ordinary labeling of $P_m \odot nK_1$

In this graph $q = m(n + 1) - 1$.
Define $f : V(P_m \odot nK_1) \to \{0, 1, 2, \ldots, 2k + 2(q-1)d\}$ by

Case (i) m is even and for any n, k and d.

For $1 \leq i \leq m-1$,

$$f(u_i) = \begin{cases} 2k + 2d(i-1)(n+1) - 1, & \text{if } i \text{ is odd} \\ 2k + 2d[2n + (n+1)(i-2)] + 1, & \text{if } i \text{ is even.} \end{cases}$$

$f(u_m) = 2k + 2d[m(n+1) - 2]$.

For $1 \leq i \leq m$ and $1 \leq j \leq n$,

$$f(u_{ij}) = \begin{cases} 2k + 2d[(i-1)(n+1) + 2j - 2] + 1, & \text{if } i \text{ is odd} \\ 2k + 2d[(i-2)(n+1) + 2j] - 1, & \text{if } i \text{ is even.} \end{cases}$$

Then the induced edge labels are as follows:

For $1 \leq i \leq m-1$,

$$f^*(e_i) = \begin{cases} 2k + 2d[n + (n+1)(i-1)], & \text{if } i \text{ is odd} \\ 2k + 2d[2n + 1 + (n+1)(i-2)], & \text{if } i \text{ is even.} \end{cases}$$

For $1 \leq i \leq m$ and $1 \leq j \leq n$; $f^*(e_{ij}) = 2k + 2d[(n+1)(i-1) + j - 1]$.

Therefore, $f^*(E(P_m \odot nK_1)) = \{2k, 2k+2d, 2k+4d, \ldots, 2k+2(q-1)d\}$.

So, f is a (k, d)-even mean labeling and hence, $P_m \odot nK_1$ is a (k, d)-even mean graph.

The examples for (2, 1)-EML of $P_4 \odot 10K_1$ and (2, 3)-EML of $P_4 \odot 5K_1$ are shown in Figure 2 and Figure 3 respectively.

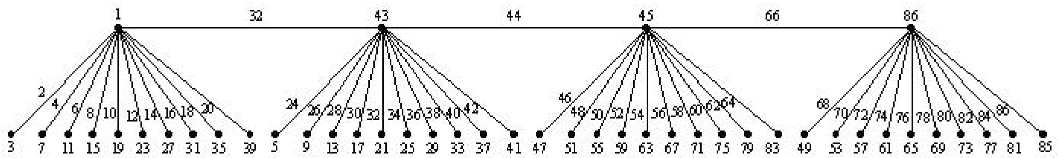

Figure 2: The examples for (2,1)-EML of $P_4 \odot 10K_1$

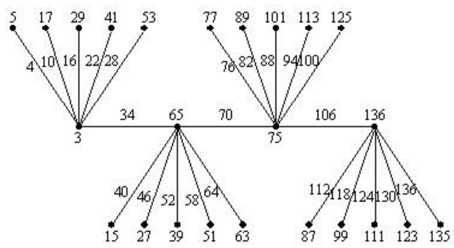

Figure 3: The examples for (2,3)-EML of $P_4 \odot 5K_1$

Case (ii) m is odd, n is odd and for any k, d.

The vertex labels are as follows:

For $1 \leq i \leq m-1$,
$$f(u_i) = \begin{cases} 2k + 2d(i-1)(n+1) - 1, & \text{if } i \text{ is odd} \\ 2k + 2d[2n + (n+1)(i-2)] + 1, & \text{if } i \text{ is even.} \end{cases}$$
$f(u_m) = 2k - 1 + 2d[m(n+1) - 2]$.
For $1 \leq i \leq m-1$ and $1 \leq j \leq n$,
$$f(u_{ij}) = \begin{cases} 2k + 2d[(i-1)(n+1) + 2j - 2] + 1, & \text{if } i \text{ is odd} \\ 2k + 2d[(i-2)(n+1) + 2j] - 1, & \text{if } i \text{ is even.} \end{cases}$$

$$f(u_{mj}) = 2k + 2d[n + (n+1)(m-3) + 2j - 1], \quad 1 \leq j \leq \frac{n-1}{2}.$$
$$f(u_{mj}) = 2k + 2d[n + (n+1)(m-3) + 2j + 1], \quad \frac{n+1}{2} \leq j \leq n.$$

Then the induced edge labels are as follows:
For $1 \leq i \leq m-2$,
$$f^*(e_i) = \begin{cases} 2k + 2d[n + (n+1)(i-1)], & \text{if } i \text{ is odd} \\ 2k + 2d[2n + 1 + (n+1)(i-2)], & \text{if } i \text{ is even.} \end{cases}$$
$f^*(e_{m-1}) = 2k + d[2m(n+1) - n - 5]$.
For $1 \leq i \leq m-1$ and $1 \leq j \leq n$,
$f^*(e_{ij}) = 2k + 2d[(n+1)(i-1) + j - 1]$.
$$f^*(e_{mj}) = \begin{cases} 2k + 2d[n(m-1) + m + j - 3], & 1 \leq j \leq \frac{n-1}{2} \\ 2k + 2d[n(m-1) + m + j - 2], & \frac{n+1}{2} \leq j \leq n. \end{cases}$$
Therefore, $f^*(E(P_m \odot nK_1)) = \{2k, 2k+2d, 2k+4d, \ldots, 2k+2(q-1)d\}$.
So, f is a (k,d)-even mean labeling and hence, $P_m \odot nK_1$ is a (k,d)-even mean graph.

The examples for (1, 2)-EML of $P_3 \odot 7K_1$ and (2, 3)-EML of $P_3 \odot 9K_1$ are shown in Figures 4 and 5 respectively.

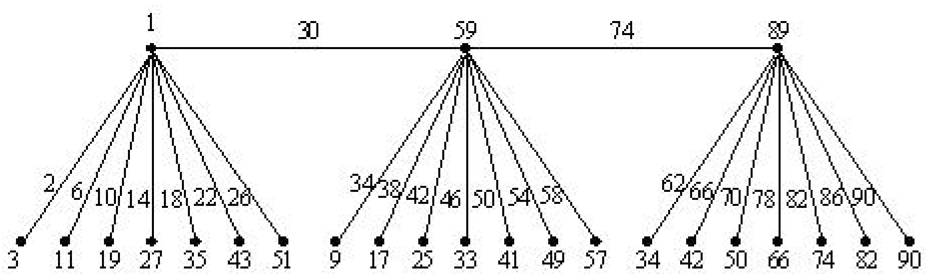

Figure 4: (1, 2)-EML of $P_3 \odot 7K_1$

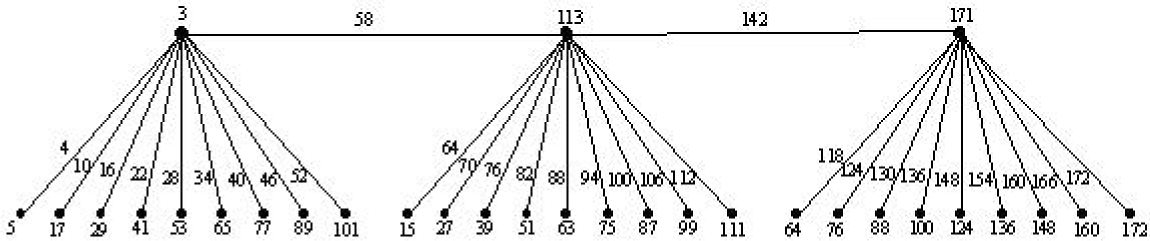

Figure 5: (2,3)-EML of $P_3 \odot 9K_1$

Case (iii) m is odd, n is even and for any n, $k \geq d$.

The vertex labels are as follows:

For $1 \leq i \leq m-1$,

$$f(u_i) = \begin{cases} 2k + 2d[(i-1)(n+1) - 1] + 1, & \text{if } i \text{ is odd} \\ 2k - 1 + 2d[2n + 1 + (n+1)(i-2)], & \text{if } i \text{ is even.} \end{cases}$$

$f(u_m) = 2k + 2d[m(n+1) - 2]$.

For $1 \leq i \leq m-1$ and $1 \leq j \leq n$,

$$f(u_{ij}) = \begin{cases} 2k - 1 + 2d[(n+1)(i-1) + 2j - 1], & \text{if } i \text{ is odd} \\ 2k + 2d[(n+1)(i-2) + 2j - 1] + 1, & \text{if } i \text{ is even.} \end{cases}$$

$$f(u_{mj}) = \begin{cases} 2k + 2d[(n+1)(m-2) + 2(j-1)], & 1 \leq j \leq \frac{n}{2} \\ 2k + 2d[(n+1)(m-2) + 2j], & \frac{n+2}{2} \leq j \leq n-1. \end{cases}$$

$f(u_{mn}) = 2k - 1 + 2d[m(n+1) - 2]$.

Then the induced edge labels are as follows:

For $1 \leq i \leq m-2$,

$$f^*(e_i) = \begin{cases} 2k + 2d[n + (n+1)(i-1)], & \text{if } i \text{ is odd} \\ 2k + 2d[2n + 1 + (n+1)(i-2)], & \text{if } i \text{ is even.} \end{cases}$$

$f^*(e_{m-1}) = 2k + d[2m(n+1) - n - 4]$.

For $1 \leq i \leq m-1$ and $1 \leq j \leq n$,

$f^*(e_{ij}) = 2k + 2d[(n+1)(i-1) + j - 1]$.

$$f^*(e_{mj}) = \begin{cases} 2k + 2d[n(m-1) + m + j - 3], & 1 \leq j \leq \frac{n}{2} \\ 2k + 2d[m(n+1) - n + j - 2], & \frac{n+2}{2} \leq j \leq n-1. \end{cases}$$

$f^*(e_{mn}) = 2k + 2d[m(n+1) - 2]$.

Therefore, $f^*(E(P_m \odot nK_1)) = \{2k, 2k+2d, 2k+4d, \ldots, 2k+2(q-1)d\}$.

So, f is a (k, d)-even mean labeling and hence, $P_m \odot nK_1$ is a (k, d)-even mean graph.

The example for (4,2)-EML of $P_5 \odot 4K_1$ is shown in Fig. 6.

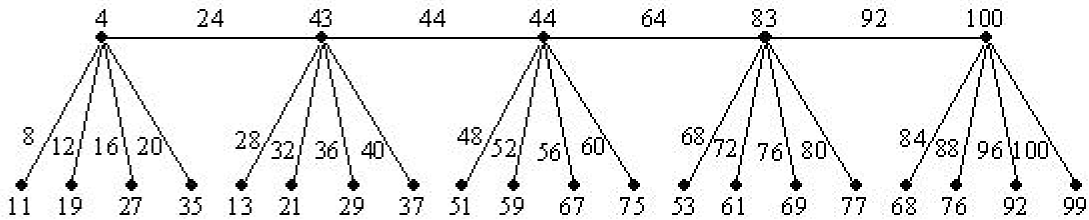

Figure 6: (4, 2)-EML of $P_5 \odot 4K_1$.

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References

[1] G.S. Bloom, S.W. Golomb, *Applications of numbered undirected graph*s, Proc. IEEE, 65 (1977), 562–570.

[2] G.S. Bloom, S.W. Golomb, *Numbered complete graphs, unusual rulers and assorted applications*, Theory and Applications of Graphs - Lecture notes in Math., Springer-Verlag, New York, 642 (1978), 53-65.

[3] G.S. Bloom, D.F. Hsu, *On graceful digraphs and a problem in network addressing*, Congressus Numerantium, 35 (1982), 91-103.

[4] A. Gallian, *A dynamic survey of graph labeling*, Electronic Journal of Combinatorics, 17 (2010) # DS6.

[5] B. Gayathri, K. Amuthavalli, *k-odd mean labeling of crown graphs*, International Journal of Mathematics and Computer Science, 3(2) (2007), 253-259.

[6] B. Gayathri, K. Amuthavalli, *(k, d)-odd mean labeling of some graphs*, Bulletin of Pure and Applied Sciences, 26E(2) (2007), 263-267.

[7] B. Gayathri, K. Amuthavalli, *k-odd mean labeling of $\langle K_{1,n} : K_{1,m}\rangle$*, Acta Ciencia Indica, XXXIVM(2) (2008), 827-834.

[8] B. Gayathri, R. Gopi, *k-even mean labeling of $D_{m,n}@C_n$*, International Journal of Engineering Science, Advanced Computing and Bio-Technology 1(3) (2010), 137-145.

[9] F. Harary, *Graph theory*, Addison-Wesley, Reading, Massachusetts, 1972.

[10] K. Manickam, M. Marudai, *Odd mean labelings of graphs*, Bulletin of Pure and Applied Sciences, 25E(1) (2006), 149-153.

[11] A. Rosa, *On certain valuations of the vertices of a graph*, Theory of Graphs (Internat. Symposium, Rome, July 1966). Gordon and Breach, N.Y. and Dunod, Paris (1967) 349-355.

[12] S. Somasundaram, R. Ponraj, *Mean labelings of graphs*, Natl. Academy Science Letter, 26(7-8) (2003), 10-13.