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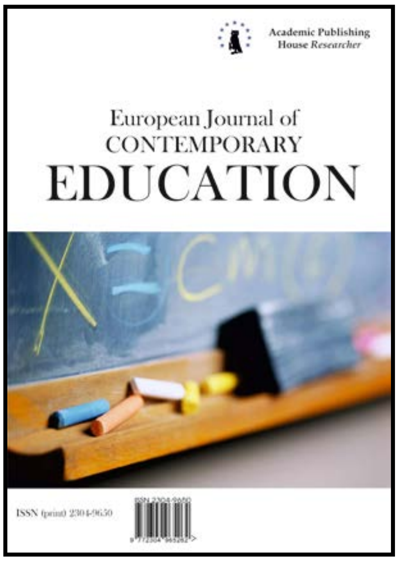

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Geometrical similarity transformations in Dynamic Geometry Environment GeoGebra

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Abstract

The subject of the article is usage of modern computer technologies through the example of interactive geometry environment Geogebra as an innovative technology of representing and studying of geometrical material which involves such didactical opportunities as vizualisation, simulation and dynamics. There is shown a classification of geometric similarity transformations of the plane, computer tools of interactive geometry environment GeoGebra which are used for realization of similarity transformations. It is illustrated an opportunity of usage of these represented tools while studying of concerns and properties of geometric transformations, theorem proving, solving of construction tasks.

Keywords: dynamic mathematics software, interactive geometry environment GeoGebra, computer tools, geometric similarity transformations.

Introduction

During the stage of education modernization in Russia questions of usage of information and communication technology are becoming very actual. Necessity of computer support in the educational process is defined the requirements of the federal educational standard of the general education:

generation and developing of competence in the area of use of information and communication technology (ICT- technology) [1; 7].

In the modern society ICT-competence is considered as one of basic competences of a school graduate since it is represented as *is the capacity* to use of information and communication technology for information search, its processing, estimation and transmission, sufficient to successfully life and work in the environment of modern society.

Modern information and communication technology enable to involve the pupil in various kinds of activity: research, creative, design and others opening new possibilities for generation ITC-competence. For this reason the main pedagogical task of education at the modern stage with usage of ITC consists not only in the delivering of current knowledge but in creating of conditions for getting it independently, for experience, “opening” new knowledge, for updating of pedagogical technology under the conditions of active usage of ICT means.

Interactive means of education on the base of information and communication technology which include dynamic geometry environment (DGE) or systems of dynamic geometry (SDG) are widely spread in modern school.

All dynamic geometry programs variety can be divided into two kinds:

– two-dimensional geometry programs (2D), for example, Cabri Geometry, The Geometer's Sketchpad (the Russian version is “Living mathematics”), GeoGebra, GeoNext;

– three-dimensional geometry programs (3D), for example, Archimedes Geo3D, Geometria, Geogebra (from version 5.0).

Dynamic geometry environment have a range of advantages comparing with traditional educational technologies, among them are the following:

✓ attraction of computer tools to performance of constructions while saving with pupils right imagination about geometrical generation methods;

✓ making of dynamic drawings with an opportunity of further research work;

✓ wide range for active independent work of pupils;

✓ usage of dynamic geometry programs at school and at home in any time.

Among didactic opportunities of dynamic geometry environment as information technology we emphasize the following:

✓ *visualization* – a pictorial rendition of educational information about geometric objects which develops “active mathematical seeing” of objects and their features [2];

✓ *simulation* –experimental observing the behavior of geometric objects and detection of unknown features and facts [3];

✓ *dynamics* –a realization moving effect of an illustrative object with computing means [4].

Thus, dynamic geometry environment is represented as an innovating type of educational product which involves didactical opportunities new in quality comparing with traditional illustrative means. When working in dynamic mathematics software, on the one hand, a pupil uses a *new innovating technology of studying* the material, and on the other, a *combined information processing technology* which is usual and natural for the modern pupil [5]. Therefore learning of dynamic geometry programs opportunities, their methodic tracking, applying in the educational process are interesting for many researchers.

Actual investigations analyzing. Analyzing of scientific and methodological literature regarding the improvement of mode of an instruction in mathematics from the point of view of usage in the educational process means of information technology allows to say that a great amount of methodological works are devoted to this question. Usage of dynamic geometry environment in the educational process is one of the actual directions of an investigating activity of scientists and instructors:

✓ creation of models and training materials in the environment «Mathematic constructor» [6];

✓ developing of flexibility of thinking through the organizing of creative workshops in the environment «Mathematic constructor» [7];

✓ developing of creativity of pupils while teaching mathematics in 5-6 forms using dynamic geometric environment [8];

- ✓ usage of the dynamic geometry environment GeoGebra in different stages of work with a theorem [9];
- ✓ usage of dynamic geometry GeoGebra as a means of computer simulation [10];
- ✓ functional opportunities GeoGebra in the context of educational and methodic support [11] etc.

Dynamic environments, in particular GeoGebra, have a wide range of tools which enable to use such opportunities as visualization, simulation and dynamics while studying geometric transformations in a plane and space. In school mathematic workbooks there is a little place for geometric transformations in a plane and space, besides with a small quantity of tasks and minima of visualization.

Incidentally it should be mentioned "Geometric Transformation" is one of the key, interesting and the most beautiful themes of geometry which allows developing "visual thinking", spatial perception and geometric literacy of pupils. Usage of concepts and features of the studied theme simplifies a theorem proving and opens a new method of the solution of many tasks on construction.

Research objective: to show an opportunity of environment GeoGebra tools using while studying geometric similarity transformations with the aim of visualization of educational information about studied concepts and developing of "active mathematic vision" of objects and their features.

Discussion

Dynamic geometry environment GeoGebra is freely distributed software which has a Russian version. The main feature of GeoGebra is a double representation of objects: in the form of algebraic and geometric models (**geo**metry+al**gebra**); for each of them is given an individual window thereby it is emphasized an unbroken link of different parts of mathematics.

The list of computing instruments in the dynamic geometry GeoGebra includes standard set of tools which enables to create main geometric objects (a point, a line, a circle, a vector, a polygon, an angle) and another tools realizing additional operations on geometric objects (segment division in halves, angle division in *n* equal parts, measurement of segment length, measurement of the angle and etc.) Lets pay attention to the tools of the environment whereby one can study and use geometric similarity transformations for problems solving.

In modern school programs there is too little place given to the concept of geometric transformation: pupils are taught definitions of such transformations as a turn, a parallel shift, symmetries. This material is studied at the end of the 9 form for short, with minima of visualization and similarity transformations are regarded only during studying of triangles similarity features [12].

Thus, similarity transformation or analogy is the transformation from one figure to another when the distance between two points is changing into the same number of times that is called the similarity coefficient. If the similarity coefficient is equal to one, the transformation is called motion [Fig. 1.].

French mathematician (geometer) of 19-th century Mishel Sharl enunciated the classification of motions: *Any motion is either the parallel transfer or the symmetry, or the composition of the symmetry and the transfer into the vector parallel to the symmetry axis (the last kind of symmetry is called slipping symmetry)*[13].

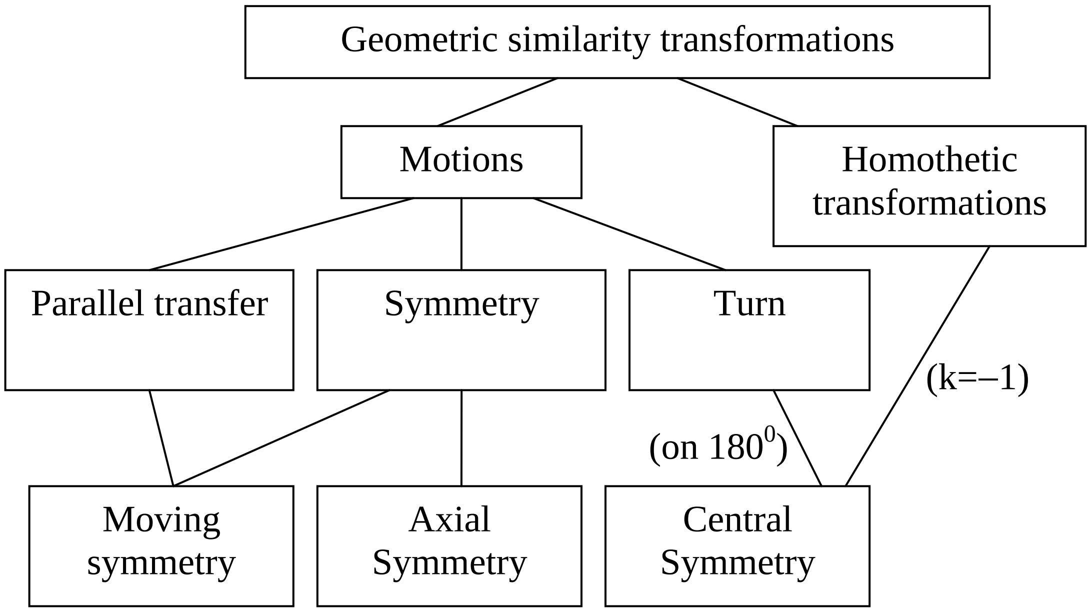

Fig. 1. Geometric similarity transformations

Let us introduce the classification of similarity transformations in a plant after main invariants. The *invariant* of the transformations multitude is called a figure characteristic saved in the course of influence on it any transformation from the pointed multitude.

Transformations	Saves distance	Saves angles	Kinds
Motions	yes	yes	turn, transfer, central and axial symmetry
Similarities but no motions	no	yes	Homothetic transformations

Dynamic geometry environment can be used not only for the illustration of studied geometric transformations but for studying their characteristics, for the theorem proving, for solving construction problems thanks to the environmental tools.

Computer tools	**Concept of Motion**
Reflection regarding to the line	Axial symmetry is a motion regarding to the line when a figure is mapped into itself
Reflection regarding to the point	Central symmetry is a motion regarding to the point when the figure turns into itself
Turn around the point	Motion around the point O through the angle α, when every point M turns into the same point M_1, that is $OM=OM_1$ and the angle $MOM_1=\alpha$
Parallel transfer along the vector	Motion to the vector $\vec{a}$, when every point M turns into the point M_1 in this case the vector $\overrightarrow{MM_1} = \vec{a}$.
Homothetic transformations regarding to the point	Homothetic transformations with the centre in the dot O and coefficient $k \neq 0$ is a geometric transformation which turns any point A into the same point A', that is $\overrightarrow{OA'} = k \cdot \overrightarrow{OA}$

Geometric Transformations –Motions

Motions are connected with the concept of figures equality (congruence): two figures F and G on the plane α are named equal if there is a motion of this plane, which turns the first figure into the second.

Axial Symmetry

Two points A and A1 are called symmetric regarding to the line *a* if this line passes through the middle point of a segment AA1 and is perpendicular to it. Two figures F and F1 are called symmetric regarding to the line if every point of one figure has a symmetric point of another figure.

Example 1. There is a polygon ABCDE and a line *f*. Make a figure which is symmetric to the given one regarding to the line *f*. Prove symmetry of figures using the definition. Show that axial symmetry maintains distances but does not change the orientation that is the direction of sense of rotation into opposite. [Fig. 2].

Steps of construction	Computer tools
Construct a polygon ABCDE	A polygon
Construct a line passing through two points	A line
Construct mirror reflection of the polygon regarding to the line	Reflection regarding to the line
Link tops of the polygon ABCDE together with tops of the polygon A′B′C′D′E′	A segment
Point middles of the made segments	Middle or centre
Measure sizes of angles between segments and the reflection line	Angle

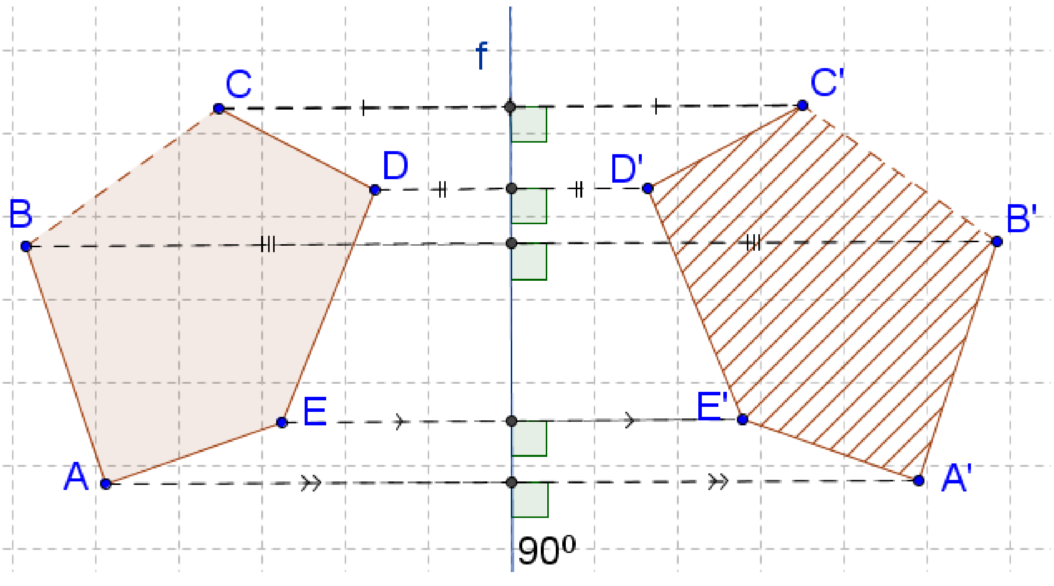

Fig. 2. Symmetry regarding to the line

Example 2. Equal circles S_1 and S_2 internally tangent the circle S in points A_1 and A_2. Any point C of the circle S is connected by segments with points A_1 and A_2. These segments cross $S_1$and S_2 in points B_1 and B_2. Prove that $A_1A_2 \| B_1B_2$ [13; 362].

Solution. When making the drawing be sure in the truth of the statement $A_1A_2 \| B_1B_2$ [Fig. 3].

Steps of construction	Computer tools
Make a circle S	Circle on the centre and radius
Sketch a diameter of the circle AB having chosen it as an axis if symmetry	Segment
Put point A_1 on the circle S	Point on the object
Sketch a tangent line to the circle S in the point A_1	Tangent
Make a circle S_1 passing through the point A_1	Circle through the centre and the point
Put the point A_2 and the circle S_2 which are the mirror reflection regarding the diameter regarding AB the point A_1 and the circle S_1	Reflection regarding to the line
Choose any point C on the circle S	Point on the object
Link the point C by segments with points A_1 и A_2, mark crossing points of segments made with circles S_1 and S_2 through B_1 и B_2	Segment
Link points A_1 и A_2 with the segment	Segment
Make the parallel line A_1A_2 passing through the point B_1. Make sure that the point B_2 belongs to the made line	Parallel line

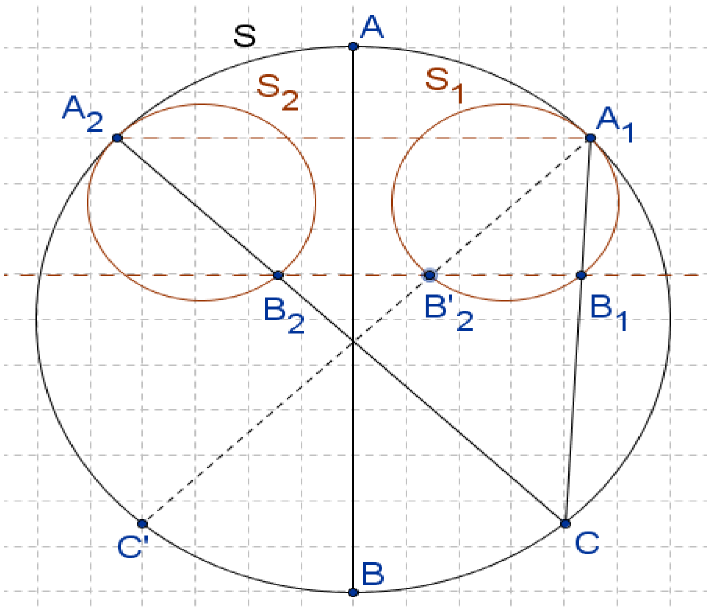

Fig. 3. Solution of the problem using the symmetry

Proof. Lets put points C′ and B′₂ symmetric to points C and B_2 in relation to a diameter AB using the tool “Reflection regarding to the line”. Since points A_1 and A_2 are symmetric regarding the diameter and the point C′ is symmetric to the point C regarding to the same diameter, then $A_1A_2 \| CC'$.

Circles S and S_1 are homothetic with the centre of homothetic transformations in the point A_1. The line $B_1B'_2$ turns into the line CC′, this means that lines are parallel. Since the circle S_1 is symmetric to the circle S_2 regarding to the diameter AB, the point B'_2 is symmetric to the point B2, the point C is symmetric to the point C′, then $B_2B'_2 \| CC'$, hence points B_1, B'_2, B_2 lay on one line B_1B_2 which is parallel to the line CC′.

We get $A_1A_2 \| CC'$ и $B_1B_2 \| CC'$, this means, $A_1A_2 \| B_1B_2$.

We see that tools of dynamic geometric similarities are convenient means of searching the problem solution result but do not free from proving of the obtained result especially by solving proof problems.

Parallel transfer

Parallel transfer on the vector $\vec{a}$ is called a mapping into itself when every point M is transferred into the point M_1, that is the vector $\overrightarrow{MM_1} = \vec{a}$.

Example 3. It is given a triangle ABC and a vector $\overrightarrow{DE}$. Make a figure which will come out from the initial one through a parallel transfer onto the vector $\overrightarrow{DE}$. Show that the parallel transfer saves distances and an orientation. [Fig. 4].

Steps of construction	Computer tools
Make the triangle ABC	Rigid polygon
Mark the vector DE	Vector
Make a figure through a parallel transfer of the triangle ABC onto the vector DE	Parallel transfer onto the vector
Mark vectors from points A, B, C which are equal and equally directed with the vector DE	Mark the vector
Define vectors length	Distance and length

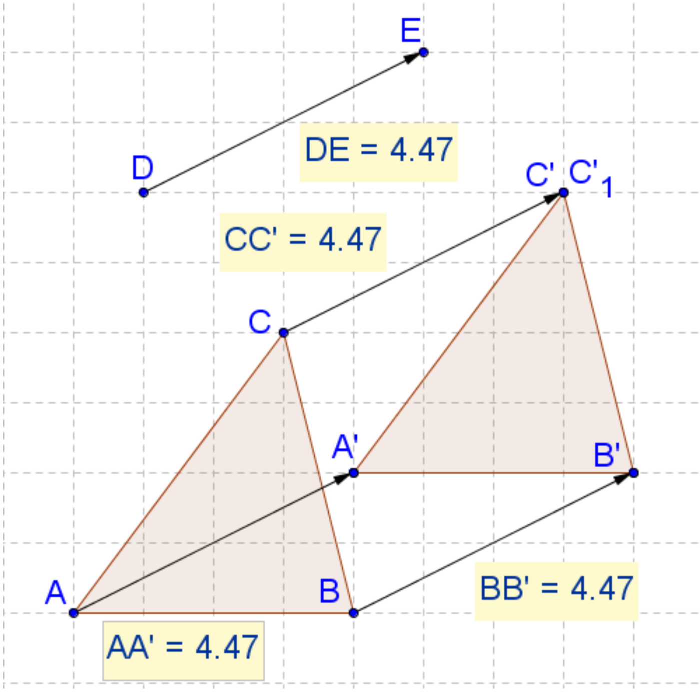

Fig. 4. Parallel transfer

Example 4. In the trapezium ABCD sides BC и AD are foundations, point M is a crossing point of angles bisectors A and B, N is a point of angles bisectors C and D [Fig. 5]. Prove that $2MN = |AB + CD - BC - AD|$ [13; 346].

Steps of construction	Computer tools
Make a trapezium ABCD	Polygon
Make angles bisectors A и B, C и D	Angles bisector
Mark a crossing point of bisectors A and B like M, C and D like N	Point on the object
Sketch a perpendicular line BC through the point M, mark a crossing point like E	Perpendicular line Point on the object
Make a circle which touch with sides AB, BC and AD, with the centre in the point M, passing through the point E	Circle on the center and a point
Put a triangle CND parallel to foundations so that N′ will coincide with the point M and the side C′D′ will be touch with the circle	Parallel transfer onto the vector
Find length of trapezium sides and the segment MN	Distance and length

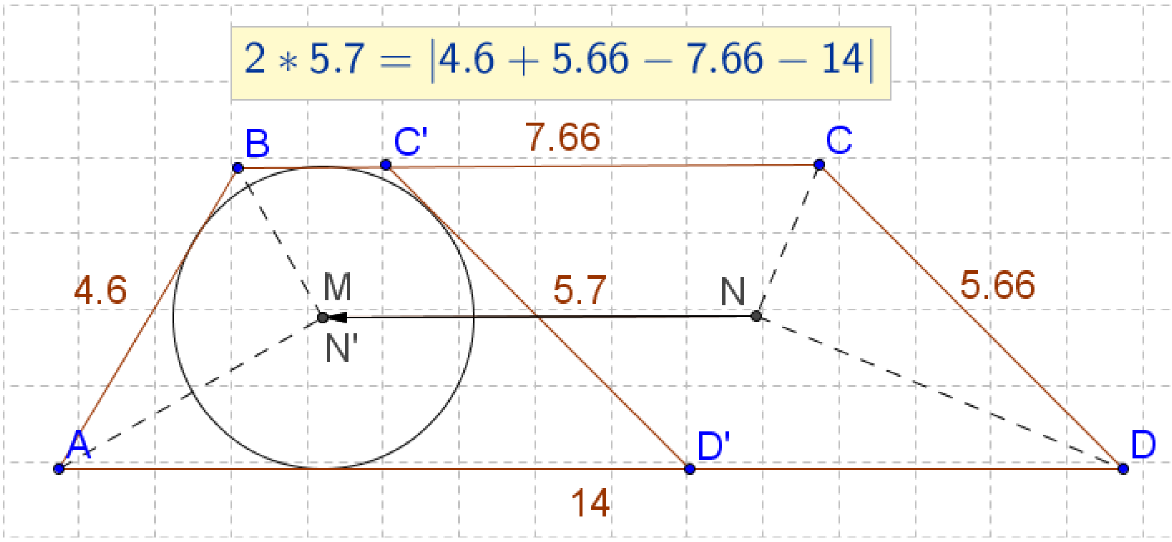

Fig. 5. Solving of problems with usage of parallel transfer

Proof. For the described trapezium ABCD the following congruence is true AB+C′D′=AD′+BC′, this can be written like 2MN′=│AB+C′D′–AD′–BC′│. If to adjoin to the left part of the congruence 2N′N and to the right one – CC′+DD, then we get a statement which we must prove.

Parquet

Parquet on the plane is the filling of the plane with polygons when any two polygons have either a common side or a common top or do not have any points in common.

Parquets on the plane is a wonderful creative material for involving pupils into an interesting cognitive activity. The easiest kind of the parquet is such a parquet where a plane is filled with figures thanks to a parallel transfer, for example, there is a task to make the parquet from triangles equal to the given triangle [Fig. 6].

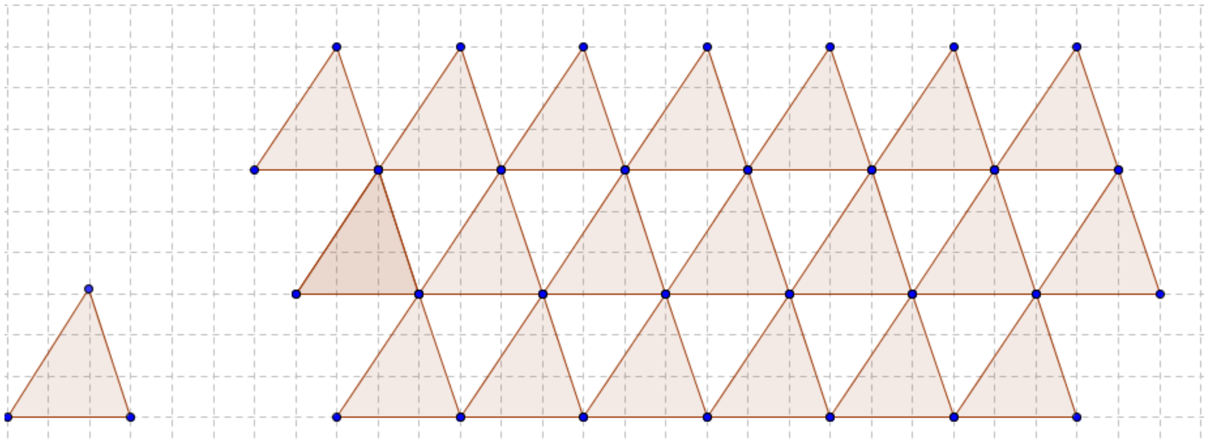

Fig. 6. Parquet

Turn

Turn of the plane around the point O on the angle α is called a mapping of the plane into itself when every point M is mapped into such point M_1, that $OM=OM_1$ и $\angle MOM_1 = \alpha$.

Example 5. It is given a circle. Make a figure which is made from the original one through the turn into the angle of 90^0, 180^0, 270^0 around the point. Show that the turn saves distances [Fig. 7].

Steps of construction	Computer tools
Make a segment AB	Segment
Make a half circle through points A и B	Half circle through two points
Make a turn around point A of a half circle and a segment into angle 90^0, 180^0, 270^0	Turn around the point
Measure sizes of angles between segments AB and AB_1, AB_1 and AB_2, AB_2 and AB_3	Angle
Measure lengths of segments AB, AB_1, AB_2, AB_3	Distance or length

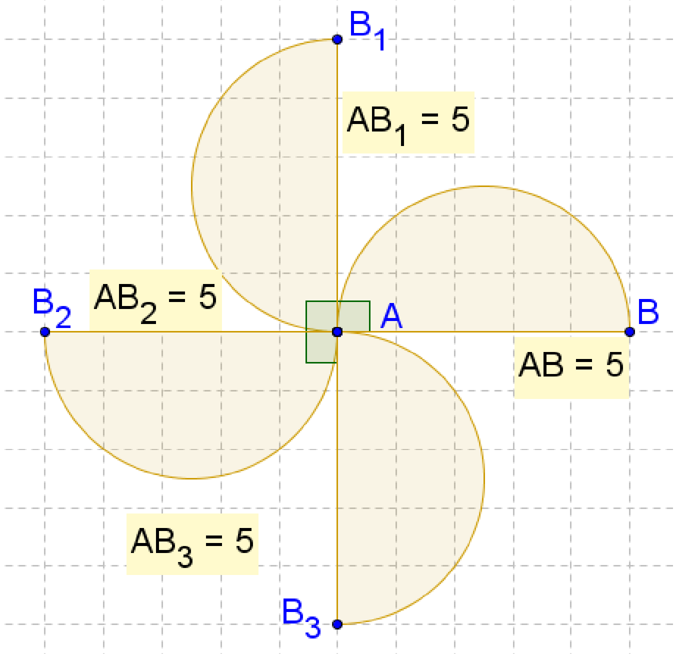

Fig. 7. Turn

Central Symmetry

Two points A and A_1 are called symmetric regarding to the point O if O is a middle of the segment AA_1 (point O is a symmetry centre). A figure is called symmetric regarding to the point O if for every point of this figure another point regarding to the point O belongs to this figure too [Fig. 8].

The concept of the central symmetry is a common for such concepts like turn and homothetic transformations and enables to establish equalextention relationship between such concepts like «turn to180^0» and « homothetic transformations with the coefficient k=–1».

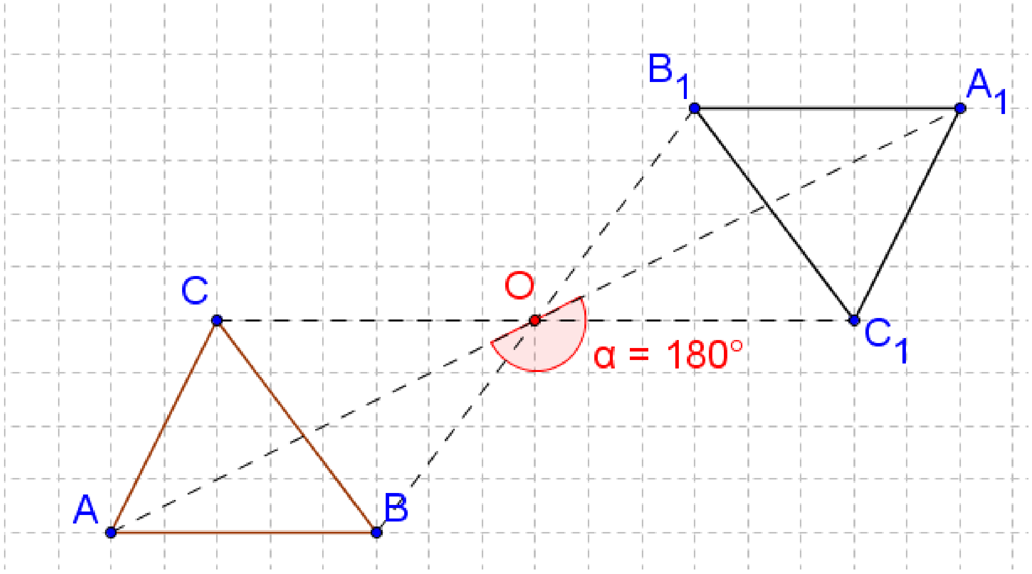

Fig. 8. Central Symmetry

Geometric similarity transformations – homothetic transformations

Two bodies are similar if one of them is made from another through increasing or decreasing all its sizes (rectilinear) in the same ratio. The most easiest similarity transformation is *homothetic* which enables to get increased or decreased copy of the figure maintaining angles and increasing lengths to the same extent.

Homothetic transformation with the centre in the point O and the coefficient k different from zero is called the transformation turning every point A into the point A' lying on the line OA and satisfying the statement OA′=k·OA. This definition leads to the fact that homothetic transformation maintains the shape but not sizes of the figure.

For making similar figures with the similarity coefficient k is used a tool *Homothetic transformations regarding to the point*. Firstly it is named the designed object, then the centre of the homothetic transformations and the homothetic transformations coefficient in the appeared dialog box [Fig. 9].

We note that homothetic transformations with the similarity coefficient k=–1 is a central symmetry, when k>0 points A и A' are lying to the one side from the point O, when k<0 they are to the different sides. For studying of features of homothetic transformations depending from the coefficient it is suitable to use the tool *Slider*.

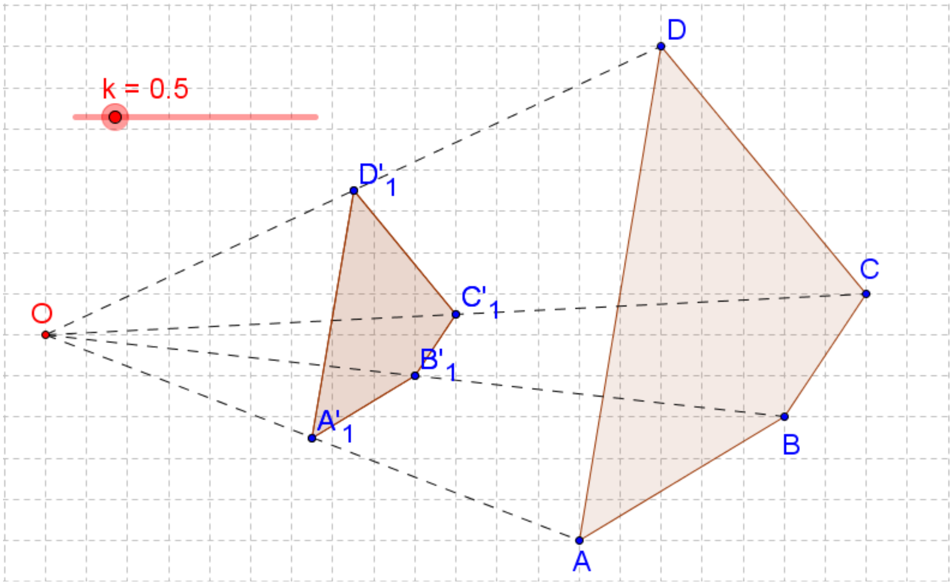

Fig. 9. Homothetic transformations

Slider is a computer tool containing a point-slider free moving on some line. With this point is connected some quantity which is used like a parameter. While moving the slider-box from less quantity to the bigger one, pupils note changes in features of the studied object [Fig. 10].

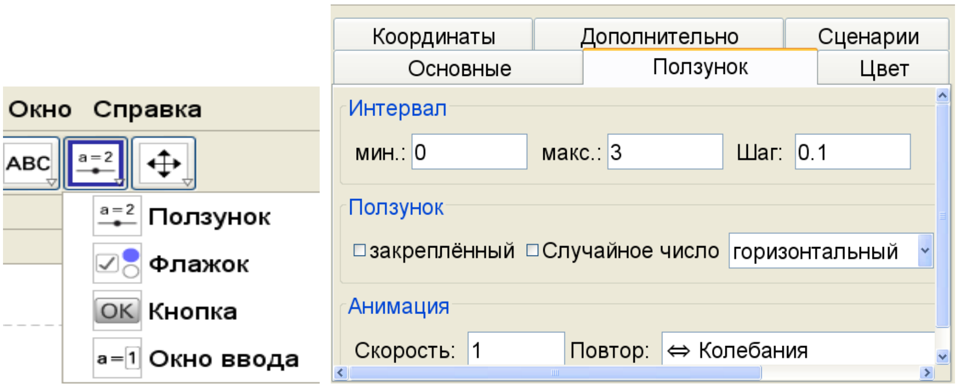

Fig. 10 Computer tool *Slider*

Compositions of similarity transformations

There no similarity transformations in school textbooks, that is transformations which are formed as a result of consequent fulfilling of some transformations. One of this compositions is *moving symmetry*: symmetry composition regarding to the line and parallel transfer in the direction of the same line (besides taken in any order) [Fig. 11].

Set of all points where come points of some figure F while moving symmetry, makes a figure F′, appeared from the moving symmetry from the figure F.

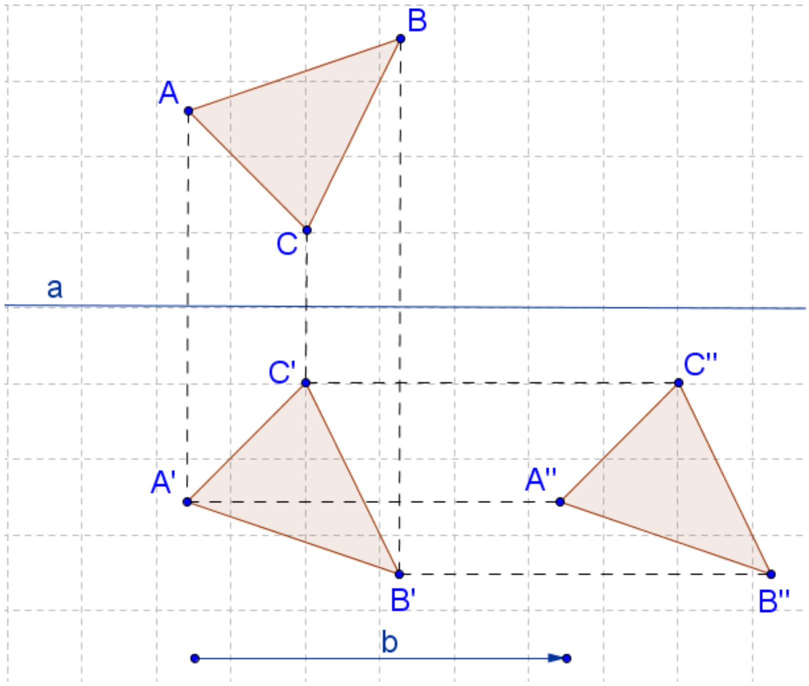

Fig. 11. Moving Symmetry

Among the transfer compositions we can distinguish the following:

✓ *turn homothetic transformations* (special similarity) is a composition of homothetic transformations with the centre in the point O and the coefficient k, different from 1, and a turn around the point;

✓ *mirror similarity* is a composition of the axial symmetry and homothetic transformations with the centre on the axe.

Studying of transfer composition and their use at the solution of tasks on the proof and construction represents a very attractive material.

Conclusions

Dynamic geometry environment is an innovation kind of the educational product which enables to change traditional attitude to the studying of many difficult questions of geometry like it was shown in the example of geometric similarity transformations. Comparing with traditional technology dynamic geometry environment is an innovation technology of geometric material studying with new in qualities didactic opportunities among the last we can note visualization, simulation, dynamics. Presence of different tools, which includes the tools for making of geometric similarity transformations, enables to make changes into traditional process of reproducing of the above mentioned concepts, gives opportunities to the developing “active mathematic vision” of objects and their features.

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