A Discrete Time Queueing Approach to Model and Evaluate Slotted Ring Network Buffer using Matrix Geometric Method

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ABSTRACT

Assorted analytical methods have been proposed for evaluating the performance of a slotted ring network. This paper proposes MGM (Matrix Geometric Method) to analyze the station buffer of a slotted ring for DT (Discrete-Time) queueing. The slotted ring is analyzed for infinite station buffer as a late arrival DT system. Utilizing the characteristics of 2-D Markov chain, various performance measures are validated with their corresponding results such as, throughput and MPAD (Mean Packet Access Delay) as well as the packet rejection probability for finite station buffer. The presented results prove efficacy of the method.

Key Words: Access Control, MGM, Slotted Ring Network, Throughput, MPAD.

1.   INTRODUCTION

Due to increasing trend of digitalization in the telecommunication networks, a DT analysis plays an important role for the evaluation of slotted ring networks. It is utmost advantageous to treat and analyze such networks on the basis of some digital assumption using advanced solutions other than those traditional techniques [1]. A ring network consists of a number of nodes joined by point-to-point links to form a closed loop as shown in Fig.1.

The way of transmitting or receiving a data from the stations of the same or other networks and the way of inserting into or removing from a particular station are the salient features governing a ring network as a communication network [2].

Bhuyan, L.N., et. al. [3] studied the performance models of ring network protocols and approximated the network with each node having infinite buffer and symmetric ring structure. Also in [4] analyzes single-buffer model of the slotted ring network by using simple approximation of large and small networks. In [5], author has analyzed high-speed slotted ring networks using a queueing model and evaluated the system against different performance measures.

The evaluation of these performance measures, including stationary state probabilities, requires solution of a set of

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higher order differential equations, which needs using various numerical methods. Therefore, for large systems, in which the states can be reached to the infinite state, the number of these equations also increases. In this paper, we use MGM modelling technique to evaluate the performance of the slotted ring network.

Rest of the paper is organized as follows: Sections 2, 3, and 4 discuss the slotted ring network, DT queueing, and MGM respectively. Section 5 presents the analytical model of the slotted ring network. Section 6 presents slotted ring networks with infinite station buffer, Section 7 presents the performance measures used to evaluate the slotted ring network with infinite buffer. The Sections 8 and 9 present the results and conclusion respectively.

2.   SLOTTED RING NETWORK

In slotted ring networks the ring comprises of number of fixed length slots continuously circulating around the ring. Each slot has a marking bit in its header, which indicates the status of the slot whether it is empty or full as shown in Fig. 2. Slotted ring networks provide high throughput [6-7]. The empty slot principle is the control strategy of the slotted ring. Initially all slots are marked empty. If a station on a ring wants to send a data then the data

processing unit of the station must breaks this data to fix it properly in the slot. After processing the data station must wait for the arrival of an empty slot and when an empty slot arrives, the station makes this empty slot as full and then it inserts the data in to the slot [4,8]. To remove the data from a slot (emptying the slot), to reuse the same slot by other stations of the network or even by the same station, either source removal or destination removal policy is used on reaching the data to its destination. To ensure the fairness among all network stations a suitable fairness policy is also implemented to avoid the hogging of the ring by one station. The ring latency of the slotted ring network determines the total number of bits in the circulated slots. In the design of a slotted ring, ring latency has an important influence.

3.   DISCRETE TIME QUEUEING SYSTEM

Queueing theory is used to analyze the performance of any communication network buffers in terms of queues. In a basic queueing system, it consists of a queue showing the capacity or number of waiting places to accommodate the data prior to its transmission, a data unit demanding service for processing and the unit known as the server to provide service to incoming data units in a predefined
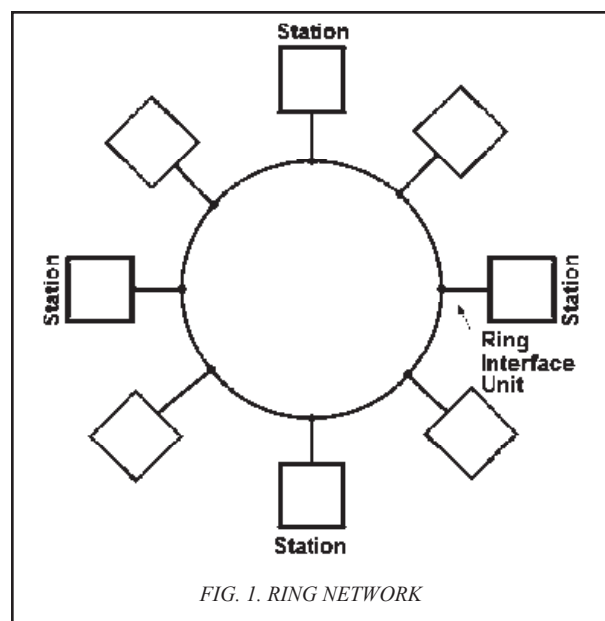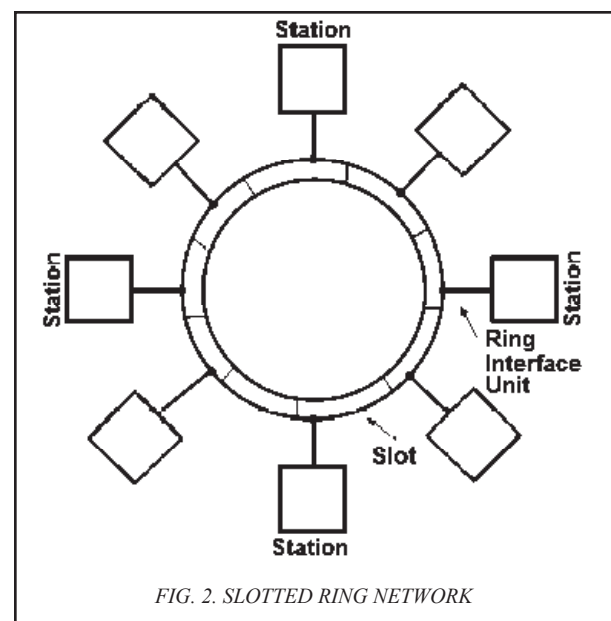

FIG. 1. RING NETWORK

FIG. 2. SLOTTED RING NETWORK

service discipline. In general, from sending station to receiving station, a communication system comprises of different stages which are inter-related to each other [8].

In broad sense each stage triggers its own event. For an example, in a packet based communication system, packets arrival at the receivers, waiting of these packets in the station buffers and processing them before they are allowed to transmit. Each of these stages represent a real queueing system. Out of these stages, buffering is one which receives a major attention as it stores the data on the network before transmitting it to other stations of network. In comparison with the continuous time queueing system, a digital system is needed to be treated with some specialized techniques in order to understand and obtain its exact behaviour.

DT queueing is one of the approaches used to obtain the performance measures of such systems [4]. Operation of digital systems is based on time slotting and it finds many applications particularly in communication networks such as packet communication or digital communication systems, satellite communication and in computer networks, fixed size data units (bits, bytes, fixed length packets) on a communication channel widely known as slotted ring networks. Due to time slot characteristics, DT queueing system has a potential applications and it provides flexibility to model a fixed length packet system.

DT queueing is based on the assumption that the time is divided into fixed length of intervals (equidistant)

called slots as shown in Fig. 3. In these systems, both arriving packets and departing packets are geometrically distributed, as shown in Fig. 4. A DT system allows multiple events in a single time unit called a slot. The common causes of the change that takes place in any queueing system are the events when a packet arrives and/or departs from the system during a given slot. An exact system can only be modelled according to the occurrence of an arriving packet in a given slot at some observing point.

Mainly, the choice of the point from where a system can be observed is a slot edge or boundary as shown in Fig. 3. In these systems, starting edges of slots are called arrival epochs, where any arriving data begin to enter in the system, whereas, the data can only departs from the system at the ending edges of slots, called departure epochs. Hence, depending on the time of arrival of a packet, DT queueing systems can be modelled in any of the two ways, Early Arrival System or Late Arrival System respectively.

There is a difference in analyzing and a way for obtaining the performance metrics between these two systems, depending on the assumptions made, whether a packet arrives before service completion epoch or just after the service completion epoch of a given slot, that is, a DT queueing system can be solved and modelled by approaching it either through an early arrival modelling approach or by a late arrival system modelling approach. This paper is concerned only with the late arrival system.
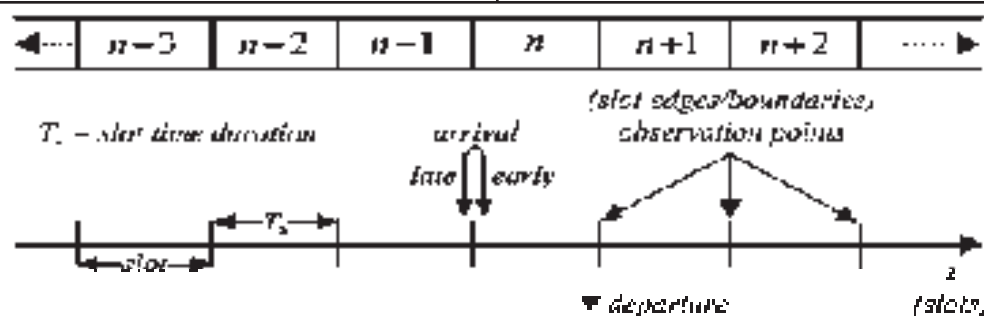
3.1 Late Arrival System

FIG. 3. SLOTTED TIME WITH LATE AND EARLY ARRIVAL EPOCH

The late arrival system assumes a packet arrives just before the end of slot's boundary, that is, just after the departure instance (departure epoch). Fig. 4 depicts the late arrival system.

Fig. 5 shows the state diagram of a system with infinite buffers. When a packet arrives and finds no departure or service completion, queue length increases for all consecutive slots, except an idle (empty) system state.

For an idle state, queue length increases whenever an arrival occurs, otherwise system remains in that state. The system remains in that state with probabilities of either there is departure and an arrival event takes place in a given slot or there is no any arrival and departure event occurred in a given slot. The system reverts back to its previous state (queue length decreases) only in the event when there arrives no packet and a packet departs from the buffer. The system remains in the same state in case of full buffer with no packet departure.

4. MATRIX GEOMETRIC METHOD

It is an analytical method based on the matrices that utilizes the special characteristics of Markov process to determine the steady state probabilities of a process in the vector form. These matrices describe different Markov processes in the form of vector state [9].

The stationary state probability of the vector state Markov chain is solved by exploiting the repetitive structure of the Markov process. In this method Markov chain is divided into two parts, boundary set and a repetitive set. States of the underlying chain are ordered lexicographically [10]. Elements of the Q matrix are grouped into sub-matrices which describe the boundary and repetitive levels of the chain.

The sub-matrices of boundary and repetitive levels are denoted by the B_i and A_i, respectively. To solve the stationary probabilities we must compute the rate matrix R with the help of sub-matrices of repetitive level by
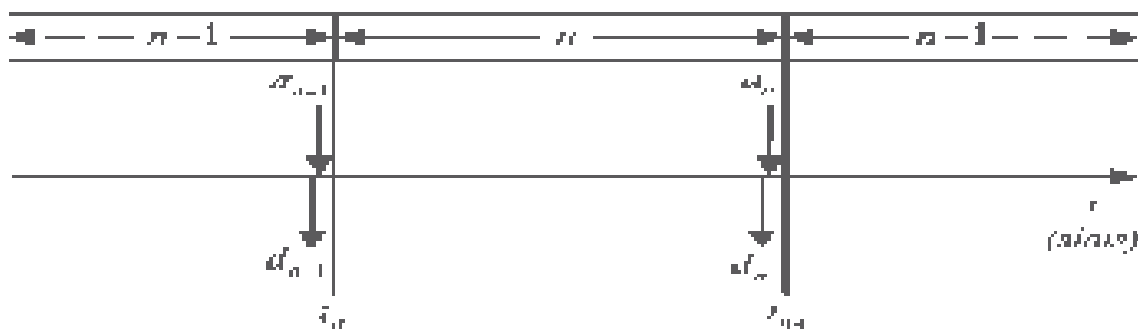

FIG. 4. LATE ARRIVAL SYSTEM: TIME AXIS
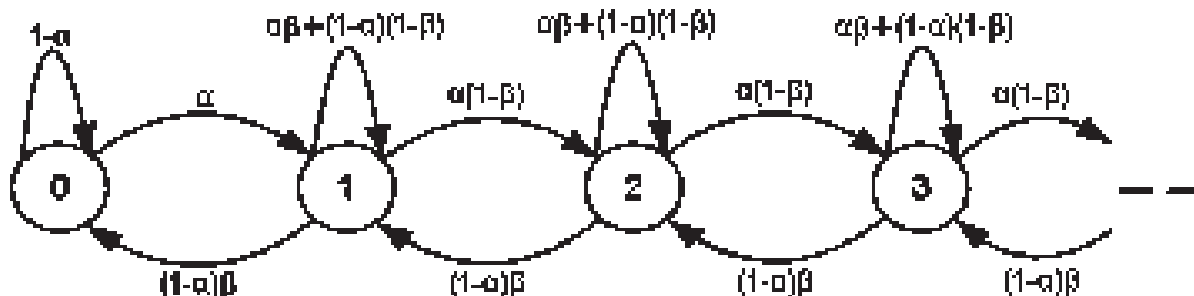FIG. 5. LATE ARRIVAL SYSTEM: INFINITE MARKOV CHAIN MODEL

FIG. 6. INFINITE STATION BUFFER MARKOV CHAIN MODEL
FIG. 7. THROUGHPUT AGAINST PACKET ARRIVAL PROBABILITY, WHEN N=8,14 AND 20

following these steps:

(a)   Queueing model

(b)   State transition diagram

(c)   Transition matrix in lexicographical order

(d)   Obtaining sub-matrices

(e)   Block form transition matrix

(f)   Equations for boundary and repetition processes

(g)   Computation of rate matrix (R)

(h)   Computation of initial steady state probability vector using R

Depending on whether the system is finite or infinite, an algorithm is chosen for computing the rate matrix 'R' [11-15]. Logarithmic reduction algorithm and classical iteration method are commonly used for infinite systems, whereas, cyclic reduction and folding algorithm are used for finite systems [16-17].

5.   NETWORK MODEL

A slotted ring network model is shown in Fig. 2. Following assumptions were made in analysing the network:

(a)   Ring uses more than one station

(b)   All stations are equidistant

(c)   Probability of an arrival is given by α

(d)   Probability of departure is given by β

(e)   There are N number of slots on the ring

(f)   Packet size is equal to the size of the slot

(g)   Arrival of packets follows Bernoulli process

(h)   Time is measured in slots

(i)   Isolation method is used in solving the model

(j)   No monitoring station is considered in the ring

6.   INFINITE BUFFER

We analyze the DT queueing with infinite capacity station buffer in the case of late packet arrival modelling technique. For simplicity reasons, we use isolation method in which the behaviour of single station superimposed on other stations of the ring. Fig. 6 shows the Markov chain model of the infinite station buffer.

The system state space of the Markov chain is represented by the order pair (i,j), where i represents the number of
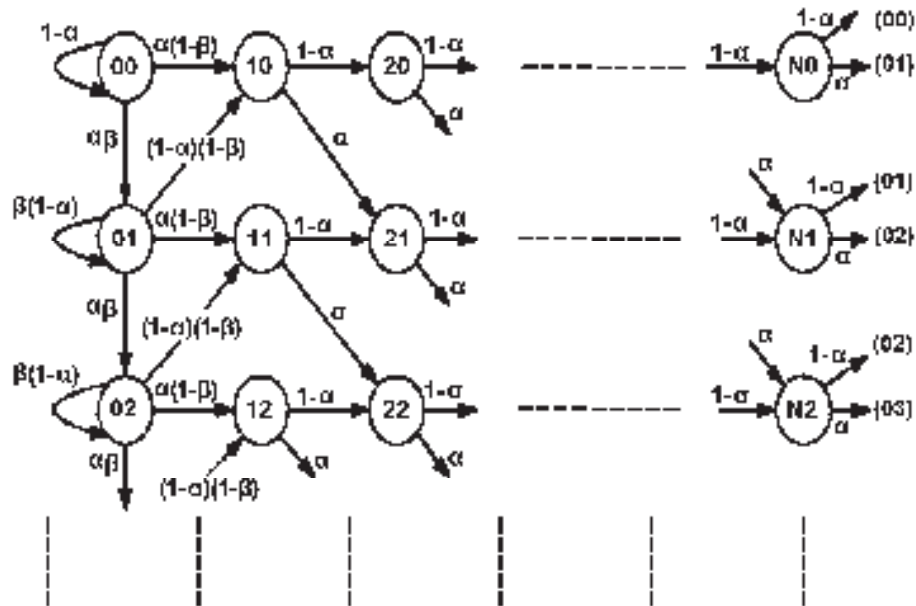

FIG. 8. MPAD AGAINST THROUGHPUT,
WHEN N=8, 14 AND 20

ring slots and j represents the buffer capacity. We use to express the probability of an arrival or no arrival during one time slot as α and 1-α respectively, whereas the probability of a departure or no departure in a time slot is given by β and 1-β, respectively. The Markov model has the same structure as the finite buffer Markov chain except the buffer size is infinite. After constructing the Markov chain, the transition matrix of the model is developed by arranging the system states in a lexicographical order as shown in Equation (1).

$$
Q = \begin{bmatrix}
f & a & \cdots & 0 & b & 0 & \cdots & 0 & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & f & 0 & 0 & \cdots & g & \cdots \\
f & 0 & \cdots & 0 & g & 0 & \cdots & 0 & \cdots \\
0 & c & \cdots & 0 & a & d & \cdots & 0 & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & f & \cdots \\
0 & 0 & \cdots & 0 & f & 0 & \cdots & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{bmatrix} \tag{1}
$$

Where a=α(1-β), b=αβ, c=(1-α)(1-β), d=β(1-α), e=β, f=(1-α), g=α. This transition matrix represents all the system transitions. By analyzing this matrix, one obtains the set of sub-matrices as shown by Equation (2). The basic rate matrix is the main requirement for obtaining the steady state probability vector of the system, and is computed from these sub-matrices.

$$
B_0 = \begin{bmatrix} f & a & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & f \\ f & 0 & \cdots & 0 \end{bmatrix}
A_0 = \begin{bmatrix} 0 & c & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}
$$
$$
A_1 = \begin{bmatrix} a & d & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & f \\ f & 0 & \cdots & 0 \end{bmatrix}
A_2 = \begin{bmatrix} b & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & g \\ g & 0 & \cdots & 0 \end{bmatrix} \tag{2}
$$

Where B_0, A_0, A_1 and A_2 represent state transitions of the initial boundary, forward state transitions of the repeating portion, local state transitions of the repeating portion and backward state transitions of the repeating portion of the Markov chain, respectively.

7. PERFORMANCE MEASURES

For the analysis of any queueing system, we require to obtain its performance measures under certain condition and assumptions. In this paper, we obtained different performance measures of a discrete time queueing system, such as, throughput, mean number, mean packet access delay.

To evaluate the performance of the slotted ring network with infinite buffer capacity, MPAD is measured in terms of the 's' and can be computed through the following formulas. The steady state probability that a station is in transmit mode is given by:

$$
P_T = \sum_{j=0}^{\infty} \sum_{i=1}^{N} \pi_j \tag{3}
$$

Where N is the number of slots and π_{ij} is the probability of the transition from state i to state j. Equation (4) gives the mean number of occupied slots.

$$
E(s) = (N-1)\, P_T \tag{4}
$$

7.1 Throughput

The rate of the number of successful service completions per slot is defined as throughput of the system in discrete-time domain. It is an amount of useful information carried per unit time by a network component, such as, a link, buffer or a switch. In steady state, since the average input rate and the average output rate are equal for a given network, throughput is the average number of customers, measured in slots, either entering or leaving the network.

In general, throughput is the number of customers that receive service and successfully departs from the system in a given time. It is important to note that the throughput of the system is only considered for calculation by multiplying the probability of service to those states of the system in which any customer is subject to receive service. It is denoted by 's' and is a dimensionless quantity. Equation (5) gives the probability of a slot being occupied.

$$s = \frac{(N-1)P_T}{N}$$

(5)

7.2 Mean Buffer Length

The birth and death process can be used as a fundamental model for the study of any telecommunication and queueing system. By choosing the corresponding birth and death probabilities, the system state probability distribution for a discrete-time queueing system is determined under steady-state conditions. As each state of a system represents a specific number of customers in a given system, therefore, the mean number in the system can be obtained by weighting those amounts with the system state probabilities of the corresponding states and summing up the results.

Mean number of data packets in the buffer or mean buffer length of a network station for the infinite buffer is described by Equations (6).

$$E(Q) = \sum_{j=1}^{\infty} \sum_{i=0}^{N} \pi_j$$

(6)

7.3 Mean Packet Access Delay

Equation (7) describes the MPAD in terms of the mean number of data packets in the buffer of a network station to the throughput of the system.

$$MPAD - \frac{E(Q)}{s}$$

(7)

8. RESULTS AND DISCUSSION

An analytical program of MGM is written in visual C++ language which utilizes all features of the method under discussion. It computes matrix R for infinite station buffer model and the boundary equation is used to evaluate the initial probability vector.

Figs. 7-8 show the throughput and MPAD for station buffer having infinite capacity. Fig. 7 shows that the throughput increases linearly with the increasing number of slots in the ring for varying number of slots, typically the number of slots 'N' are taken as 8, 14 and 20.

Fig. 8 shows that after nearly achieving 70-80% throughput, MPAD increases swiftly and the throughput does not go beyond 90% for varying number of slots.

9. CONCLUSIONS
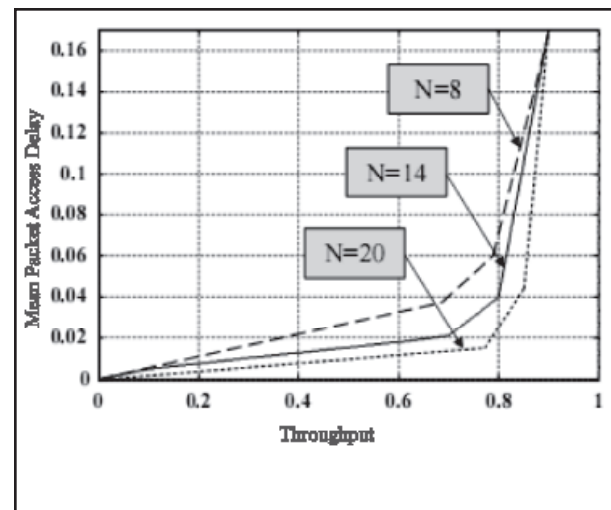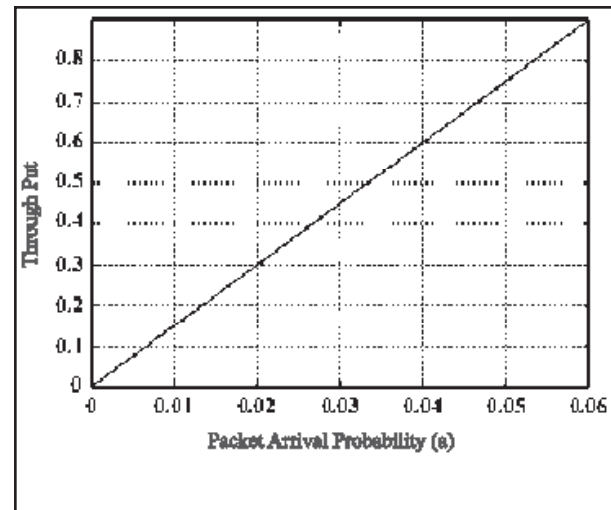
In this paper, we used MGM to evaluate the performance of a slotted ring network. Since MGM utilizes the special characteristics of the Markov chain, it is efficient in estimating the system performance parameters.

The DT queueing analysis is used in contrast with MGM technique to evaluate infinite buffer station of slotted ring networks. Various performance parameters for infinite buffer station are measured and their results are presented. In terms of performance analysis of the networks which operates on some discrete assumptions, the DT Queueing with MGM technique can be used more effectively and quickly in solving the large networks that complex and requires solution of a set of higher order differential equations and needs enormous numerical methods.

ACKNOWLEDGEMENTS

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