ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REŞIȚA
ANUL XXI, NR. 1, 2014, ISSN 1453-7397

Zoltan-Iosif Korka, Ion-Cornel Mituletu

## A Review of Dynamic Models Used in Simulation of Gear Transmissions

The investigation of relevant scientific literature regarding gear modeling enabled us to discover a significant number of papers dating back several decades and continuing to the present. The purpose of the $d y$ namic models was quite diverse, but all modeling efforts share the goal of replicating the complex physics of power transmission through gear interaction. This paper investigates the relevant aspects regarding the dynamic modeling of gear transmissions, starting with the simplest model (1DOF), then developing it into a model with three degrees of freedom (3DOF) and finishing with six degrees of freedom model (6DOF).

Keywords: Dynamic model, simulation, gear transmission

## 1. Introduction

Because the gears are critical components of any rotating machine, they have received a considerable amount of attention regarding their dynamic modeling, being published a significant number of papers concerning this problem [1], [2]. The objectives of dynamic modeling of gear transmissions varied past five decades, from vibration controlling and noise analysis, to the study of transmission error and stability analyses [3], [4]. The final scopes of dynamic modeling of gears could be summarized as follows:

- Analysis of contact and bending stress;
- Reduction of superficial wear as for example pitting;
- Study of transmission efficiency;
- Study of noise radiation;
- Influence on other parts of the transmission, particularly bearings;
- Natural frequencies of the system;
- Studies regarding the vibratory motion of the system;
- Studies of reliability and life cycle.


## 2. Dynamic Model with One Degree of Freedom (1DOF)

Figure 1 show a typical dynamic model with one degree of freedom (1DOF) used for the mesh investigation of a gear pair system. The gear transmission is modeled as a pair of discs, connected along the mesh line by a spring and a damper.

The model takes into account influences of the static transmission error which is simulated by a displacement excitation $\mathrm{e}(\mathrm{t})$ at the mesh. This transmissions error arises from several sources, such as tooth deflection under load, non-uniform tooth spacing, tooth profile errors caused by machining errors as well as pitting, scuffing of teeth flanks. The mesh stiffness $c_{z}(t)$ is expressed as a time-varying function. The gear pair is assumed to operate under high torque condition with zero backlash. Effects of friction forces at the meshing interface are neglected on the basis that in particular, the coefficient of friction is low (approx. 6\%, according to [5]). Furthermore, the viscous damping coefficient of the gear mesh $d_{z}$ is assumed to be constant.


Figure 1. 1DOF dynamic model for a gear pair system
The differential equations of motion for this system can be expressed in the form:

$$
\begin{gathered}
J_{1} \ddot{\varphi}_{1}+r_{b 1} c_{z}(t)\left[r_{b 1} \varphi_{1}+r_{b 2} \varphi_{2}+e(t)\right]+r_{b 1} d_{z}\left[r_{b 1} \dot{\varphi}_{1}+r_{b 2} \dot{\varphi}_{2}+\dot{e}(t)\right]=M_{1}(t), \\
J_{2} \ddot{\varphi}_{2}+r_{b 2} c_{z}(t)\left[r_{b 1} \varphi_{1}+r_{b 2} \varphi_{2}+e(t)\right]+r_{b 2} d_{z}\left[r_{b 1} \dot{\varphi}_{1}+r_{b 2} \dot{\varphi}_{2}+\dot{e}(t)\right]=M_{2}(t),
\end{gathered}
$$

where $\varphi_{i}, \dot{\varphi}_{i}, \ddot{\varphi}_{i}(\mathrm{i}=1,2)$ are rotation angle, angular velocity, angular
acceleration of the input pinion and the output wheel respectively. $\mathrm{J}_{1}$ and $\mathrm{J}_{2}$ are the mass moments of inertia of the gears. $M_{1}(t)$ and $M_{2}(t)$ denote the external torques load applied on the system. $r_{b 1}$ and $r_{b 2}$ represent the base radii of the gears.

By introducing the composite coordinate

$$
\begin{equation*}
q=r_{b 1} \varphi_{1}+r_{b 2} \varphi_{2}, \tag{3}
\end{equation*}
$$

equations. (1), (2) yield a single differential equation in the following form:

$$
\begin{equation*}
m_{\text {red }} \ddot{q}+c_{z}(t) q+d_{z} \dot{q}=F(t)-c_{z}(t) e(t)-d_{z} \dot{e}(t), \tag{4}
\end{equation*}
$$

where: $m_{\text {red }}=\frac{J_{1} J_{2}}{J_{1} r_{b 2}^{2}+J_{2} r_{b 1}^{2}}$, and $F(t)=m_{\text {red }}\left(\frac{M_{1}(t) r_{b 1}}{J_{1}}+\frac{M_{2}(t) r_{b 2}}{J_{2}}\right)$.
For a specific gear-pair, the mesh stiffness $\mathrm{c}_{2}(\mathrm{t})$ can be approximately represented by a truncated Fourier series:

$$
\begin{equation*}
c_{z}(t)=c_{0}+\sum_{k=1}^{K} c_{k} \cos \left(k \omega_{z} t+\gamma_{k}\right), \tag{6}
\end{equation*}
$$

where $\omega_{z}$ is the gear meshing angular frequency and K is the number of terms of the series.

Generally, the components of the meshing error are not identical to each gear tooth and consequentially, they will produce excitation movements periodical to the rotation speed of the wheel (repeated every time the respective tooth is in contact). Therefore, the excitation function $\mathrm{e}(\mathrm{t})$ can be represented by a Fourier series with the mains frequency corresponding to the rotation speed of the wheel. If it is considered that the errors are located only at teeth of the pinion, $\mathrm{e}(\mathrm{t})$ can be written as:

$$
\begin{equation*}
e(t)=\sum_{i=1}^{I} e_{i} \cos \left(i \omega_{l} t+\alpha_{i}\right), \tag{7}
\end{equation*}
$$

where $\omega_{1}$ is the angular speed (rotation frequency) of the pinion.
When it is assumed that:
$\dot{\varphi}_{1}=\omega_{1}=$ const, $\dot{\varphi}_{2}=\omega_{2}=$ const, $d_{z}=0, c_{2}(t)=c_{0}$, the dynamic transmission error of the gear pair q is equal to the static deformation of the teeth under the constant load $\mathrm{q}_{\mathrm{o}}$.

Therefore:

$$
\begin{equation*}
q=r_{b 1} \varphi_{1}+r_{b 2} \varphi_{2}=q_{0}, \tag{8}
\end{equation*}
$$

So that, Eq. (4) becomes:

$$
\begin{equation*}
m_{r e d} \ddot{q}+c_{z}(t) q+d_{z} \dot{q}-f(t)=0 . \tag{9}
\end{equation*}
$$

Based on the above, the vibration of a gear pair can be written as a differential equation of the form:

$$
\begin{equation*}
m_{r e d} \ddot{q}+c_{z}(t) q+d_{z} \dot{q}-f(t)=0, \tag{10}
\end{equation*}
$$

where:

$$
\begin{equation*}
f(t)=c_{0} q_{0}-\left[c_{z}(t)-c_{0}\right] e(t)-d_{z} \dot{e}(t) . \tag{11}
\end{equation*}
$$

Taking into account four dominant coefficients $\mathrm{c}_{0}, \mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{c}_{3}$ in the Fourier series of the mesh stiffness, Eq. (6) can be written as:

$$
\begin{equation*}
c_{z}(t)=c_{0}+\sum_{k=1}^{3} c_{k} \cos \left(k \omega_{z} t+\gamma_{k}\right)=c_{0}+\sum_{k=1}^{3}\left(\hat{c}_{k} \cos k \omega_{z} t+\hat{s}_{k} \sin k \omega_{z} t\right), \tag{12}
\end{equation*}
$$

where $\omega_{z}=z_{1} \omega_{1}$.
If the excitation function $e(t)$ is expressed by its first two terms of the Fourier series, we have:

$$
\begin{equation*}
e(t)=\sum_{k=1}^{2} e_{k} \cos \left(k \omega_{l} t+\alpha_{k}\right) . \tag{13}
\end{equation*}
$$

Substituting the expressions (12) and (13) in equation (11), we obtain:

$$
\begin{gather*}
f(t)=c_{0} q_{0}+d_{z} e_{1} \omega_{1} \sin \left(\omega_{1} t+\alpha_{l}\right)+2 d_{z} e_{2} \omega_{1} \sin \left(2 \omega_{1} t+\alpha_{2}\right)-\frac{e_{1}}{2} \sum_{k=1}^{3}\left\{\hat{c}_{k} \cos \left(k z_{1}-1\right) \omega_{1} t-\alpha_{1}\right]+ \\
\left.\left.\left.+\hat{s}_{k} \sin \left[k z_{1}-1\right) \omega_{1} t-\alpha_{1}\right]++\hat{c}_{k} \cos \left(k z_{l}+1\right) \omega_{1} t+\alpha_{l}\right]+\hat{s}_{k} \sin \left(k z_{1}+1\right) \omega_{l} t+\alpha_{l}\right\}-  \tag{14}\\
\left.-\frac{e_{2}}{2} \sum_{k=1}^{3}\left\{\hat{c}_{k} \cos \left[\left(k z_{1}-2\right) \omega_{l} t-\alpha_{2}\right]+\hat{s}_{k} \sin \left(k z_{1}-2\right) \omega_{l} t-\alpha_{2}\right]+\hat{c}_{k} \cos \left(k z_{l}+2\right) \omega_{l} t+\alpha_{2}\right]+ \\
+\hat{s}_{k} \sin \left[\left(k z_{1}+2\right) \omega_{l} t+\alpha_{2}\right\}
\end{gather*}
$$

Based on the analytical form of the functions $c_{z}(t)$, respective $f(t)$ and using the harmonic balance method, the solution of the differential equation (10) can be approximated by the expression:

$$
\begin{align*}
& q(t)=a_{0}+\sum_{k=1}^{3}\left(a_{k} \cos \left(\omega_{l} t+b_{k} \sin k \omega_{l} t\right)+\sum_{k=1}^{3}\left[a_{k z_{1}-2} \cos \left(k z_{1}-2\right) \omega_{1} t+b_{k z_{l}-2} \sin \left(k z_{1}-2\right) \omega_{l} t+\right.\right. \\
& +a_{k z_{1}-1} \cos \left(k z_{1}-1\right) \omega_{1} t+b_{k z_{1}-1} \sin \left(k z_{1}-1\right) \omega_{1} t+a_{k z_{1}} \cos \left(k z_{1}\right) \omega_{1} t+b_{k z_{l}} \sin \left(k k_{1}\right) \omega_{1} t+  \tag{15}\\
& \left.+a_{k z_{1}+1} \cos \left(k z_{1}+1\right) \omega_{1} t+b_{k z_{1}+1} \sin \left(k z_{l}+1\right) \omega_{l} t+a_{k z_{1}+2} \cos \left(k z_{1}+2\right) \omega_{l} t+b_{k z_{1}+2} \sin \left(k z_{1}+2\right) \omega_{1} t\right]
\end{align*}
$$

## 3. Dynamic Model with Three Degrees of Freedom (3DOF)

When the stiffness respective the elasticity of the shafts and the bearings cannot be neglected, a dynamic model with tree degrees of freedom has to be considered.


Figure 2. Dynamic model with three degrees of freedom (3DOF)
Such a model is shown in figure 2. Same as at the 1 DOF model, the gear mesh is modelled as a pair of discs, connected along the mesh line by a spring and a damper. In addition to the 1 DOF model, the shafts and the bearings are considered elastic, each of the discs being supported by a spring and a damper, having elastic constants $c_{1}$ and $c_{2}$, respective the viscous damping coefficients $d_{1}$ and $d_{2}$.Forthermore, the backlash between the teeth of the pinion and the gear is noted with 2 b . The gear mesh stiffness $\mathrm{c}_{\mathrm{z}}(\mathrm{t})$ and the static transmission error $\mathrm{e}(\mathrm{t})$ are considered time varying, while the viscous damping is noted with $\mathrm{d}_{\mathrm{z}}$.

Dynamic transmission error is defined:

$$
\begin{equation*}
y_{d}(t)=y_{1}+r_{b 1} \varphi_{1}(t)-y_{2}-r_{b 2} \varphi_{2}(t) \tag{16}
\end{equation*}
$$

The difference between the dynamic transmission error $y_{d}(t)$ and the static transmission error $e(t)$ is given by the relation:

$$
\begin{equation*}
y(t)=y_{1}+r_{b 1} \varphi_{1}(t)-y_{2}-r_{b 2} \varphi_{2}(t)-e(t) \tag{17}
\end{equation*}
$$

The meshing force can be written as:

$$
\begin{equation*}
F_{y}=c_{z}(t) \cdot f(y)+d_{z} \cdot \dot{y}(t) \tag{18}
\end{equation*}
$$

where $f(y)$ is a nonlinear function used for the description of the gear pair with backlash.

Assuming an equal repartition of the gap between teeth, $f(y)$ can be written as:

$$
f(y)=\left\{\begin{array}{cc}
y-b & y>b  \tag{19}\\
0 & |y| \leq b \\
y+b & y>-b
\end{array}\right.
$$

The differential equations of motion can be written as follows:

$$
\left\{\begin{array}{l}
m_{1} \ddot{y}_{1}+d_{1} \dot{y}_{1}+d_{z} \dot{y}+c_{1} y_{1}+c_{z}(t) f(y)=0  \tag{20}\\
m_{2} \ddot{y}_{2}+d_{2} \dot{y}_{2}-d_{z} \dot{y}+c_{2} y_{2}-c_{z}(t) f(y)=0 \\
m_{\text {red }} \ddot{y}-m_{\text {red }} \ddot{y}_{1}+m_{\text {red }} \ddot{y}_{2}+d_{z} \dot{y}+c_{z}(t) f(y)=F
\end{array},\right.
$$

where:

$$
\left\{\begin{array}{l}
m_{r e d}=\frac{J_{1} J_{2}}{J_{1} r_{b 2}^{2}+J_{2} r_{b 1}^{2}}  \tag{21}\\
F(t)=m_{r e d}\left(\frac{M_{1}(t) r_{b 1}}{J_{1}}+\frac{M_{2}(t) r_{b 2}}{J_{2}}\right)-m_{r e d} \ddot{e}(t)
\end{array}\right.
$$

The system of equations (20) can be written in matrix form as follows:

$$
\left[\begin{array}{ccc}
m_{1} & 0 & 0  \tag{22}\\
0 & m_{2} & 0 \\
-m_{\text {red }} & m_{\text {red }} & m_{\text {red }}
\end{array}\right] \cdot\left[\begin{array}{c}
\ddot{y}_{1} \\
\ddot{y}_{2} \\
\ddot{y}_{2}
\end{array}\right]+\left[\begin{array}{ccc}
d_{1} & 0 & d_{z} \\
0 & d_{2} & -d_{z} \\
0 & 0 & d_{z}
\end{array}\right] \cdot\left[\begin{array}{c}
\dot{y}_{1} \\
\dot{y}_{2} \\
\dot{y}_{1}
\end{array}\right]+\left[\begin{array}{ccc}
c_{1} & 0 & c_{z}(t) \\
0 & c_{2} & -c_{z}(t) \\
0 & 0 & c_{z}(t)
\end{array}\right] \cdot\left[\begin{array}{c}
y_{1} \\
y_{2} \\
f(y)
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
f
\end{array}\right] .
$$

According to [2], a dimensionless form of the Eq. (22) can be obtained, by assuming following simplifications. Let:

$$
\begin{align*}
& e(t)=e_{a} \cos \left(\omega t+\varphi_{a}\right)+e_{b} \cos \left(\omega t+\varphi_{b}\right)  \tag{23}\\
& c_{z}(t)=c_{m}+c_{a} \cos (\omega t+\varphi), \quad \overline{c_{a}}=\frac{c_{a}}{c_{m}} \tag{24}
\end{align*}
$$

where $\omega$ is the main excitation frequency of the transmission error respective of the stiffness of the gear transmission.

$$
\begin{gather*}
\overline{f\left(y_{i}\right)}=\overline{y_{i}}(t)=\frac{y_{i}(t)}{b} ; \quad \omega_{i}=\sqrt{\frac{c_{i}}{m_{i}}} \quad \mathrm{i}=1,2  \tag{25}\\
\bar{y}(t)=\frac{y(t)}{b} ; \quad \omega_{n}=\sqrt{\frac{c_{m}}{m_{r e d}}} \quad ; \quad \bar{t}=\omega_{n} t  \tag{26}\\
\zeta_{11}=\frac{d_{1}}{2 m_{1} \omega_{n}} ; \zeta_{13}=\frac{d_{z}}{2 m_{1} \omega_{n}} ; \xi_{22}=\frac{d_{2}}{2 m_{2} \omega_{n}} ; \xi_{23}=\frac{d_{z}}{2 m_{2} \omega_{n}} ; \xi_{33}=\frac{d_{z}}{2 m_{r e d} \omega_{n}} ;  \tag{27}\\
c_{11}=\frac{\omega_{1}^{2}}{\omega_{n}^{2}} ; c_{22}=\frac{\omega_{2}^{2}}{\omega_{n}^{2}} ; c_{13}=\frac{m_{r e d}}{m_{1}} ; c_{23}=\frac{m_{r e d}}{m_{2}} ; c_{33}=1+\bar{k}_{a} \cos (\overline{\omega t}+\varphi) ; \tag{28}
\end{gather*}
$$

with the excitation frequency vector $\quad \bar{\omega}=\frac{\omega}{\omega_{n}}$

$$
\begin{equation*}
\bar{F}_{m}=\frac{F_{m}}{m_{r e d} b \omega_{n}^{2}} ; \quad \quad F_{a}=\frac{e_{a}}{b}(\bar{\omega})^{2} ; \quad \quad F_{b}=\frac{e_{b}}{b}(\bar{\omega})^{2} \tag{29}
\end{equation*}
$$

So that:

$$
\begin{equation*}
\bar{F}=\overline{F_{m}}+\overline{F_{a}} \cos \left(\overline{\omega t}+\varphi_{a}\right)+\overline{F_{b}} \cos \left(\overline{\omega t}+\varphi_{b}\right) \tag{31}
\end{equation*}
$$

Eq. (22) can thus be written in dimensionless form:

$$
\left[\begin{array}{ccc}
1 & 0 & 0  \tag{32}\\
0 & 1 & 0 \\
-1 & 1 & 1
\end{array}\right] \cdot\left[\begin{array}{l}
\frac{\ddot{y}_{1}}{}(t) \\
\overline{\ddot{y}_{2}}(t) \\
\overline{\ddot{y}}(t)
\end{array}\right]+2 \cdot\left[\begin{array}{ccc}
\xi_{11} & 0 & \xi_{13} \\
0 & \xi_{22} & -\xi_{23} \\
0 & 0 & \xi_{33}
\end{array}\right] \cdot\left[\begin{array}{c}
\overline{\dot{y}}_{1}(t) \\
\frac{\dot{y}_{2}(t)}{\dot{\dot{y}}^{\prime}(t)}
\end{array}\right]+\left[\begin{array}{ccc}
c_{11} & 0 & c_{13} c_{33} \\
0 & c_{22} & -c_{23} c_{33} \\
0 & 0 & c_{33}
\end{array}\right] \cdot\left[\frac{\overline{f\left(y_{1}\right)}}{\overline{f\left(y_{2}\right)}} \overline{f(y)}\right]=\left[\begin{array}{c}
0 \\
0 \\
\bar{F}
\end{array}\right],
$$

with:

$$
\overline{f(y)}=\frac{f(y)}{b}=\left\{\begin{array}{cc}
y-1 & \bar{y}>1  \tag{33}\\
0 & -1 \leq \bar{y} \leq 1 \\
y+1 & \bar{y}>-1
\end{array}\right.
$$

For simplifying reasons, the "-" sign above the variables in the Eq. (32) and (33) will be neglected and, therefore, the equation (32) becomes:

$$
\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
-1 & 1 & 1
\end{array}\right] \cdot\left[\begin{array}{l}
\ddot{y}_{1}(t) \\
\ddot{y}_{2}(t) \\
\ddot{y}(t)
\end{array}\right]+2 \cdot\left[\begin{array}{ccc}
\xi_{11} & 0 & \xi_{13} \\
0 & \xi_{22} & -\xi_{23} \\
0 & 0 & \xi_{33}
\end{array}\right] \cdot\left[\begin{array}{c}
y_{1}(t) \\
y_{2}(t) \\
y(t)
\end{array}\right]+\left[\begin{array}{ccc}
c_{11} & 0 & c_{13} c_{33} \\
0 & c_{22} & -c_{23} c_{33} \\
0 & 0 & c_{33}
\end{array}\right] \cdot\left[\begin{array}{l}
f\left(y_{1}\right) \\
f\left(y_{2}\right) \\
f(y)
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
\frac{F}{F}
\end{array}\right] \text { (34) }
$$

As a first step, the three second order differential equation are converted in six first order differential equations by using the Runge-Kutta method.

For this purpose, $q$ variable is introduced as follows:

$$
\begin{gather*}
q=\left\{\begin{array}{lllll}
q_{1}, & q_{2}, & q_{3}, & q_{4}, & q_{5}, \\
q_{6}
\end{array}\right\}^{\top}=\left\{\begin{array}{lll}
y_{1}, & \dot{y}_{1}, & y_{2}, \dot{y}_{2}, \\
\text {, } & y, \dot{y}
\end{array}\right\}^{T}  \tag{35}\\
\text { Thus, Eq. (34) can be written in matrix form as follows: } \\
\dot{q}=H \cdot q+c_{33} \cdot f\left(q_{5}\right) \cdot A+B
\end{gather*}
$$

e nonlinear coefficients:

$$
\begin{equation*}
A=\left\{0,-c_{13}, 0, c_{23}, 0,-c_{13}-c_{23}-c_{33}\right\}^{\top} \tag{37}
\end{equation*}
$$

$B$ is the vector of load (forces):

$$
B=\left\{\begin{array}{llllll}
0 & 0 & 0 & 0 & 0 & F \tag{38}
\end{array}\right\}^{\top}
$$

and H is the matrix of the linear coefficients:

$$
H=\left[\begin{array}{cccccc}
0 & 1 & 0 & 0 & 0 & 0  \tag{39}\\
-c_{11} & -2 \xi_{11} & 0 & 0 & 0 & -\xi_{13} \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -c_{22} & -2 \xi_{22} & 0 & \xi_{23} \\
0 & 0 & 0 & 0 & 0 & 1 \\
-c_{11} & -2 \xi_{11} & c_{22} & 2 \xi_{22} & 0 & -\xi_{13}-\xi_{23}-2 \xi_{33}
\end{array}\right]
$$

Eq. (36) can be solved by using the MATLAB software, with which can be also performed a simulation by using the facilities offered by the SIMULINK tool.

## 4. Dynamic Model with Six Degrees of Freedom (6DOF)

When the influence of the driving, respective driven machine cannot be neglected, it has to be choosing a mathematical model with six degrees of freedom.

Such a model is shown in figure 3.


Figure 3. Dynamic model with six degrees of freedom (6DOF)
Generalizing those presented in the chapters 2 and 3 of this paper, as well as on the theoretical considerations of the forced damped vibration, it can be concluded that the equation of motion of a dynamic system, which includes a gear transmission can be written in following form:

$$
\begin{equation*}
[M] \cdot\{\ddot{\bar{G}}\}+[D] \cdot\{\dot{\bar{G}}\}+[C] \cdot\{\bar{q}\}=\{\bar{F}\} \tag{40}
\end{equation*}
$$

where:
[M]- Matrix of masses;
[D]- Matrix of damping's;
[C]- Matrix of stiffness;
$\bar{q}$ - vector of deplacements; $\bar{F}$-vector of force.
For the 6 DOF dynamic model, the elements of Eq. (40) can be written, according to [5], as follows:

$$
[M]=\frac{1}{m_{\text {red }}}\left[\begin{array}{cccccc}
\frac{4 \cdot J_{M_{0}}}{r_{b 1}^{2}} & 0 & 0 & 0 & 0 & 0  \tag{41}\\
0 & \frac{4 \cdot J_{1}}{r_{b 1}^{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{4 \cdot J_{2}}{r_{b 1}^{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{4 \cdot J_{M a}}{r_{b 1}^{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & m_{1} & 0 \\
0 & 0 & 0 & 0 & 0 & m_{2}
\end{array}\right] \text {, }
$$



$$
[C]=\frac{1}{c_{z}}\left[\begin{array}{cccccc}
\frac{4 \cdot C_{M o}}{r_{b 1}^{2}} & -\frac{4 \cdot C_{M O}}{r_{b 1}^{2}} & 0 & 0 & 0 & 0 \\
& \left(\frac{4 \cdot C_{M o}}{r_{b 1}^{2}}+\frac{c_{z}}{4}\right) & -\frac{c_{z} \cdot r_{b 2}}{4 \cdot r_{b 1}} & 0 & \frac{c_{z}}{2} & -\frac{c_{z}}{2} \\
& & \left(\frac{4 \cdot C_{M a}}{r_{b 1}^{2}}+\frac{c_{z} \cdot r_{b 2}}{4 \cdot \cdot r_{b 1}}\right) & -\frac{4 \cdot C_{M a}}{r_{b 1}^{2}} & -\frac{c_{z} \cdot r_{b 2}}{2 \cdot r_{b 1}} & \frac{c_{z} \cdot r_{b 2}}{2 \cdot r_{b 1}} \\
& & \frac{4 \cdot C_{M a}}{r_{b 1}^{2}} & 0 & 0 \\
& - \text { symmetric - } & & & c_{1}+c_{z} & -c_{z} \\
& & & & & c_{2}+c_{z}
\end{array}\right] ;
$$

$$
\bar{q}=\left\{\begin{array}{llllll}
\varphi_{M 0} & \varphi_{1} & \varphi_{2} & \varphi_{M a} & y_{1} & y_{2} \tag{44}
\end{array}\right\}^{\top} .
$$

## 5. Conclusions

Mathematical models attempt to include the essential parameters of natural phenomena in systems of equations or in systems of differential equations in order to predict the evolution of the observed system.

The basic principle in formulating a scientific model (modeling) is to reduce complexity, by trying to make the truth describable and understandable through simplicity.

The present paper presented relevant aspects regarding the dynamic modeling of gear transmissions. Starting with the simplest model (1DOF), developing it by considering factors as bearing, shaft, driving and driven machine, until the mathematical model with six degrees of freedom (6 DOF) was reached.

## Acknowledgment

The work has been funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/132395.

## References

[1] Buechner S., Zschaeck S., Amthor A., Ament C., Dynamic model and identification of a spur gear system, Proceeding of $15^{\text {th }}$ IASTED International Conference on Control and Applications (CA 2013), pp. 43-48, 2013.
[2] Diehl J.E., Tang J., DeSmidt H., Gear fault modeling and vibration response analysis, ASME 2012 5th Annual Dynamic Systems and Control Conference, Florida, 2012.
[3] Gillich G.R., Machine dynamics- Modelling of technical systems, ISBN 973-8466-51-2, Editura AGIR, Bucureşti, 2003.
[4] Korka Z., An overview of mathematical models used in gear dynamics, Romanian Journal of Acoustics and Vibration, vol. IV, No. 1, pp. 43-50, 2007.
[5] Korka Z., Research on vibration reduction in operation of cylindrical gearboxes, Ph.D. Thesis, "Eftimie Murgu" University, Reşiţa, 2009.

## Addresses:

- Lect. PhD. Eng. Zoltan Iosif Korka, "Eftimie Murgu" University of Resita, Piata Traian Vuia, nr. 1-4, 320085, Resita, z.korka@uem.ro
- PhD. Eng. Ion-Cornel Mituletu, "Eftimie Murgu" University of Resita, Piata Traian Vuia, nr. 1-4, 320085, Resita, mituic@yahoo.com

