

# Generalized Inequalities Related to the Classical Euler's Gamma Function

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**Abstract** This paper presents some inequalities concerning certain ratios of the classical Euler's Gamma function. The results generalized some recent results.

**Keywords:** Gamma function, q-Gamma function, k-Gamma function, (p,q)-Gamma function, (q,k)-Gamma function, inequality

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## 1. Introduction

We begin by outlining the following basic definitions well-known in literature.

The celebrated classical Euler's Gamma function,  $\Gamma(t)$  is defined for  $t > 0$  as

$$\Gamma(t) = \int_0^{\infty} e^{-x} x^{t-1} dx.$$

The q-Gamma function,  $\Gamma_q(t)$  is defined for  $q \in (0,1)$  and  $t > 0$  as (see [2])

$$\Gamma_q(t) = (1-q)^{1-t} \prod_{n=1}^{\infty} \frac{1-q^n}{1-q^{t+n}}.$$

Also, the k-Gamma function,  $\Gamma_k(t)$  was defined by Diaz and Pariguan [1] for  $k > 0$  and  $t > 0$  as

$$\Gamma_k(t) = \int_0^{\infty} e^{-\frac{x}{k}} x^{t-1} dx.$$

Diaz and Teruel [5] further defined the (q,k)-Gamma function  $\Gamma_{(q,k)}(t)$  for  $q \in (0,1)$ ,  $k > 0$  and  $t > 0$  as

$$\Gamma_{(q,k)}(t) = \frac{(1-q^k)_{q,k}^{t-1}}{(1-q)^{\frac{t-1}{k}}},$$

where

$$\begin{aligned} (t)_{n,k} &= t(t+k)(t+2k)\cdots(t+(n-1)k) \\ &= \prod_{j=0}^{n-1} (t+jk) \end{aligned}$$

is the k-generalized Pochhammer symbol.

Furthermore, Krasniqi and Merovci [4] defined the (p,q)-Gamma function  $\Gamma_{(p,q)}(t)$  for  $p \in \mathbb{N}$ ,  $q \in (0,1)$  and  $t > 0$  as

$$\Gamma_{(p,q)}(t) = \frac{[p]_q^t [p]_q!}{[t]_q [t+1]_q \cdots [t+p]_q},$$

where

$$[p]_q = \frac{1-q^p}{1-q}.$$

The psi function,  $\psi(t)$  otherwise known as the digamma function is defined as the logarithmic derivative of the Gamma function. That is,

$$\psi(t) = \frac{d}{dt} \ln \Gamma(t) = \frac{\Gamma'(t)}{\Gamma(t)}.$$

The q-digamma function, k-digamma function, (p,q)-digamma function and (q,k)-digamma function are similarly defined as follows:

$$\psi_q(t) = \frac{d}{dt} \ln \Gamma_q(t) = \frac{\Gamma'_q(t)}{\Gamma_q(t)},$$

$$\psi_k(t) = \frac{d}{dt} \ln \Gamma_k(t) = \frac{\Gamma'_k(t)}{\Gamma_k(t)},$$

$$\psi_{(p,q)}(t) = \frac{d}{dt} \ln \Gamma_{(p,q)}(t) = \frac{\Gamma'_{(p,q)}(t)}{\Gamma_{(p,q)}(t)}$$

and

$$\psi_{(q,k)}(t) = \frac{d}{dt} \ln \Gamma_{(q,k)}(t) = \frac{\Gamma'_{(q,k)}(t)}{\Gamma_{(q,k)}(t)}.$$

It is common knowledge that these functions exhibit the following series characterizations (see also [7-12]):

$$\psi_q(t) = -\ln(1-q) + (\ln q) \sum_{n=1}^{\infty} \frac{q^{nt}}{1-q^n} \tag{1}$$

$$\psi_k(t) = \frac{\ln k - \gamma}{k} - \frac{1}{t} + \sum_{n=1}^{\infty} \frac{t}{nk(nk+t)} \tag{2}$$

$$\psi_{(p,q)}(t) = \ln[p]_q + (\ln q) \sum_{n=1}^p \frac{q^{nt}}{1-q^n} \tag{3}$$

$$\psi_{(q,k)}(t) = \frac{-\ln(1-q)}{k} + (\ln q) \sum_{n=1}^{\infty} \frac{q^{nkt}}{1-q^{nk}} \tag{4}$$

where  $\gamma = 0.577215664901532\dots$  represents the Euler-Mascheroni's constant.

Of late, the following double inequalities were presented in [7] by the use of some monotonicity properties of some functions related with the Gamma function.

$$\begin{aligned} \frac{(1-q)^{-t} \Gamma_q(\alpha)}{[p]_q^t \Gamma_{(p,q)}(\alpha)} &\geq \frac{\Gamma_q(\alpha+t)}{\Gamma_{(p,q)}(\alpha+t)} \\ &\geq \frac{(1-q)^{1-t} \Gamma_q(\alpha+1)}{[p]_q^{t-1} \Gamma_{(p,q)}(\alpha+1)} \end{aligned} \tag{5}$$

for  $t \in (0,1)$ ,  $\alpha > 0$ ,  $p \in N$  and  $q \in (0,1)$ .

$$\begin{aligned} \frac{(1-q)^{-t} \Gamma_q(\alpha)}{(1-q)^{-\frac{t}{k}} \Gamma_{(q,k)}(\alpha)} &\geq \frac{\Gamma_q(\alpha+t)}{\Gamma_{(q,k)}(\alpha+t)} \\ &\geq \frac{(1-q)^{1-t} \Gamma_q(\alpha+1)}{(1-q)^{\frac{1}{k}(1-t)} \Gamma_{(q,k)}(\alpha+1)} \end{aligned} \tag{6}$$

for  $t \in (0,1)$ ,  $\alpha > 0$ ,  $q \in (0,1)$  and  $k \geq 1$ .

$$\begin{aligned} \frac{\alpha k^{\frac{t}{k}} e^{-\frac{\gamma t}{k}} \Gamma_k(\alpha)}{(\alpha+t)[p]_q^t \Gamma_{(p,q)}(\alpha)} &< \frac{\Gamma_k(\alpha+t)}{\Gamma_{(p,q)}(\alpha+t)} \\ &< \frac{(\alpha+1)k^{\frac{t-1}{k}} e^{-\frac{\gamma(1-t)}{k}} \Gamma_k(\alpha+1)}{(\alpha+t)[p]_q^{t-1} \Gamma_{(p,q)}(\alpha+1)} \end{aligned} \tag{7}$$

for  $t \in (0,1)$ ,  $\alpha > 0$ ,  $p \in N$ ,  $q \in (0,1)$  and  $k > 0$ .

$$\begin{aligned} \frac{\alpha k^{\frac{t}{k}} e^{-\frac{\gamma t}{k}} \Gamma_k(\alpha)}{(\alpha+t)(1-q)^{-\frac{t}{k}} \Gamma_{(q,k)}(\alpha)} &< \frac{\Gamma_k(\alpha+t)}{\Gamma_{(q,k)}(\alpha+t)} \\ &< \frac{\alpha k^{\frac{t}{k}} e^{-\frac{\gamma t}{k}} \Gamma_k(\alpha+1)}{(\alpha+t)(1-q)^{\frac{1-t}{k}} \Gamma_{(q,k)}(\alpha+1)} \end{aligned} \tag{8}$$

for  $t \in (0,1)$ ,  $\alpha > 0$ ,  $q \in (0,1)$  and  $k > 0$ .

Results of this form can also be found in [8,9,10,11,12]. By utilizing similar techniques as in the previous results, this paper seeks to provide some generalizations of the above inequalities. We present our results in the following sections.

## 2. Supporting Results

We begin with the following Lemmas.

**Lemma 2.1.** Suppose that  $\alpha > 0$ ,  $\beta > 0$ ,  $u \geq w > 0$ ,  $t > 0$ ,  $p \in N$  and  $q \in (0,1)$ . Then,

$$\begin{aligned} &u \ln(1-q) + w \ln[p]_q \\ &+ u \psi_q(\alpha + \beta t) - w \psi_{(p,q)}(\alpha + \beta t) \leq 0. \end{aligned}$$

**Proof.** From the characterization in equations (1) and (3) we obtain,

$$\begin{aligned} &u \ln(1-q) + w \ln[p]_q + u \psi_q(t) - w \psi_{(p,q)}(t) \\ &= (\ln q) \left[ u \sum_{n=1}^{\infty} \frac{q^{nt}}{1-q^n} - w \sum_{n=1}^p \frac{q^{nt}}{1-q^n} \right] \leq 0. \end{aligned}$$

We conclude the proof by substituting  $t$  by  $\alpha + \beta t$ .

**Lemma 2.2.** Suppose that  $\alpha > 0$ ,  $\beta > 0$ ,  $u \geq w > 0$ ,  $t > 0$ ,  $q \in (0,1)$  and  $k \geq 1$ . Then,

$$\begin{aligned} &u \ln(1-q) - w \frac{\ln(1-q)}{k} \\ &+ u \psi_q(\alpha + \beta t) - w \psi_{(q,k)}(\alpha + \beta t) \leq 0. \end{aligned}$$

**Proof.** From the characterization in equations (1) and (4) we obtain,

$$\begin{aligned} &u \ln(1-q) - w \frac{\ln(1-q)}{k} + u \psi_q(t) - w \psi_{(q,k)}(t) \\ &= (\ln q) \sum_{n=1}^{\infty} \left[ u \frac{q^{nt}}{1-q^n} - w \frac{q^{nkt}}{1-q^{nk}} \right] \leq 0. \end{aligned}$$

We conclude the proof by substituting  $t$  by  $\alpha + \beta t$ .

**Lemma 2.3.** Suppose that  $\alpha > 0$ ,  $\beta > 0$ ,  $u > 0$ ,  $w > 0$ ,  $t > 0$ ,  $k > 0$ ,  $p \in N$  and  $q \in (0,1)$ . Then,

$$\begin{aligned} &w \ln[p]_q - u \frac{\ln k}{k} + \frac{u \gamma}{k} + \frac{u}{\alpha + \beta t} \\ &+ u \psi_k(\alpha + \beta t) - w \psi_{(p,q)}(\alpha + \beta t) > 0. \end{aligned}$$

**Proof.** From the characterization in equations (2) and (3) we obtain,

$$\begin{aligned} &w \ln[p]_q - u \frac{\ln k}{k} + \frac{u \gamma}{k} + \frac{u}{t} + u \psi_q(t) - w \psi_{(p,q)}(t) \\ &= u \sum_{n=1}^{\infty} \frac{t}{nk(nk+t)} - w \sum_{n=1}^p \frac{q^{nt}}{1-q^n} > 0. \end{aligned}$$

We conclude the proof by substituting  $t$  by  $\alpha + \beta t$ .

**Lemma 2.4.** Suppose that  $\alpha > 0$ ,  $\beta > 0$ ,  $u > 0$ ,  $w > 0$ ,  $t > 0$ ,  $q \in (0,1)$  and  $k > 0$ . Then,

$$\begin{aligned} &-\frac{\ln(k^u (1-q)^w)}{k} + \frac{u \gamma}{k} + \frac{u}{\alpha + \beta t} \\ &+ u \psi_k(\alpha + \beta t) - w \psi_{(q,k)}(\alpha + \beta t) > 0. \end{aligned}$$

**Proof.** From the characterization in equations (2) and (4) we obtain,

$$\begin{aligned}
 & -\frac{\ln(k^u(1-q)^w)}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha + \beta t} + u\psi_q(t) - w\psi_{(q,k)}(t) \\
 & = u \sum_{n=1}^{\infty} \frac{t}{nk(nk+t)} - w(\ln q) \sum_{n=1}^{\infty} \frac{q^{nkt}}{1-q^{nk}} > 0.
 \end{aligned}$$

We conclude the proof by substituting  $t$  by  $\alpha + \beta t$ .

### 3. Main Results

We now present our results in the following Theorems.

**Theorem 3.1.** Define a function  $E$  for  $p \in N$  and  $q \in (0,1)$  by

$$E(t) = \frac{(1-q)^{u\beta t} \Gamma_q(\alpha + \beta t)^u}{[p]_q^{-w\beta t} \Gamma_{(p,q)}(\alpha + \beta t)^w}, t \in (0, \infty) \tag{9}$$

where  $u, w, \alpha, \beta$  are positive real numbers such that  $u \geq w$ . Then,  $E$  is non-increasing on  $t \in (0, \infty)$  and the inequalities:

$$\begin{aligned}
 & \frac{(1-q)^{-u\beta t} \Gamma_q(\alpha)^u}{[p]_q^{w\beta t} \Gamma_{(p,q)}(\alpha)^w} \geq \frac{\Gamma_q(\alpha + \beta t)^u}{\Gamma_{(p,q)}(\alpha + \beta t)^w} \\
 & \geq \frac{(1-q)^{u\beta(1-t)} \Gamma_q(\alpha + \beta)^u}{[p]_q^{w\beta(1-t)} \Gamma_{(p,q)}(\alpha + \beta)^w}
 \end{aligned} \tag{10}$$

are valid for each  $t \in (0,1)$ .

**Proof.** Let  $\lambda(t) = \ln E(t)$  for every  $t \in (0, \infty)$ . Then

$$\begin{aligned}
 \lambda(t) &= \ln \frac{(1-q)^{u\beta t} \Gamma_q(\alpha + \beta t)^u}{[p]_q^{-w\beta t} \Gamma_{(p,q)}(\alpha + \beta t)^w} \\
 &= u\beta t \ln(1-q) + w\beta t \ln[p]_q \\
 &\quad + u \ln \Gamma_q(\alpha + \beta t) - w \ln \Gamma_{(p,q)}(\alpha + \beta t)
 \end{aligned}$$

Then,

$$\begin{aligned}
 \lambda'(t) &= u\beta \ln(1-q) + w\beta \ln[p]_q \\
 &\quad + u\beta \psi_q(\alpha + \beta t) - w\beta \psi_{(p,q)}(\alpha + \beta t) \\
 &= \beta \left[ u \ln(1-q) + w \ln[p]_q \right. \\
 &\quad \left. + u\psi_q(\alpha + \beta t) - w\psi_{(p,q)}(\alpha + \beta t) \right] \leq 0
 \end{aligned}$$

as a result of Lemma 2.1. That implies  $\lambda$  is non-increasing on  $t \in (0, \infty)$ . Consequently,  $E$  is non-increasing on  $t \in (0, \infty)$  and for each  $t \in (0,1)$  we have,

$$E(0) \geq E(t) \geq E(1)$$

yielding equation (10).

**Theorem 3.2.** Define a function  $F$  for  $q \in (0,1)$  and  $k \geq 1$  by

$$F(t) = \frac{(1-q)^{u\beta t} \Gamma_q(\alpha + \beta t)^u}{(1-q)^{\frac{w\beta t}{k}} \Gamma_{(q,k)}(\alpha + \beta t)^w}, t \in (0, \infty) \tag{11}$$

where  $u, w, \alpha, \beta$  are positive real numbers such that  $u \geq w$ . Then,  $F$  is non-increasing on  $t \in (0, \infty)$  and the inequalities:

$$\begin{aligned}
 & \frac{(1-q)^{-u\beta t} \Gamma_q(\alpha)^u}{(1-q)^{\frac{w\beta t}{k}} \Gamma_{(q,k)}(\alpha)^w} \geq \frac{\Gamma_q(\alpha + \beta t)^u}{\Gamma_{(q,k)}(\alpha + \beta t)^w} \\
 & \geq \frac{(1-q)^{u\beta(1-t)} \Gamma_q(\alpha + \beta)^u}{(1-q)^{\frac{w\beta}{k}(1-t)} \Gamma_{(q,k)}(\alpha + \beta)^w}
 \end{aligned} \tag{12}$$

are valid for each  $t \in (0,1)$ .

**Proof.** Let  $\eta(t) = \ln F(t)$  for every  $t \in (0, \infty)$ . Then

$$\begin{aligned}
 \eta(t) &= \ln \frac{(1-q)^{u\beta t} \Gamma_q(\alpha + \beta t)^u}{(1-q)^{\frac{w\beta t}{k}} \Gamma_{(q,k)}(\alpha + \beta t)^w} \\
 &= u\beta t \ln(1-q) - \frac{w\beta t}{k} \ln(1-q) \\
 &\quad + u \ln \Gamma_q(\alpha + \beta t) - w \ln \Gamma_{(q,k)}(\alpha + \beta t)
 \end{aligned}$$

Then,

$$\begin{aligned}
 \eta'(t) &= u\beta \ln(1-q) - \frac{w\beta}{k} \ln(1-q) \\
 &\quad + u\beta \psi_q(\alpha + \beta t) - w\beta \psi_{(q,k)}(\alpha + \beta t) \\
 &= \beta \left[ u \ln(1-q) - w \frac{\ln(1-q)}{k} \right. \\
 &\quad \left. + u\psi_q(\alpha + \beta t) - w\psi_{(q,k)}(\alpha + \beta t) \right] \leq 0
 \end{aligned}$$

as a result of Lemma 2.2. That implies  $\eta$  is non-increasing on  $t \in (0, \infty)$ . Consequently,  $F$  is non-increasing on  $t \in (0, \infty)$  and for each  $t \in (0,1)$  we have,

$$F(0) \geq F(t) \geq F(1)$$

yielding equation (12).

**Theorem 3.3.** Define a function  $G$  for  $t \in (0, \infty)$ ,  $p \in N$ ,  $q \in (0,1)$  and  $k > 0$  by

$$G(t) = \frac{(\alpha + \beta t)^u k^{\frac{u\beta t}{k}} e^{\frac{u\beta \gamma t}{k}} \Gamma_k(\alpha + \beta t)^u}{[p]_q^{-w\beta t} \Gamma_{(p,q)}(\alpha + \beta t)^w} \tag{13}$$

where  $u, w, \alpha, \beta$  are positive real numbers. Then,  $G$  is increasing on  $t \in (0, \infty)$  and the inequalities:

$$\begin{aligned}
 & \frac{\alpha^u k^{\frac{u\beta t}{k}} e^{\frac{u\beta \gamma t}{k}} \Gamma_k(\alpha)^u}{(\alpha + \beta t)^u [p]_q^{w\beta t} \Gamma_{(p,q)}(\alpha)^w} < \frac{\Gamma_k(\alpha + \beta t)^u}{\Gamma_{(p,q)}(\alpha + \beta t)^w} \\
 & < \frac{(\alpha + \beta)^u k^{\frac{u\beta(1-t)}{k}} e^{\frac{u\beta \gamma(1-t)}{k}} \Gamma_k(\alpha + \beta)^u}{(\alpha + \beta t)^u [p]_q^{w\beta(1-t)} \Gamma_{(p,q)}(\alpha + \beta)^w}
 \end{aligned} \tag{14}$$

are valid for each  $t \in (0,1)$ .

**Proof.** Let  $\mu(t) = \ln G(t)$  for every  $t \in (0, \infty)$ . Then

$$\mu(t) = \ln \frac{(\alpha + \beta t)^u k^{\frac{u\beta t}{k}} e^{\frac{u\beta \gamma t}{k}} \Gamma_k(\alpha + \beta t)^u}{[p]_q^{-w\beta t} \Gamma_{(p,q)}(\alpha + \beta t)^w}$$

$$= u\ln(\alpha + \beta t) - \frac{u\beta t}{k} \ln k + \frac{u\beta\gamma t}{k} + w\beta t \ln [p]_q + u\ln \Gamma_k(\alpha + \beta t) - w\ln \Gamma_{(p,q)}(\alpha + \beta t).$$

Then,

$$\begin{aligned} \mu'(t) &= w\beta \ln [p]_q - u\beta \frac{\ln k - \gamma}{k} + \frac{u\beta}{\alpha + \beta t} \\ &+ u\beta \psi_k(\alpha + \beta t) - w\beta \psi_{(p,q)}(\alpha + \beta t) \\ &= \beta \left[ w\ln [p]_q - u \frac{\ln k}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha + \beta t} \right. \\ &\quad \left. + u\psi_k(\alpha + \beta t) - w\psi_{(p,q)}(\alpha + \beta t) \right] > 0 \end{aligned}$$

as a result of Lemma 2.3. That implies  $\mu$  is non-increasing on  $t \in (0, \infty)$ . Consequently,  $G$  is non-increasing on  $t \in (0, \infty)$  and for each  $t \in (0, 1)$  we have,

$$G(0) < G(t) < G(1)$$

yielding equation (14).

**Theorem 3.4.** Define a function  $H$  for  $t \in (0, \infty)$ ,  $q \in (0, 1)$  and  $k > 0$  by

$$H(t) = \frac{(\alpha + \beta t)^u e^{\frac{u\beta\gamma t}{k}} \Gamma_k(\alpha + \beta t)^u}{k^{\frac{u\beta t}{k}} (1-q)^{\frac{w\beta t}{k}} \Gamma_{(q,k)}(\alpha + \beta t)^w} \quad (15)$$

where  $u, w, \alpha, \beta$  are positive real numbers. Then,  $H$  is increasing on  $t \in (0, \infty)$  and the inequalities:

$$\begin{aligned} &\frac{\alpha^u e^{\frac{u\beta\gamma t}{k}} \Gamma_k(\alpha)^u}{(\alpha + \beta t)^u k^{\frac{u\beta t}{k}} (1-q)^{\frac{w\beta t}{k}} \Gamma_{(q,k)}(\alpha)^w} \\ &< \frac{\Gamma_k(\alpha + \beta t)^u}{\Gamma_{(q,k)}(\alpha + \beta t)^w} \\ &< \frac{(\alpha + \beta t)^u e^{\frac{u\beta\gamma(1-t)}{k}} \Gamma_k(\alpha + \beta t)^u}{(\alpha + \beta t)^u k^{\frac{u\beta(1-t)}{k}} (1-q)^{\frac{w\beta(1-t)}{k}} \Gamma_{(q,k)}(\alpha + \beta t)^w} \end{aligned} \quad (16)$$

are valid for each  $t \in (0, 1)$ .

**Proof.** Let  $\delta(t) = \ln H(t)$  for every  $t \in (0, \infty)$ . Then

$$\begin{aligned} \delta(t) &= \ln \frac{(\alpha + \beta t)^u e^{\frac{u\beta\gamma t}{k}} \Gamma_k(\alpha + \beta t)^u}{k^{\frac{u\beta t}{k}} (1-q)^{\frac{w\beta t}{k}} \Gamma_{(q,k)}(\alpha + \beta t)^w} \\ &= u\ln(\alpha + \beta t) + \frac{u\beta\gamma t}{k} - \frac{u\beta t}{k} \ln k - \frac{w\beta t}{k} \ln(1-q) \\ &\quad + u\ln \Gamma_k(\alpha + \beta t) - w\ln \Gamma_{(q,k)}(\alpha + \beta t). \end{aligned}$$

Then,

$$\begin{aligned} \delta'(t) &= -\beta \frac{\ln(k^u(1-q)^w)}{k} + \frac{u\beta\gamma}{k} + \frac{u\beta}{\alpha + \beta t} \\ &\quad + u\beta \psi_k(\alpha + \beta t) - w\beta \psi_{(q,k)}(\alpha + \beta t) \\ &= \beta \left[ -\frac{\ln(k^u(1-q)^w)}{k} + \frac{u\gamma}{k} + \frac{u}{\alpha + \beta t} \right. \\ &\quad \left. + u\psi_k(\alpha + \beta t) - w\psi_{(q,k)}(\alpha + \beta t) \right] > 0 \end{aligned}$$

as a result of Lemma 2.4. That implies  $\delta$  is non-increasing on  $t \in (0, \infty)$ . Consequently,  $H$  is non-increasing on  $t \in (0, \infty)$  and for each  $t \in (0, 1)$  we have,

$$H(0) < H(t) < H(1)$$

yielding equation (16).

### 4. Conclusion

If we fix  $u = w = \beta = 1$  in inequalities (10), (12), (14) and (16), then we respectively obtain the inequalities (5), (6), (7) and (8) as special cases. By this, the previous results [7] have been generalized.

### Competing Interests

The authors have no competing interests.

### References

- [1] R. Diaz and E. Pariguan, *On hypergeometric functions and Pachhammer k-symbol*, Divulgaciones Matematicas 15(2)(2007), 179-192.
- [2] T. Mansour, *Some inequalities for the q-Gamma Function*, J. Ineq. Pure Appl. Math. 9(1)(2008), Art. 18.
- [3] F. Merovci, *Power Product Inequalities for the  $\Gamma_k$  Function*, Int. Journal of Math. Analysis, 4(21)(2010), 1007-1012.
- [4] V. Krasniqi and F. Merovci, *Some Completely Monotonic Properties for the (p, q)-Gamma Function*, Mathematica Balkanica, New Series 26(2012), 1-2.
- [5] R. Diaz and C. Teruel, *q,k-generalized gamma and beta functions*, J. Nonlin. Math. Phys. 12(2005), 118-134.
- [6] V. Krasniqi, T. Mansour and A. Sh. Shabani, *Some Monotonicity Properties and Inequalities for  $\Gamma$  and  $\zeta$  Functions*, Mathematical Communications 15(2)(2010), 365-376.
- [7] K. Nantomah, *On Certain Inequalities Concerning the Classical Euler's Gamma Function*, Advances in Inequalities and Applications, Vol. 2014 (2014) Art ID 42.
- [8] K. Nantomah and M. M. Iddrisu, *Some Inequalities Involving the Ratio of Gamma Functions*, Int. Journal of Math. Analysis 8(12)(2014), 555-560.
- [9] K. Nantomah, M. M. Iddrisu and E. Prempeh, *Generalization of Some Inequalities for the Ratio of Gamma Functions*, Int. Journal of Math. Analysis, 8(18)(2014), 895-900.
- [10] K. Nantomah and E. Prempeh, *Generalizations of Some Inequalities for the p-Gamma, q-Gamma and k-Gamma Functions*, Electron. J. Math. Anal. Appl. 3(1)(2015), 158-163.
- [11] K. Nantomah and E. Prempeh, *Some Sharp Inequalities for the Ratio of Gamma Functions*, Math. Aeterna, 4(5)(2014), 501-507.
- [12] K. Nantomah and E. Prempeh, *Generalizations of Some Sharp Inequalities for the Ratio of Gamma Functions*, Math. Aeterna, 4(5)(2014), 539-544.