
Meta-Heuristic Cuckoo Search Algorithm for the Correction of Faulty Array Antenna

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ABSTRACT

In this article, we introduce a CSA (Cuckoo Search Algorithm) for compensation of faulty array antenna. It is assumed that the faulty elemental location is also known. When the sensor fails, it disturbs the power pattern, owing to which its SLL (Sidelobe Level) raises and nulls are shifted from their required positions. In this approach, the CSA optimizes the weights of the active elements for the reduction of SLL and null position in the desired direction. The meta-heuristic CSA is used for the control of SLL and steering of nulls at their required positions. The CSA is based on the necessitated kids' bloodsucking behavior of cuckoo sort in arrangement with the Levy flight manners. The fitness function is used to reduce the error between the preferred and probable pattern along with null constraints. Imitational consequences for various scenarios are given to exhibit the validity and presentation of the proposed method.

Key Words: Cuckoo Search Algorithm, Chebyshev Array, Null Position, Linear Array, Sidelobes.

1. INTRODUCTION

In array antenna, null placement in the direction of interferers and beam steering in the desired direction are very important area of research. Its demand has increased rapidly during the last decade. In array antenna, the main beam is placed in the desired direction and nulls are placed in the direction of interferers. It has appliance in sonar, mobile and satellite [1]. In this paper, various methods are referred to the problem of side lobes reduction and null positioning in failed array antenna [2-5]. The condition becomes more severe and complex when a particular sensor fails in the antenna array. In case of sensor failure, the SLL raises high and nulls are damaged, which is very critical. It is very costly in terms of time and resources to restore the malfunctioning element frequently. In case of satellite, it is not possible to restore the faulty sensors. Hence the weights of the remaining sensors in

the array should be recalculated and readjusted to generate a new pattern close to the original pattern. In case of failure correction some algorithms [6-8] have been planned to accurate the faulty pattern of the antenna array.

In the field of detection and correction of faulty arrays researchers are working to develop methods which are computationally efficient in case of detecting the faulty element and provide the required pattern in case of correction. In literature [9-10] using the mete-heuristic computing technique, the method recalculates and adjusts the weights of the active array to get the pattern near the original one. By recalculating the weights of the faulty array using heuristic technique, the array will improve its capabilities in the desired directions. In [11-13] the symmetrical sensor malfunction method is used to get the

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required null depth level and first null beam width and in [14] the failed elements signal is reconstructed from the symmetrical counterpart element in the array by taking its conjugate. Firefly algorithm is used for finding the defective element position [15]. Evolutionary computing algorithms are stochastic population based methods that have established to be dominant and forceful to solve optimization problems.

In this article, we are introduce a CSA for correction of faulty elements in linear array antenna. The CSA optimizes the weights of the active elements for the reduction of SLL and null position in the desired direction. Its problems include the raise in SLL and dislocation of nulls from their original positions. We propose a CSA that provides improved results in terms of the reduced SLL and null position in the required direction. The technique which is based on the meta-heuristic computing algorithm is proposed, namely CSA is used for the reduction of side lobes and position of nulls in the required direction and null broadening. Imitational results for different scenarios are provided to verify the presentation of the planned method.

The set-up of the article is prepared as follows. We will discuss the design of array factor in Section 2, while antenna pattern for different techniques in Section 3 and proposed methodology is in Section 4. Section 5 describes the simulations, Section 6 will talk about the results and at the end Section 7 will conclude the paper and propose some future work.

2. DESIGN OF ARRAY FACTOR FOR LINEAR ARRAY

Let us consider an array of 30 sensors in which all the sensors are positioned along x-axis as shown in Fig. 1. The power pattern for these working elements with uniformly spaced sensors weight will be [16]:

$$AF(\theta_i) w^H s(\theta_i) \tag{1}$$

where $w = [w_{-M}, w_{-M+1}, w_{-M+2}, \dots, w_{-1}, \dots, w_1, \dots, w_{M-2}, w_{M-1}, w_M]^T$ is the weight vector and $s(\theta_i)$ is the steering vector given by:

$$s(\theta_i) = \left[\exp - j \left(\frac{2M-1}{2} \right) kd \sin \theta_i, \dots, \exp - j \left(\frac{1}{2} \right) kd \sin \theta_i, \right. \\ \left. \exp j \left(\frac{1}{2} \right) kd \sin \theta_i, \dots, \exp j \left(\frac{2M-1}{2} \right) kd \sin \theta_i \right]^T$$

The array factor for $N=2M$ number of elements can be written as:

$$AF(\theta_i) = \sum_{n=1}^N w_n \cos \left[\left(\frac{2n-1}{2} \right) kd \sin \theta_i \right] \tag{2}$$

Where w_n is the complex weight of n th antenna element selected to steer a desired array pattern, whereas $n=1, 2, \dots, N$. The d , is the distance between the elements and θ_i is the angle. $k=2\pi/\lambda$ is the wave number and λ is the wavelength. Now if any m th element gets failed in the array, radiation pattern is disturbed in the terms of SLL and diminishing of nulls, then the output of the beam former for the non-healthy setup will be:

$$AF(\theta_i) = \sum_{\substack{n=1 \\ n \neq m}}^N w_n \cos \left[\left(\frac{2n-1}{2} \right) kd \sin \theta_i \right] \tag{3}$$

The power pattern for 3rd sensor damage is given by:

$$AF(\theta_i) = \sum_{\substack{n=1 \\ n \neq 3}}^N w_n \cos \left[\left(\frac{2n-1}{2} \right) kd \sin \theta_i \right] \tag{4}$$

We consider that the 3rd sensor is failed in the array specified in Fig. 1. From Fig. 2 it is clear that the power pattern is disturbed in terms of SLL and null are damaged due to malfunctioning of sensors. In this article, our main work is to restore the damage pattern in terms of SLL, and null steering in the direction of interferer. Different meta-heuristic optimization techniques are provided in literature to correct the damage pattern of element failure, however, in this work; we use CSA to optimize the weights of the remaining element with minimum SLL and nulls in the direction of interferer.

3. ANTENNA PATTERN FOR DIFFERENT SCHEMES

The conventional beam forming has equal magnitude weights. The phases are preferred to turn the array in the

desired direction and nulls in the interferer direction. The Taylor weights are optimized in the sense that it leaves the conventional pattern undistorted except for a reduction of the first n side lobes below a prescribed level. The Dolph Chebyshev pattern provides a constant SLL of equal weightage. Fig. 3 shows different pattern of linear array of 21 elements. The conventional pattern is taken as a reference pattern. The SLL is selected in such a way that the 3 dB beam width of all patterns is approximately equal.

4. PROPOSED SOLUTION

In this section, we develop the CSA for the correction of faulty elements in failed array antenna. In this case, as we have assumed the failure of 3rd element, the whole radiation pattern is disturbed. Its SLL gets high and nulls are displaced from their original positions as shown in Fig. 2. The CSA is used to restore the weights of the remaining active elements to reduce SLL and places nulls in the direction of interferers. When generating new solutions $w^{(t+1)}$ for a cuckoo i , a levy flight is performed:

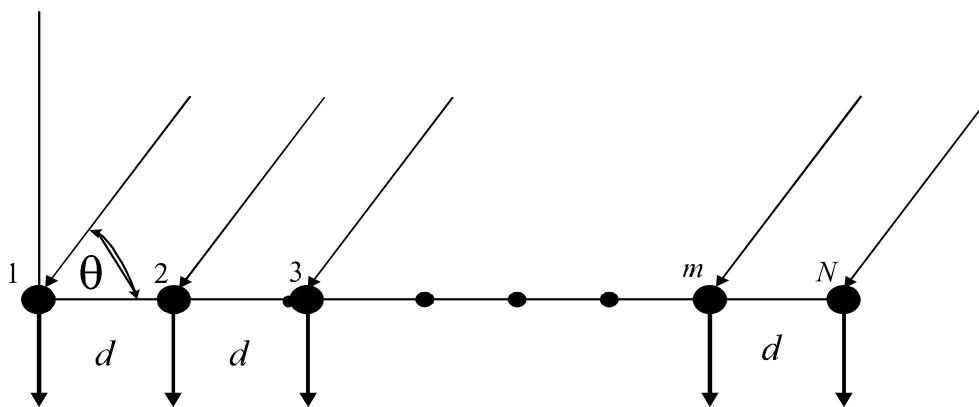


FIG. 1. THE LINEAR ARRAY OF N ELEMENTS WITH 3RD ELEMENT FAILURE

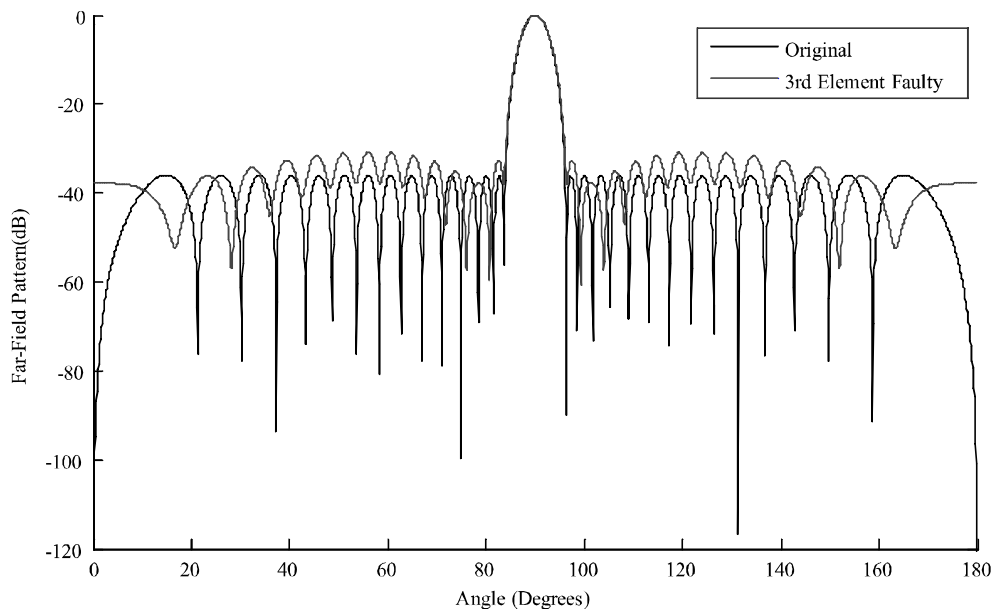


FIG. 2. THE ORIGINAL DOLPH CHEBYSHEV PATTERN AND THE 3RD FAULTY ELEMENT PATTERN

$$w_i^{t+1} = \beta w_i^t + (1 - \beta) \alpha \oplus Levy(\lambda) \quad (5)$$

Where the value of $0 < \beta < 1$ and $\alpha > 0$ is the step size which is related to the problem of interest while the product represents the entry-wise multiplications. Levy distribution for large steps applied a power law:

$$Levy = t^{-\lambda} (1 < \lambda \leq 3) \quad (6)$$

The Levy flight provides a random walk and the random step length is from a Levy distribution which has an infinite variance. By using the CSA, the weights w_n of the remaining array elements are found to achieve the minimum SLL and nulls at their required locations.

4.1 Cuckoo Search Algorithm

CSA is the latest meta-heuristic algorithm based on the breeding behaviour of brood parasitism of certain species of cuckoos. First we introduce the breeding behaviour of cuckoos and the characteristics of Levy flights of birds and fruit flies. Cuckoos are charming birds, not only because of their striking sounds, but also of their aggressive reproduction. The CSA depends on three assumptions; (i) Each cuckoo lays one egg at a time, and places its egg randomly in the chosen nest, (ii) The best nests with high quality of eggs will be carried over to the next generations, (iii) The number of available host nests

is fixed, and the cuckoo's egg is discovered by the host bird with a probability $p_a \in [0,1]$. In this case, the host bird can either throw the egg away or abandon the nest, and build a completely new nest. For simplicity, this last assumption can be approximated by the fraction p_a of the n nests replaced by new nests. In the present work, we use the simple representations that each egg in a nest represents a solution, and a cuckoo's egg represents a new solution, the job is to use the new and better solution to replace it, not so good solution in the nests. Flight behavior of animals and insects has verified the attribute of levy flight [17-20]. Levy flight is a random walk with a heavy tailed probability distribution [21]. The behavior has been emulated to global optimal search and optimization [22] with a hopeful ability [23].

In this paper, the CSA has been used for the reduction of SLL and null steering at their desired positions after failure of array elements. Fig. 4 is the flowchart which shows the steps of CSA. Depending upon the three idealized rules, the pseudo code of the CSA can be shown in Fig. 5. The parameters used for CSA is give in Table 1. Levy flight is a process in which after a huge number of steps, the distance from the source of random walk tends to a stable distribution. The levy flight is done through the stochastic procedure.

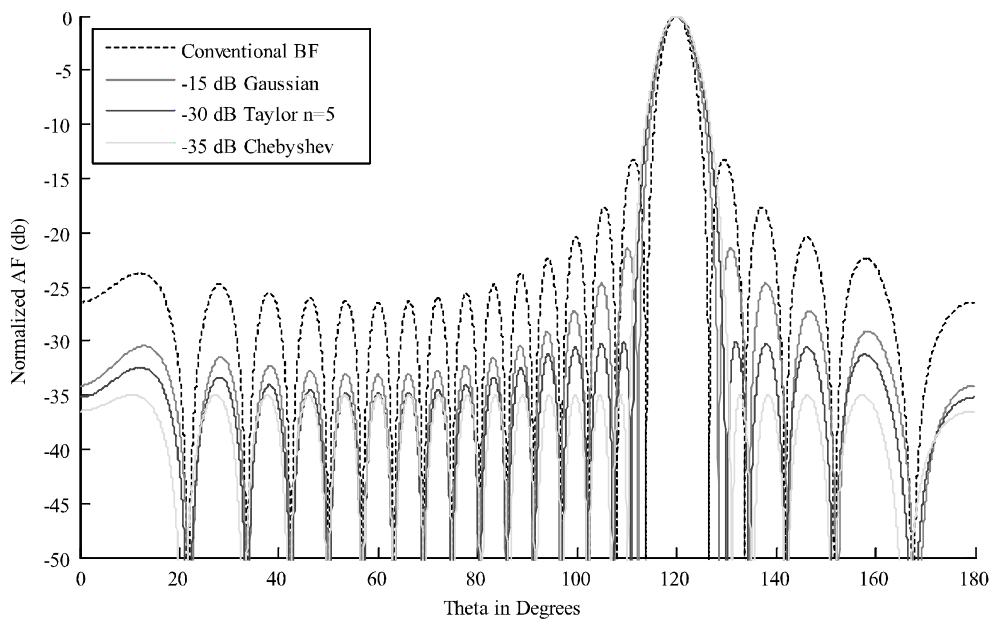


FIG. 3. LOW SIDE LOBE LEVEL PATTERN OF EQUAL BEAM WIDTH

4.2 Null Constraint

A jamming signal location at a particular angle wants to be eliminated, in case of satellite, radar and mobile communication applications. For a uninformed array, to put a null at a particular angle θ_i , we want [24],

$$AF(\theta_i) = w^H s(\theta_i) = 0 \tag{7}$$

where w is the weight vector and $s(\theta_i)$ is the steering vector. The null constraint is given as:

$$w^H s(\theta_i) = 0, \quad i = 1, 2, \dots, M_0$$

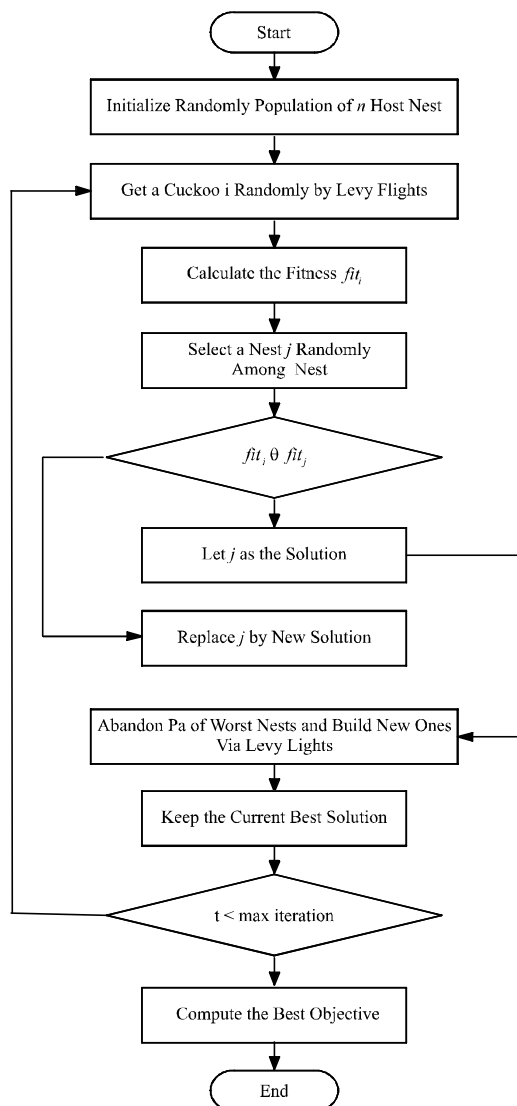


FIG. 4. THE FLOW DIAGRAM OF THE CUCKOO SEARCH ALGORITHM

Accordingly we may define the fitness function as follows:

$$fitness = \sum_{i=1}^P \beta [|AF_d(\theta_i) - AF_{CSA}(\theta_i)|]^2 + \sum_k \gamma |AF_d(\theta_k)|^2 \tag{8}$$

The fitness function of Equation (8) is to be minimized. The finest result of the fitness function will give the minimum cost of the fitness. The first value in Equation (8) is used for SLL decrease and β is the weighting factor to adjust the SLL of the corrected pattern, where $AF_d(\theta_i)$ is the preferred pattern and $AF_{CSA}(\theta_i)$ is obtained by using CSA. The second value in Equation (8) is use for controlling the null positions and γ is the factor for controlling the null depth level.

Start

Fitness function $f(w)$, $w = (w_1, w_2, \dots, w_d)^T$

Create the initial inhabitants of n swarm nests $w_i = i(1, 2, 3, \dots, n)$

while ($t < \text{max number of generaton or stop criterion}$)

 obtain a cuckoo arbitrarily by Levy flights

 compute the fitness fit_i

 select arbitrarily a nest j

 If $fit_i < fit_j$, replace j by new solution i

end

A fraction (p_a) of worst nests are abandoned and new nests are built.

Keep the best solution.

Rank the solutions and find the current best solution

end while

Postprocess results and visualization

End

FIG. 5. THE PSEUDO-CODE OF THE CUCKOO SEARCH ALGORITHM

TABLE 1. PARAMETER SETTING FOR CSA

CSA	
Parameters	Setting
Population size	25
No of Generation	500
Step size	1
Discovery rate	0.25

5. SIMULATION RESULTS

Let us consider a Chebyshev array of 30 sensors with 0.5λ element spacing between the sensors is used as the test antenna. The power pattern represents a -35 dB SLL with the nulls at the given angles. Some methods are available in literature to calculate the weights for the Chebyshev array. In case of sensor damage, the CSA is used to optimize the weights of the active elements in failed array antenna for the reduction of SLL and nulls in the direction of interferers.

Case-A: We assumed that the 3rd element in the array becomes failed. After elemental failure the whole pattern becomes destroyed in terms of SLL and diminishing of null positions. In order to recover the required pattern, we use CSA to optimize the remaining weights of the array. By applying CSA, we achieve the desired SLL and null placement in the direction of interferers. Due to 3rd element failure the SLL rises to -30.82 dB and nulls are damaged as shown in Fig. 6. After optimization by the CSA, the weights of the remaining elements are optimized and the SLL are reduced to -35.93 dB as depicted in Fig. 7.

By applying the CSA, the weights of the active elements are optimized which results in reduced the SLL and the optimized SLL are given in Table 2. If the interferer is coming from an angle of $\theta=70^\circ$, then we will place null at an angle of $\theta=70^\circ$ as shown in Fig. 8. After optimization by CSA the SLL and NDL of one null are given in Table 3.

Now if two interferers are coming from the prescribed direction, then we need to place two nulls in the required directions. The two nulls are recovered at angles of $\theta_1=45^\circ$ and $\theta_2=150^\circ$ for the 3th elemental failure in array antenna as shown in Fig. 9. The SLL and null depth level for the two nulls are given in Table 4.

Now if the interferers change their location, the nulls will be placed in the direction of interferers. In this case, two nulls are placed at angles of $\theta_1=30^\circ$ and $\theta_2=70^\circ$ as shown in Fig. 10.

Now we check the recovery of two nulls at angles of $\theta_1=70^\circ$ and $\theta_2=150^\circ$ for the 3rd element failure in array antenna is given in Fig. 11.

Now if three interferers are coming from three different directions, we place three nulls in the direction of interferers. The recovery of three nulls at angles of $\theta_1=45^\circ$, $\theta_2=70^\circ$ and $\theta_3=120^\circ$ are shown in Fig. 12. The SLL and NDL for the failed array and recovered pattern of three nulls are given in Table 5.

If the three jammers change their location, then in this case the three nulls are placed in the direction of interferers at angles of $\theta_1=25^\circ$, $\theta_2=45^\circ$ and $\theta_3=70^\circ$ are shown in Fig. 13. As the jammer changes their location, the null direction will be change in the direction of interferers.

The null broadening is very important in adaptive beam forming in failed array antenna. In this case we discuss the null broadening in failed array antenna. The broad nulls are required when the direction of interferers changes slightly. In this work, the broad nulls are placed at angles of $\theta=125^\circ$ with $\theta=5^\circ$ is obtained by adjusting the weights of the active elements along with null constraint as shown in Fig. 14.

Case-B: The main beam direction can be steered in the required position when user changes their position. Fig. 15 shows the main beam direction at an angle of $\theta_s=125^\circ$ with the nulls at the specific angle. The array factor for the main beam directed in the direction θ_s can be written by the following expression as:

$$AF(\theta_i) = \sum_{\substack{n=1 \\ n \neq m}}^N w_n \cos \left[\left(\frac{2n-1}{2} \right) kd (\sin \theta_i - \sin \theta_s) \right] \quad (9)$$

6. CONCLUSION

We have proposed a meta-heuristic CSA for the compensation of faulty arrays. The CSA is used first time for the reduction of SLL and null steering in the direction of interferers in failed array antenna. Null steering and SLL reduction has been achieved by a CSA using a proper fitness function. The replication outcome for various scenarios has been to exhibit the validity and presentation of the proposed method. The decrease in the corrected pattern comes at the charge of wider main beam. The corrected pattern has beamwidth wider than that of the

original pattern. By using the appropriate fitness function, with the decrease of the SLL, we can turn the nulls in the

desired directions. This technique can be extensive to circular arrays.

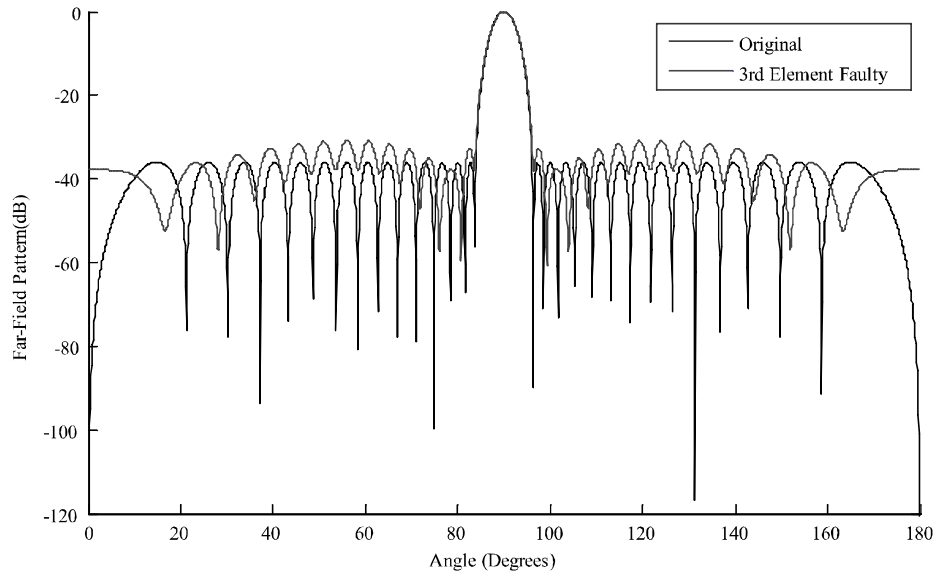


FIG. 6. THE ORIGINAL DOLPH CHEBYSHEV PATTERN AND THE 3RD ELEMENT FAULTY PATTERN.

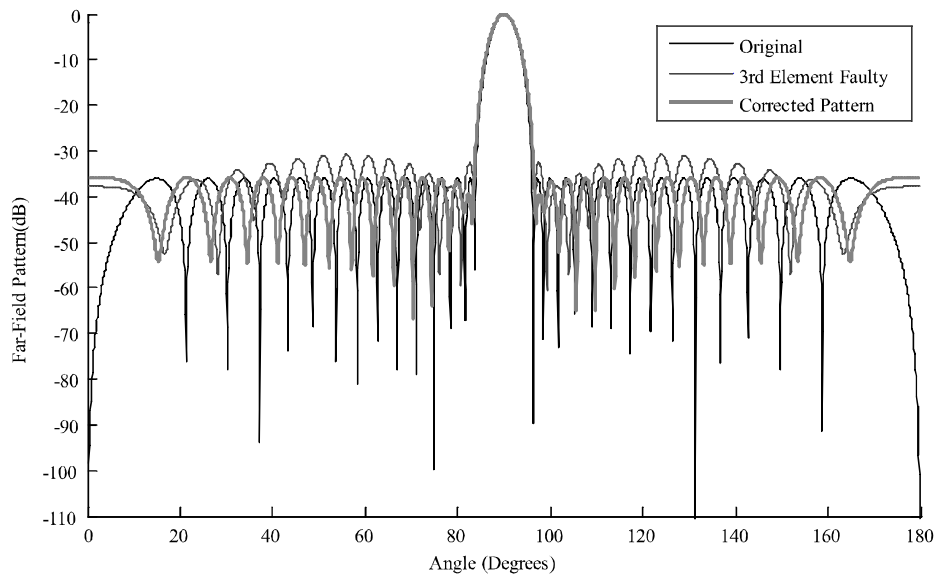


FIG. 7. THE ORIGINAL DOLPH CHEBYSHEV, 3RD ELEMENT FAULTY AND CORRECTED PATTERN

TABLE 2. RECOVERY OF THE REQUIRED PATTERN

Before and After Correction the SLL and NDL				
Before Correction		After Correction		Nulls Recovery
SLL (dB)	NDL (dB)	SLL (dB)	NDL (dB)	
-31.2	-55.03	-35.93	-66.89	Recovered pattern.

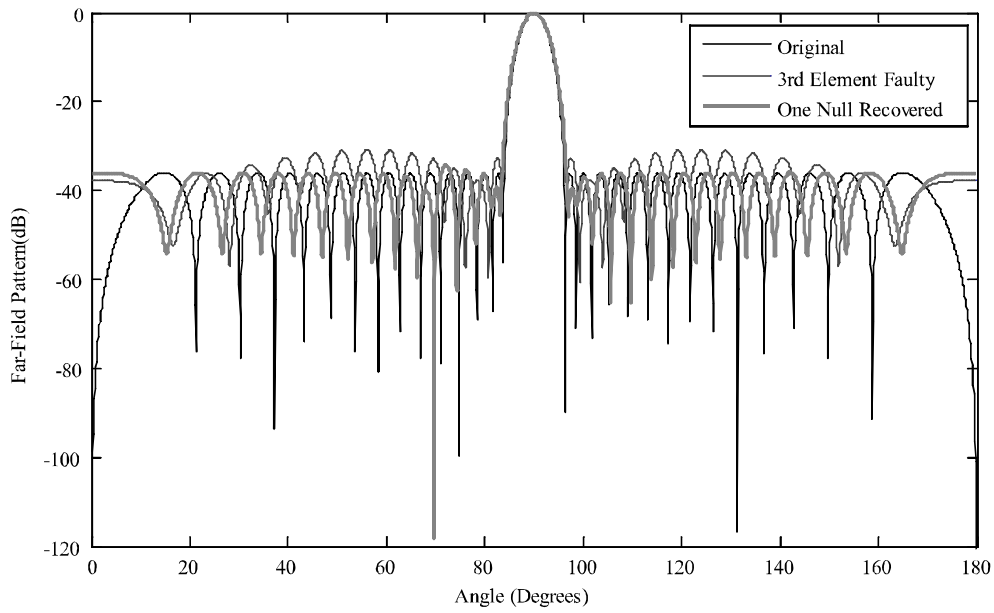


FIG. 8. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAILURE AND ONE NULL RECOVERED PATTERN

TABLE 3. RECOVERY OF ONE NULL

Before and After Correction the SLL and NDL				
Before Correction		After Correction		Nulls Recovery
SLL (dB)	NDL (dB)	SLL (dB)	NDL (dB)	
-31.6	-55.25	-40.72	-118.3	One null recovered

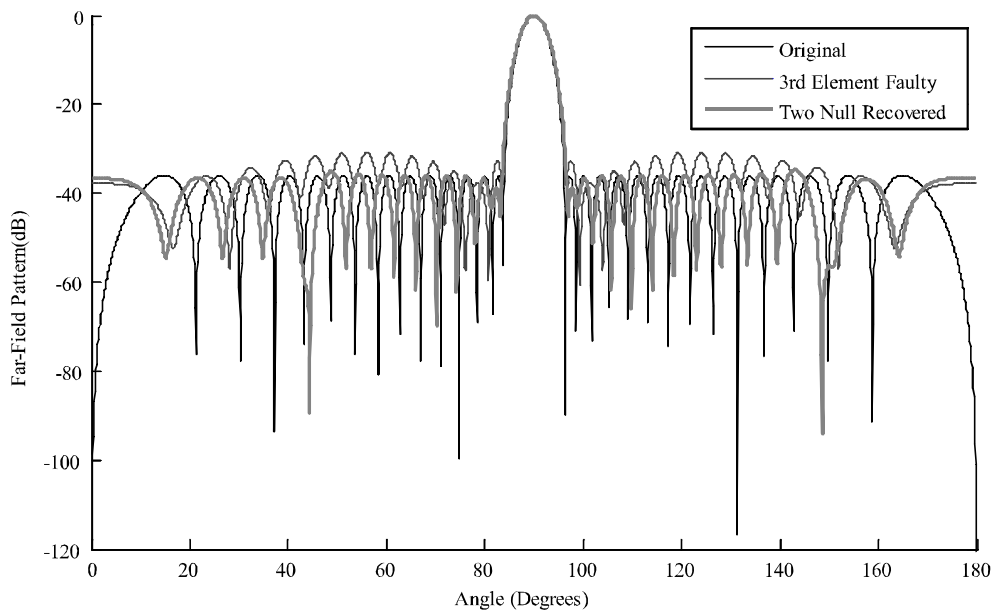


FIG. 9. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAILURE AND TWO NULL RECOVERED PATTERNS

TABLE 4. RECOVERY OF ONE NULL

Before and After Correction the SLL and NDL				
Before Correction		After Correction		Nulls Recovery
SLL (dB)	NDL (dB)	SLL (dB)	NDL (dB)	
-31.6	-53.4	-34.7	-89.7	One null recovered
-31.3	-53.55	-34.1	-93.91	Second null recovered

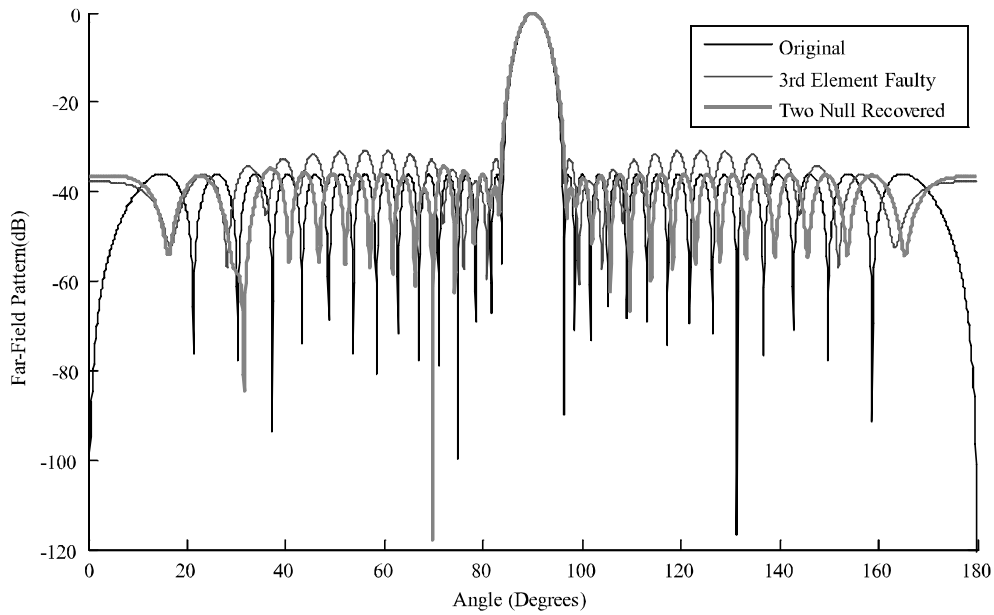


FIG. 10. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENTAL FAILURE AND TWO NULL RECOVERED PATTERNS

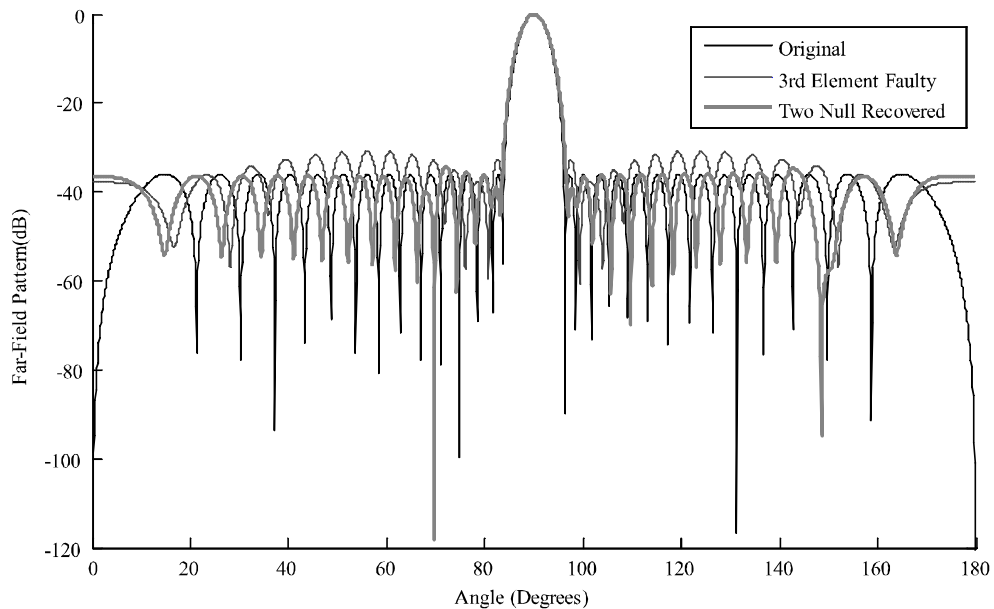


FIG. 11. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAILURE AND TWO NULL RECOVERED PATTERNS

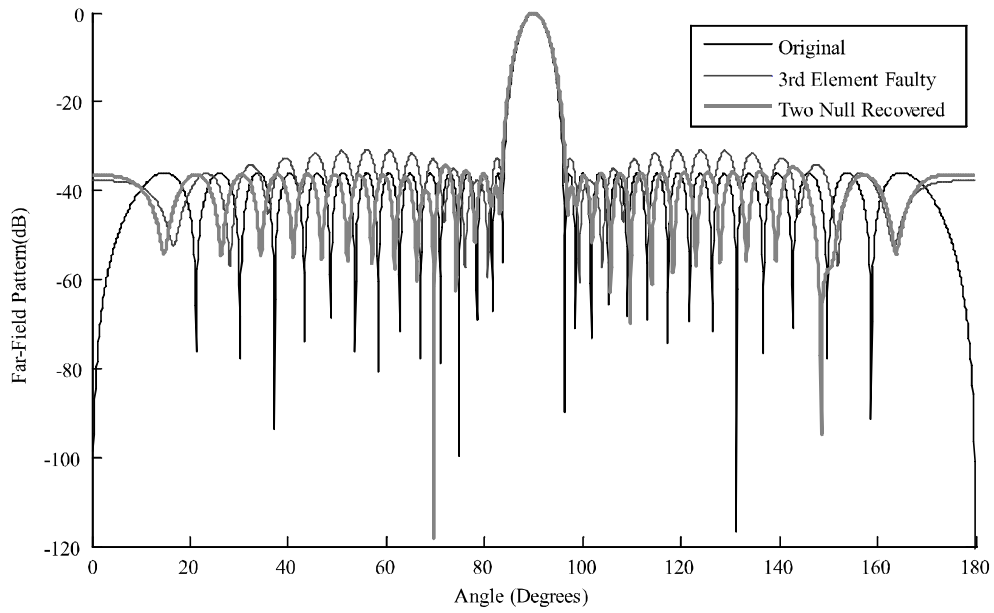


FIG. 12. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAILURE AND THREE NULL RECOVERED PATTERNS

TABLE 5. THREE NULL RECOVERED

Before and After Correction the SLL and NDL				
Before Correction		After Correction		Nulls Recovery
SLL (dB)	NDL (dB)	SLL (dB)	NDL (dB)	
-30.1	-53.4	-34.50	-100	One null recovered.
-31.3	-57.2	-34.2	-118.2	Second null recovered.
-31.5	-56	-34.7	-98.3	Third null recovered.

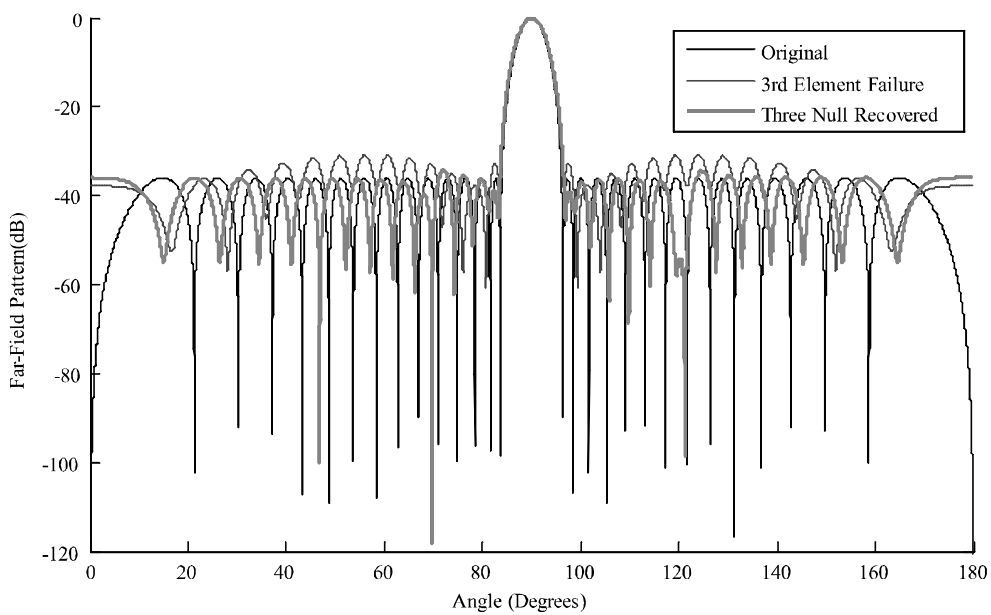


FIG. 13. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAILURE AND THREE NULL RECOVERED PATTERNS

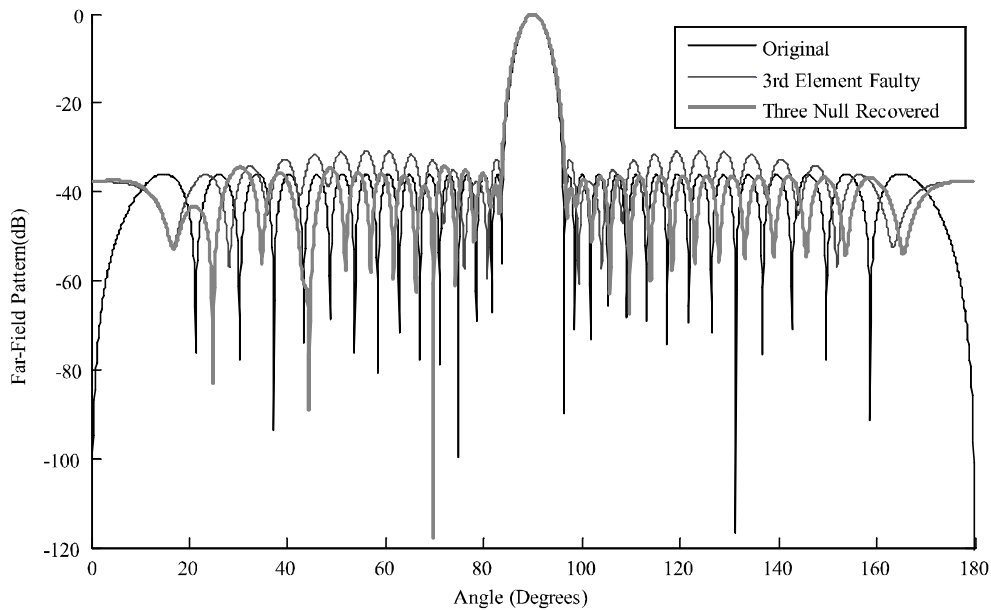


FIG.14. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAILURE AND NULL BROADENING PATTERN

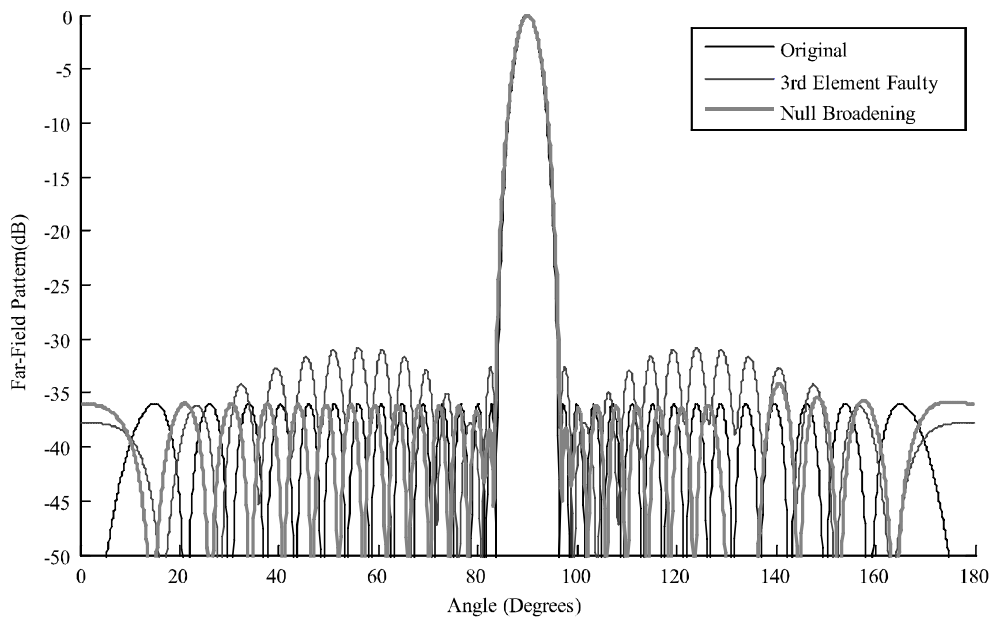


FIG.15. THE ORIGINAL DOLPH-CHEBYSHEV, THE 3RD ELEMENT FAULTY AND MAIN BEAM POINTING AT 125°

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