Validity of the Janssen Model for Layered Structures in a Silo

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RECEIVED ON 01.04.2010 ACCEPTED ON 04.10.2010

ABSTRACT

Granular materials are found every where despite they are poorly understood at microscopic level. The main hindrance is how to connect the microscopic properties with the macroscopic behavior and propose a rigorous unified theory. One method is to test the existing theoretical models in various configurations. In this connection we have performed experiments in different configurations of granules in a silo to determine the validity of the Janssen model under such arrangements. Two and four layered structures of different bead diameters are prepared. The effective mass at the bottom of the container in such cases have been measured. Moreover, the investigation of layered structures reveals that such configurations also follow well the Janssen model. An interesting percolation phenomenon was observed when smaller beads were stacked on larger ones, despite the model remained valid. Furthermore, it is demonstrated that Janssen law holds for larger bead diameters.

Key Words: **Granular Solids, Granular Systems, Static Sand Piles, Granular Compaction.**

1. INTRODUCTION

Granular materials exhibit many interesting and exceptional phenomenon that prevents them from simply perceiving among the known form of matter solid, liquid and gas [1]. Such a media has enormous industrial and geophysical applications and any advancement in understanding their properties would obviously lead to many economic benefits. There have many efforts to characterize them in fundamental terms, using kinematic models there have been some success in understanding the energetic granular gases. However, the mechanics of dense granular media still lacks such an understanding, because of the complexities found in the underlying microscopic systems [2]. For practical purposes there exist empiric engineering model which describes well the mean behavior of granular materials, Janssen model is one of them. Janssen model demonstrates that unlike hydrostatic the mass measured below a granular column does not increase indefinitely as the height increases instead after a critical height λ known as effective screening length, the top weight is screened by the side walls and the force felt at the base of container is only a fraction of total filled mass the rest is supported by side walls, this is attributed to shielding effect causing most of weight to side walls rather than to base of silo [3]. Despite of its simplicity it is still used in designing of silos [4]. The difficulty in testing experimentally lies in its strong dependency on filling procedure even the same filling mean exhibits different results [5-6]. Vanel and clement have

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devised a well controlled experimental set-up to measure accurately and reproducibly the effective mass as well as its fluctuation at the bottom of a granular column moving down with slow velocity they found their results in good agreement with the model, however in order to get better fitting of their data points they proposed two parameter model [7]. Furthermore Janssen model have also been checked in other experimental studies where the side walls were dragged upward to fully mobilize the packing these investigations have also confirmed the validity of Janssen law [8-10].

The aim of this paper is to look at the validity of the Janssen law in different configurations. Janssen has been checked for larger diameter of beads as well as for two and four horizontal layered structures. In this paper we verify that Janssen model is also valid not only for larger bead diameters 3, 4, 6, 7, 8 and 12mm but also for two and four horizontal layered structures.

2. EXPERIMENTAL SET-UP

Our experimental set-up is illustrated in Fig. 1. The grains of mass M are uniformly poured by funnel placed on top into the gap between the silos. A cylindrical piston forms the base of silo. The piston is carefully aligned so that during its motion it neither touches the walls of silo nor allows the leakage of grains. Before each experimental run piston is inserted inside a vertical cylinder to a certain height, after the grains are filled in cylinder a short relaxation time less than one minute is allowed so that elastic energy accumulated by weight sensor may be relaxed. Then the piston is allowed to descend at a slow velocity (0.02mm/s) on a total distance of 20mm. The aim of choosing a longer is to such that the slow downward motion of the piston could fully mobilize the friction force between the granules and the confining walls of silo. The bottom end of piston screwed onto the weight sensor and the force exerted by grains on the top horizontal surface of piston is measured. In such an arrangement the static pressure is entirely transmitted to a weight sensor, and a computer records data given by sensor throughout the process. The corresponding force exerted by beads on weight sensor is referred to as instantaneous effective mass M_t, since its value is different from the total filled mass M_f.

In this work six beads of different diameters are used. The grains are non cohesive, dry and mono-disperse glass beads with density 2.6 g/cm^3. The diameter of granules d used are 3, 4, 6, 7, 8 and 12mm, respectively, whereas silo having internal diameter D=43.6mm have been used. Fig. 2 displays the typical plots of instantaneous effective mass M_t as a function of filling mass M_f of 600g for different diameter of beads. It can be seen that the static pressure measured by the weight sensor exhibits comparatively larger fluctuations, in the beginning of descent of piston and eventually a relatively steady state is reached with smaller fluctuations. Here, we take the statistical average of 500 data points in the sequence of time series as depicted in the inset of Fig. 2, as a measuring value of effective mass M_e. To assure the reproducibility and also reduce the error, the experiment is done 5 times, and then the average value of them is taken as a data point M_e.

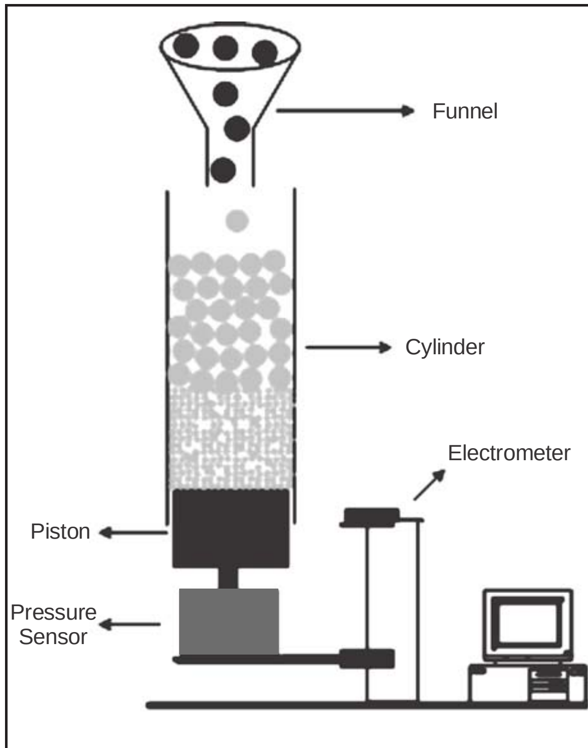

FIG. 1. SCHEMATIC DIAGRAM OF EXPERIMENTAL SET-UP MEASUREMENT PROCEDURE

3. RESULTS AND DISCUSSION

Firstly, we use same size of silo D=46.3mm and vary the bead sizes d=3, 4, 6, 7 and 12 mm. In such a configuration the variation of effective mass on the filling mass is depicted in Fig. 3. The scatters are experimental results, and the lines represent a fitting from all data with different diameters of granules. It can be observed that for low values of filling mass, M_e is approximately equal to M_f implying that in this region the hydrostatic behavior is more pronounced. For higher values of filling mass screening effect is dominant and M_e illustrates a saturated state and the state have some fluctuations. The trend of every stress saturation curves is almost the same however they branch off and increase with the increase in diameter of granular. Obviously, this phenomenon inspired us for its further investigation as the different bead sizes effects the stress transmission in the granular column.

In order to interpret the splitting of stress saturation curves, we use Janssen model. The model considers the granular medium in a silo as continuous and the friction force at wall is in coulomb yield criterion. Further it assumes that average stresses in horizontal direction are proportional to the vertical stresses.

Owing to screening effect the mean pressure at bottom is only a part of total filling mass known as effective mass the rest is screened out by the container walls. The relationship between the effective mass and the saturation mass is as:

$$M_a = M_s(1 - e^{-Mf/Ms}) \quad (1)$$

Where saturation mass is given by:

$$Ms = \rho\pi(D/2)^3/2\mu K \quad (2)$$

Here ρ represents the density of the material, D is the diameter of cylinder, μ is the coefficient of friction between the grains and the confining wall and λ is a central parameter in the theory known as characteristic length. The saturation curves in Fig. 3 are fitted using Equation (1). It is illustrated that Janssen model holds true for even larger diameter of beads. It is attributed to the explanation that with the increase in granular diameter the redirecting of vertical stresses is weakened and more stress reach at the bottom of cylinder, resulting in higher effective mass. Hence the stress saturation curve of 12mm is higher than the others.

Next we investigate the validity of Janssen model for layered structures. Therefore, in the first experimental run the two horizontal layered structures are prepared. In order to construct layered structure, the cylinder is filled with 2mm beads up to 450g at intervals of every 50g. Then 10.5mm beads are placed on following the same procedure. This configuration is represented by 2-10.5mm in Fig. 4. The reverse arrangement is represented by 10.5-2mm. Here also the first and last digit shows the bottom and top position of beads respectively. Both arrangements are depicted by the right sketch of Fig. 4. Both the curves are

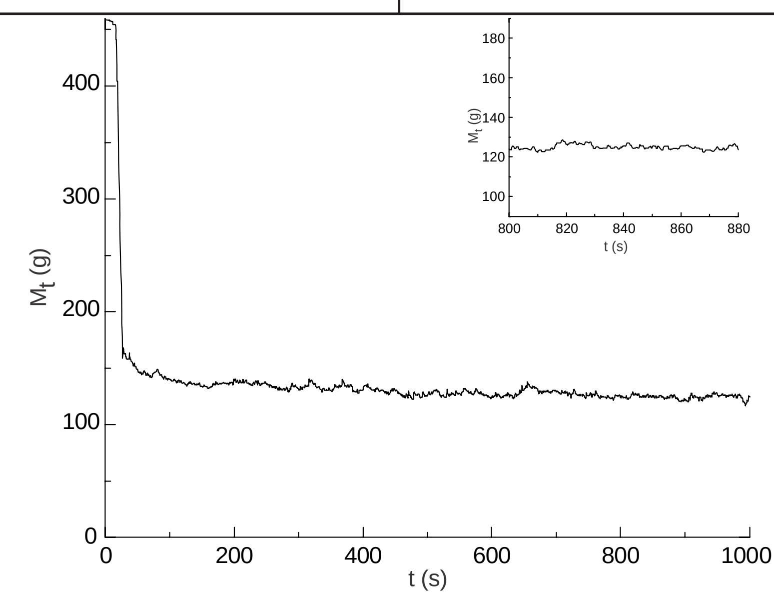

FIG. 2. TIME VARIATION OF THE INSTANTANEOUS EFFECTIVE MASS. THE FILLING MASS OF 600g FOR 4mm BEAD

fitted using Equation (1). It can be observed form the fig that for the case of 2-10.5mm the fits are in accordance with the Janssen prediction, however, for the reversal arrangement there is larger dispersion of data points.

It may be due to the reason that for the 2-10.5mm arrangement the presence of 2mm beads with the filling mass of 450g in the lower region of the silo has already attained a saturation state. Therefore, any increase of weight, even 10.5mm granules on its top does not vary the effective mass because the top weight is screened by the walls of the container in accordance with Janssen model. However, the presence of larger beads 10.5mm at the lower region of container, and pouring of 2mm granules on them results in percolation. Because the size of small beads is less than the gaps formed by the large beads, they can sieve through the gaps and most of small beads accumulate at the bottom of the container. It appears that due to the percolation there is sudden discontinuity in the stress saturation curve causing the deviation from Janssen curve.

In order to further demonstrate that it is the percolation phenomenon is responsible for dispersion of data points, we investigate the effect of percolation on the effective

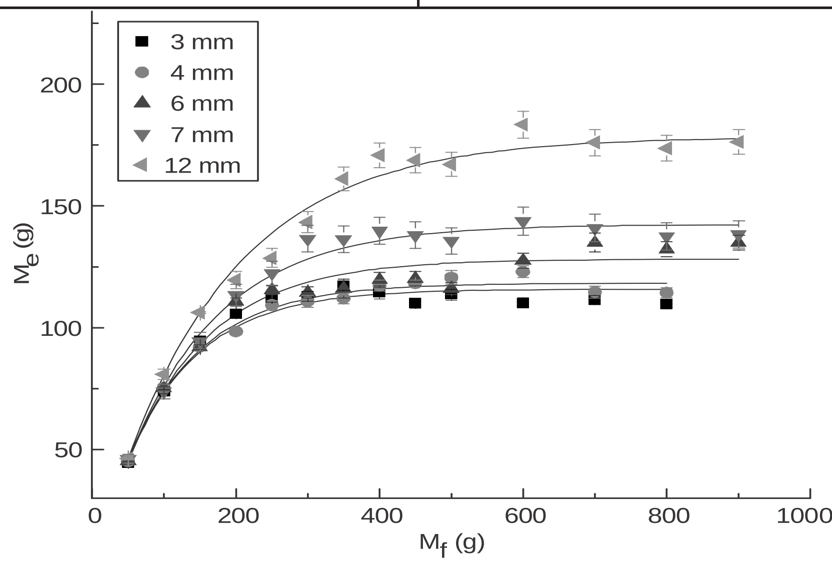

FIG. 3. EFFECTIVE MASS AS A FUNCTION OF FILLING MASS FOR VARIOUS GRAIN SIZES

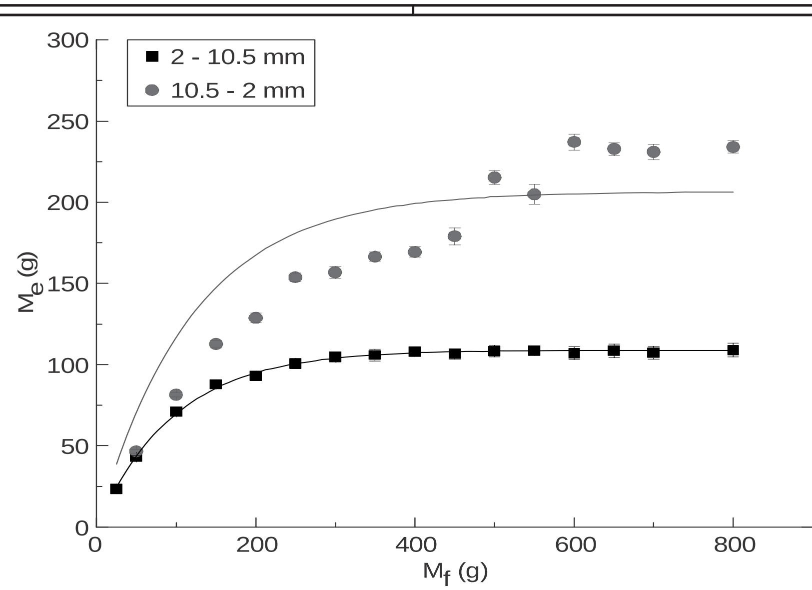

FIG. 4. EFFECTIVE MASS AS A FUNCTION OF FILLING MASS FOR TWO LAYERED CONFIGURATION

mass separately. In the investigation, each layer of granules has same mass. For instance, the filling mass of 50g means that firstly 25g of 10.5mm beads are poured in a silo, and then 2mm beads with equal mass are poured on it. Similarly for the filling mass of 100g, grains of 10.5 and 2mm each 50g are poured. In next trial a piece of paper with negligible mass is inserted at the interface of two layers to prevent the percolation in every case, while the construction process is same as mentioned above. In Fig. 5 the circles represent the occurrence of percolation and the squares indicate the absence of percolation. It is clearly illustrated in Fig. 5 that the effective mass with percolation displays higher value than that without percolation. The data lines represented in Fig. 4 represents the best fit using Equation (1). The difference of the effective mass in both the cases may be attributed to the reason that the lower region of the silo exhibits hydrostatic behavior and percolation of small beads causes more mass to be accumulated in that region. In such case the force chains among the grains are greatly perturbed due to percolation hence part of weight is not deflected towards silo wall but directly to the base. Consequently the M_e exhibits augmented effective mass in the case of percolation compared to the case without percolation. The Janssen also hold true in this arrangements of two horizontal layered structures.

Since in two layer structure each bottom layer is in saturation state, so now our aim is to investigate the validity of the law in the case where the bottom layer has not attained saturation state. This is achieved by stacking granules in four layers as depicted in Fig. 6.

The four alternate layered horizontal structure is formed in such a way that for instance, first we pour in cylinder in increment of 50gm up to 150gm of 4mm diameter glass beads, then we pour 150gm of 2mm diameter of glass beads also in increment of 50gm, forming second layer and this is repeated up to four layers in total mass of such column is 600gm. For the simplicity we can write such configuration as 4-2-4-2 where the first digit implies that beads having such diameter are placed at bottom. In the next experimental trial beads having diameter 4mm are replaced with 6, 8 and 10.5mm diameter beads respectively and same procedure is repeated, here also 2mm diameters glass bead layer retains its earlier position i.e. is immediately on top of the bottom layer. Such configuration can be represented as 4-2-4-2, 6-2-6-2, 8-2-8-2 and 10.-2-10.5-2 mm, respectively. The fitting of stress saturation curves in each case reveals that the Janssen classical theory remain valid in such an arrangement too.

4. CONCLUSION

Various experiments have been carried out to check the validity of Janssen model in different configurations. The effective mass as a function of filling mass have been measured in such cases. It is found that Janssen law fits well for the case of larger diameter of beads as well as in two and four horizontal layered structures. During these experiments an interesting phenomenon of percolation was observed and even in this case the law remained valid.

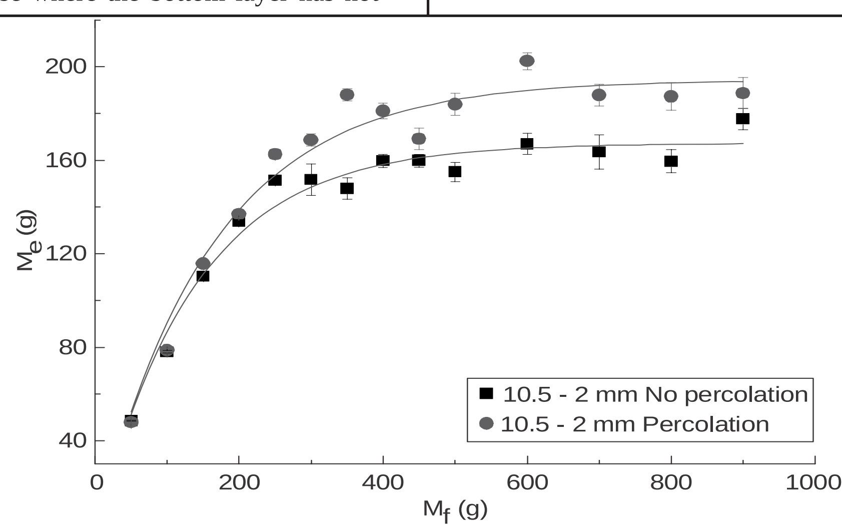

FIG. 5. EFFECTIVE MASS AS A FUNCTION OF FILLING MASS FOR THE CASE OF PERCOLATION AND NO PERCOLATION

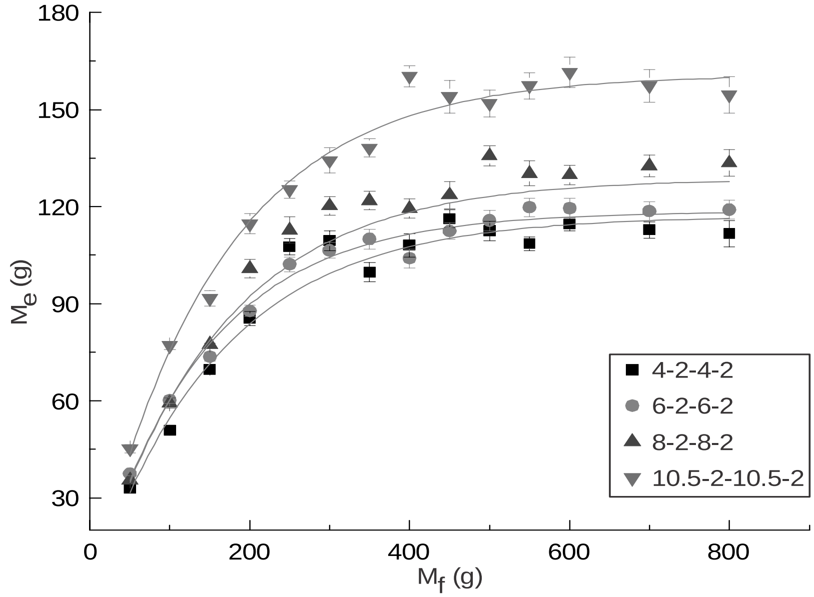

FIG. 6. EFFECTIVE MASS AS A FUNCTION OF FILLING MASS FOR ALTERNATE LAYERS

The result of these experiments suggests that force distributions in dense granular flows may be described by straightforward models over a much broader range of configurations than usually expected.

ACKNOWLEDGEMENT

This work is supported by the Chinese National Science Foundation, Project Nos. 10975014 and 10674157.

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