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Modeling and Experimental Studies on Pre-Loaded Reinforced Concrete Beams Strengthened by External Reinforcement

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Abstract: Reinforcement and strengthening of reinforced concrete beams by unbounded external reinforcement is one of the methods of fortification used after loading and prior to failure of the beams. This method is used in different forms to strengthen members of reinforced concrete structures. To investigate the effect of fortification on cracked reinforced concrete beams, numbers of reinforced concrete beams were selected for testing. Strengthening was examined by attaching external reinforcing bars on both sides of the beams, at the level of internal flexural tensile reinforcement and by means of deflectors. The investigation was carried out through experimental data analysis and modeling using ANSYS finite element structural software. The results showed that the method of fortification used has increased the flexural capacity of the beams. It was also concluded that this method is suitable for strengthening of beams under their dead loads.

Key words: Strengthening · External reinforcing bars · Pre-loaded · Reinforced concrete beams · Nonlinear analysis · ANSYS

INTRODUCTION

Building structures, in their life time span, may need fortification due to different causes like design errors, change of application, lack of proper construction practices and damages due to aging, environmental effect, war and/or earthquakes. However, due to economic and cultural considerations, strengthening of structural members has advantages over substitution or reconstruction of these members. Strengthening of buildings include strengthening of columns, beams, wall joints and structural frames.

One of the methods of strengthening of reinforced concrete beams that is being used recently is strengthening by means of unbounded external reinforcement [1]. In this method external reinforcing bars are installed on both sides of the beam in the level of internal flexural tensile reinforcement. External reinforcing bars are secured to both sides of the beam using u-shaped steel deflectors at specific locations along the beam length, to achieve the approximate flexural displacement profile (deflections) of external reinforcing bars and the beam. They are also anchored at both ends of the beam by means of steel plates and bolts.

For the first time, Farooq [1] has tested and analyzed 30 full scale simply supported reinforced concrete beams, strengthened by unbounded external reinforcing bars, loaded to complete failure. The behavior of tested reinforced concrete beams was related to parameters like the percentage of internal flexural tensile reinforcement and external reinforcement, effective depth of external reinforcing bars, the application of deflectors, cylindrical compressive strength of concrete, shear span and the beam span. The results showed that strengthening by unbounded external reinforcing bars has increased the flexural capacity of reinforced concrete beams. In addition, Abdollahi [2] has evaluated the experimental results of Farooq's investigation. He used ANSYS finite element

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structural software to model reinforced concrete beams strengthened by unbounded external reinforcing bars and used nonlinear analysis to calculate ultimate capacity and ductility of the beams.

Another independent work, Erfanain [3] has investigated the behavior of reinforced concrete beams strengthened by unbounded external reinforcing bars experimentally. In Farooq's experiment the unbounded external reinforcing bars were attached on both sides of the beams at the level of internal flexural tensile reinforcement, whereas in Erfanian's experiment external bars were located at the bottom of the beams. In the later method the results showed a remarkable increase in flexural capacity and decrease in ductility of the beam.

This research studied strengthening of cracked pre-loaded simply supported reinforced concrete beams using unbounded external reinforcing bars. The external bars were anchored at both ends of beams by means of steel plates and bolts twisted manually by ranch. This means that a certain amount of tensile load was applied to external bars and therefore an equal amount of compressive load was applied to beams at ends prior to the test. When the load is applied and the beam deflects, tensile load is applied to external bars which are translated to a compressive load applied to the ends of the beam. It is not always practical to remove the loads from reinforced concrete beams which need strengthening. Therefore, to study the effect of this method of strengthening, the experiments were carried out on the beams which were under certain amount of sustained load prior to the strengthening.

In order to investigate the effect of different parameters on the behavior of strengthened beams, present research was carried out both experimentally and by analysis of finite element modeling using ANSYS finite element structural software. To conduct experimental study of ultimate flexural capacity and ductility of beams, full scale simply supported normal reinforced concrete beams and cracked reinforced concrete beams, strengthened by unbounded reinforcing bars, were designed, fabricated and tested to failure. The beams were also modeled by ANSYS finite element structural software and analyzed by nonlinear static analysis. The important parameters in this work included: the amount of pre-load applied prior to strengthening, the amount of pre-load applied during strengthening, the percentage of internal flexural tensile reinforcement and the percentage of external reinforcing bars.

Results from experiments and analysis of modeled beams by ANSYS, showed an increase in flexural capacity of beams. They also showed the amount of strengthening was in reverse proportion with the percentage of internal flexural tensile reinforcement. In addition, the deformability of strengthened beams was also in reverse proportion with the percentage of dead load and extent of cracks in beams. Moreover, it was concluded that this method of strengthening is a proper method of strengthening reinforced concrete beams, even in presence of large dead loads and large amounts of the pre-load and cracks.

- Analytical expressions governing the structural behavior of strengthened reinforced concrete beams

When the beam is to be strengthened without the removal of its existing loads, there exist some initial deflections and longitudinal strains in the beam. Therefore, strengthening is carried out while the beam has experienced sustained deflection and longitudinal strain to some extent. After strengthening because of pre-tensile load in external reinforcing bars and therefore, pre-compressive load at both ends of the beam, the beam deflection and its longitudinal strain is reduced to some extent accordingly. For the case when external reinforcing bars are located at the level of internal flexural tensile reinforcement, the following assumptions are used to simplify the used analysis [4].

- The behavior of concrete in compression is linear elastic.
- The tensile strength of concrete is negligible.

The equilibrium condition for loads and compatibility condition for deformations should also be satisfied. To satisfy the equilibrium condition, the net force in any section of the beam under pure flexure should be zero. This is presented by equation (1). The internal forces should also be in equilibrium with flexural moment (see equation (2)). In addition, the compatibility of deformations along the beam length should be satisfied according to equation (3). From equilibrium of loads, moment and compatibility of deformations along the beam length we have:

$$A_{sb} f_{sb} + A_{sub} f_{sub} + \frac{1}{2} f_c b x = 0$$

(1) Equilibrium condition of loads

$$Z(A_{sb}f_{sb} + A_{sub}f_{sub}) = M$$

(2) Equilibrium condition of moments

$$\int_0^L \varepsilon_{sb} dl = \int_0^L (\varepsilon_{sub} + \varepsilon_{sb_0}) dl = \int_0^L \varepsilon_c dl$$

(3) Compatibility condition of deformations along the beam length at the level of internal re-bars

where:

f_{sub}, f_{sb} = Stress in unbounded and bounded reinforcement, respectively (N/mm^2),

A_{sub}, A_{sb} = Cross-sectional area of unbounded and bounded reinforcement, respectively (mm^2),

f_c = Cylindrical compressive strength of concrete (N/mm^2),

x = The distance of the extreme compressive fiber of concrete from neutral axis of the beam (*mm)*,

z = The internal lever arm *(mm)*,

M = Flexural moment in specified section *(N-mm)*,

$\varepsilon_c, \varepsilon_{sub}, \varepsilon_{sb}$ = Strains in concrete at the level of reinforcement, in unbounded reinforcement and in bounded reinforcement, respectively

L = Length of beam *(mm)*

ε_{sb_o} = Primary strain in bounded reinforcement in the beginning of strengthening [4].

By Satisfying the above three equations, the amount of stress in bounded reinforcement at service load, the amount of deflection at service load, ultimate stress in unbounded reinforcement and ultimate flexural moment of the section will be obtained [4]. The algorithm of the calculation of ultimate load and deflection at service load in these beams is reported in the literature [5].

The ultimate flexural capacity of the section can be finally derived from the following equation [6].

$$M_u = M_u^{'} f_{cu} b d^2 \tag{4}$$

where:

$$M_u^{'} = Q_{ub}\left[\frac{d_{ub}}{d} - \frac{K_2}{K_1}(Q_{ub} + 2Q - 2Q_c)\right] + Q_b\left[1 - \frac{K_2}{K_1}(Q_b - 2Q_c)\right] - Q_c\left[\frac{K_2}{K_1}Q_c + \frac{d'}{d}\right] \tag{5}$$

$$Q_c = \rho_c \cdot f_{yc} / f_{cu} \tag{6}$$

$$Q_b = \rho_b \cdot f_{yb} / f_{cu} \tag{7}$$

$$Q_{ub} = \frac{Q_t + K_3}{(1 + \frac{K_3}{K_1}\frac{d}{d_{ub}})} - \frac{K_3 d Q_b}{K_1 d_{ub} + K_3 d} + \frac{K_3 d Q_c}{K_1 d_{ub} + K_3 d} \tag{8}$$

$$K_3 = \frac{E_s \beta (\varepsilon_c - \varepsilon_{cc0}) \rho_{ub} d_{ub}}{L f_{cu}}, \quad \begin{cases} f_{cu}^e \le 30Mpa \to \beta_1 = 0/85 \\ f_{cu} > 30Mpa \to \beta_1 = 0/85 - 0/008(f_{cu}^0 - 30) \ge 0/65 \end{cases} \tag{9}$$

$$Q_t = \rho_{ub} \cdot f_{st} / f_{cu} \tag{10}$$

$$\beta = \frac{f_{sub} - f_{st}}{\frac{(\varepsilon_c - \varepsilon_{cc_0}) E_s d_{ub}}{L}\left[1 - \frac{d}{d_{ub} K_1 f_c'}(\rho_{ub} f_{sub} + (\rho_b f_{yb} - (\rho_c f_{yc})\right]} \tag{11}$$

K_1 and K_2 can be determined from both BS8110 and ACI. The detail of the parameters used in the above equations can be found in literature [6].

Knowing M_u, the values for P_u and Δ can be determined from the following equations:

$$P_u = \frac{2M_u}{Q_u} \tag{12}$$

$$\Delta = \frac{KL_1^2 M_u}{E_c I} \tag{13}$$

The above analytical expressions were not used in present research for estimation of failure load. The derived formulations may need to develop for nonlinear structures subjected in failure load.

Experimental Specimen and Procedure: In order to investigate the strength and ductility of pre-loaded reinforced concrete beams strengthened by external unbounded (reinforcing) bars, 12 beams in three different groups with different percentages of internal flexural tensile reinforcement were designed and fabricated for loading and recording the test results.

The beams were designed, fabricated and tested in three groups A, B and C, where group A beams included beams A, A_1, A_2 and A_3; group B beams included beams B, B_1, B_2 and B_3 and finally group C beams, included beams C, C_1, C_2 and C_3. The difference between beam groups was in the amount of internal flexural tensile reinforcement. The cross sectional area and beam spans were designed for beams to behave in flexure. The cross sectional area of all beams were 20 by 20 cm. Tensile flexural reinforcement were 3 number 16, 3 number 14 and 2 number 14 bars in beam groups A, B and C, respectively [7]. Beam cross sections, reinforcements and details of strengthening of beams are shown in Figure (1).

Beams A, B and C are reference beams in their group and are tested to failure without being strengthened. In this research, the behavior of 12 simply supported reinforced concrete beams in three different groups with different percentages of internal flexural tensile reinforcement were studied. One beam in each group was used as a reference beam. The other three beams in each group were strengthened by external reinforcing bars, while subjected to sustained constant loads, amounting to different percentages of their calculated ultimate capacity. The beams were then loaded to their ultimate capacity. It should be noted that 4 beams were primarily loaded to the calculated yielding of their internal flexural tensile load, unloaded to the calculated service dead load, strengthened at this dead load and finally loaded to failure [7].

Concrete mix design was used to achieve a 28 days compressive strength of 300 kg/m^2. To design external reinforcing bars it should be noted that decrease in internal flexural tensile reinforcement resulted in an increase in the capacity of the required strengthening capacity. This means that reducing internal bars resulted in an increase in the external reinforcement. To design the external bars the design diagrams presented in the literature were used [1]. Group A beams were designed for 25% increase in strength which required two number 12 bars, group B beams were designed for 40% increase in capacity requiring two number 14 bars and finally group C beams were designed for 90% capacity increase by using two number 16 bars. To study the effect of the percentage of external reinforcing bars on the strength of strengthened beams, two number 14 bars where used in C_2 beam. The specifications of the experimental beams are shown in Table 1.

The strengthening is carried out by attaching two bars on the sides of the beam at the level of internal reinforcement. Since the external bars were attached to

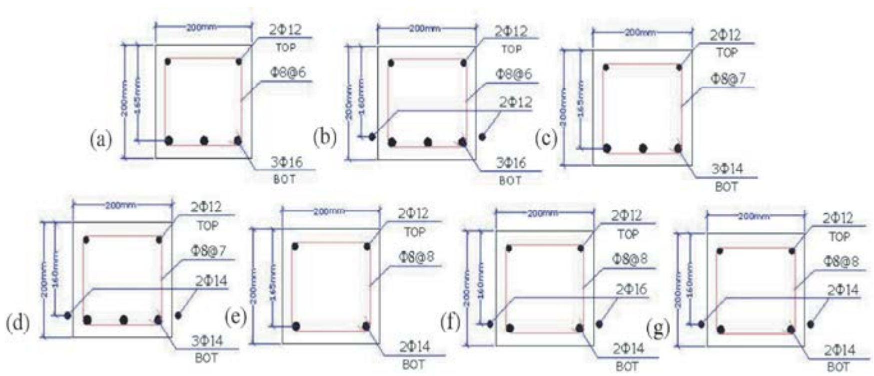

Fig. 1: Cross sections of tested beams, a) beam A, b) beams A_1, A_2 and A_3, c) beam B, d) beams B_1, B_2 and B_3, e) beam C, f) beams C_1 and C_3 and g) beam C_2.

Table 1: Specifications of experimental beams

Specimen name	Specimen type	Internal bounded reinforcing bars: Number	Diameter	Yield strength kg/cm^2	External unbounded reinforcing bars: Number	Diameter	Yield strength kg/cm^2	The compressive cylindrical strength of concrete kg/cm^2	The percentage of pre-load in the beginning of strengthening %	The percentage of primary pre- load %
A	Reference	3	16	4200	-	-	-	376.8	-	-
A_1	Strengthened	3	16	4200	2	12	3549	396.4	65.4	65.4
A_2	Strengthened	3	16	4200	2	12	3549	365.4	77	77
A_3	Strengthened	3	16	4200	2	12	3549	428.8	62	95
B	Reference	3	14	4269	-	-	-	310.4	-	-
B_1	Strengthened	3	14	4269	2	14	4269	392	68	68
B_2	Strengthened	3	14	4269	2	14	4269	389.6	77.7	77.7
B_3	Strengthened	3	14	4269	2	14	4269	341.2	68	97
C	Reference	2	14	4269	-	-	-	363.2	-	-
C_1	Strengthened	2	14	4269	2	16	4200	367.2	67	67
C_2	Strengthened	2	14	4269	2	14	4269	364.6	67	93.3
C_3	Strengthened	2	14	4269	2	16	4200	332	67	93

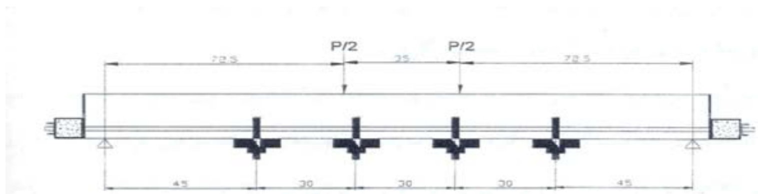

Fig. 2: The location of deflectors and applied loads

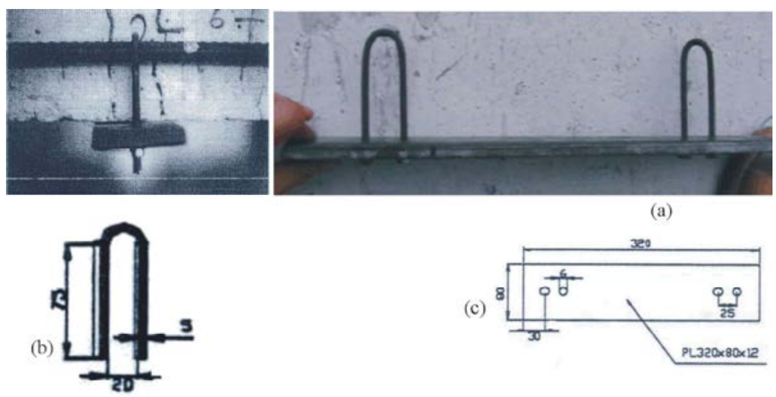

Fig. 3: Details of deflectors, a) used in the lab, b)deflector, c) MS plate

the beam at the ends only, to achieve the same deflection profile of the beam and external bars along the beam span, 4 deflectors are used to attach them to the beam [7] (Figure (2)). Figure (3) shows the details of deflectors.

The experiments were completely carried out in the structural lab of Babol Noshirvani University of Technology. After the loading of reference beams, beams A_1, B_1 and C_1 were loaded to 65 percent of the calculated ultimate load of their reference beams. At this stage, the beams were strengthened and were loaded to failure, sustaining the applied load. Beams A_2 and B_2 were also loaded in the same way, but the pre-loads (dead load) were 77 percent of the calculated ultimate loads of their reference beams. Beams A_3, B_3, C_3 and C_2 were pre-loaded to 93 percent of the calculated ultimate loads of their reference beams, unloaded to 65 percent of the calculated ultimate service loads, strengthened and finally loaded to failure.

Numerical Modeling of Beams Strengthened Beams: The ANSYS finite element structural software is used to conduct both simple analysis, like linear or elastic analysis and more complicated analysis, like nonlinear or dynamic analysis. Because of the applicability of the software to different engineering branches and in order to increase the speed of the process and reduce the space needed, the program is divided into groups and subgroups with their own finite elements, specifications and rules. This software, like other similar soft ware's, has three major sections: 1. construction of the model, 2. loading and analysis and 3. observing the results [8].

The most essential part of the model construction is the selection of proper elements. This program has 180 elements each with certain specifications and therefore, the selection of the element with needed specifications can be done rather easily. For loading and analysis parts, analysis type, loading cases and the conditions for analysis should be entered. The type of analysis is dependent upon loading and considered response. This program includes static, modal, harmonic, dynamic, transient, spectrum, semi-structural and flexural analysis. The results of analysis can be observed in two ways. One choice is to see the results of the whole model or part of it in the form of deformation of the model, table and/or colored curve and the other choice is to obtain the results corresponding to a specific point in the model in the form of curves with respect to time and also tables [9].

Modeling: In this section the geometry of the model is drawn and the types of elements are determined. A reinforced concrete beam strengthened with external reinforcement includes different elements that, the type of element, material properties and real constants should be defined for each of them. The constituent elements of this beam are [5]:

Steel support of external reinforcing bars, End plate, Steel support, MS plate, Steel loading Plate, Concrete beam, Compression reinforcement, Tension reinforcement, External reinforcing bars, Shear reinforcement, Deflectors.

Steel support of external reinforcing bars is a steel box filled with concrete for more strength and bolts are used to anchor the external bars in the lab. For modeling of this set in ANSYS, a rectangular steel plate is used which plays a role of anchorage for external reinforcing bar.

Details of this anchorage are shown in Figure (4) for two conditions.

End plate is an end plate of the area equal to that of beam cross-section and the thickness of 2 mm, as shown in Figure (5), was used to prevent stress concentration at both ends of the beam, caused by external reinforcement attachment.

At the supports, the beam was placed on the steel plate to prevent stress concentration; it is called plates steel support.

Also deflectors were used to achieve the same deflection profile in the beam and external reinforcing bar. For external reinforcing bars to attain the same deflection profile of the beam under the load, deflectors as shown in Figure (3) were used in the Lab. As shown in Figure (3), a deflector was composed of two parts, a plate which is located under the beam at specific locations and a u-shaped bar which is hanged on the external reinforcement and attached to the plate. These are called deflectors.

To model the deflectors, a plate and two reinforcing bars were used. These bars which were used to model the connector bars are called deflectors. Modeling of steel plate and its attachment to external reinforcing bars is shown in Figure (6).

To prevent concrete crack in the vicinity of the applied load, a load was placed on the steel plate which prevents stress concentration at that point and we call

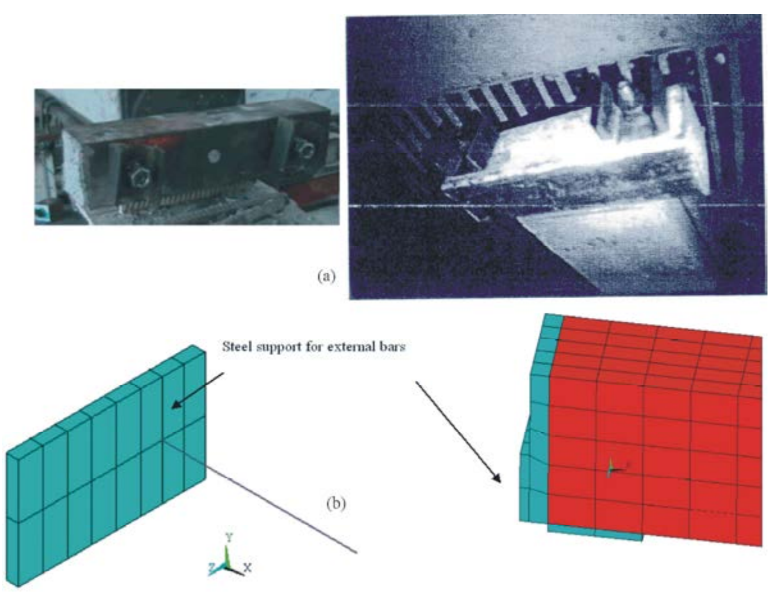

Fig. 4: Details of steel support of external reinforcing bar, a) in test beam, b) in the modeled beam

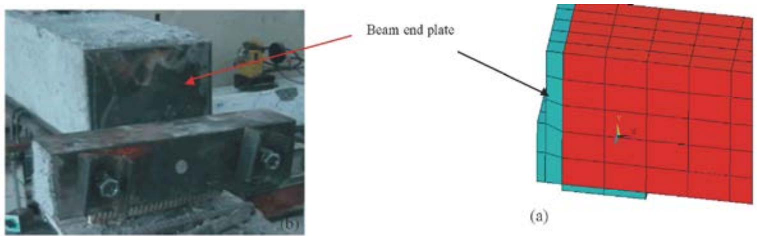

Fig. 5: Beam end plates, a) in test beams, b) in the model

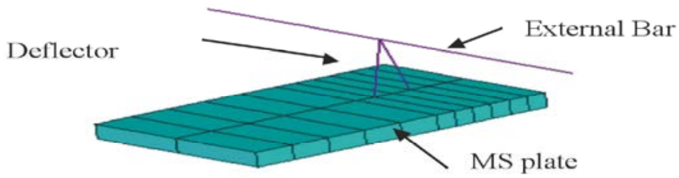

Fig. 6: ANSYS model of deflector

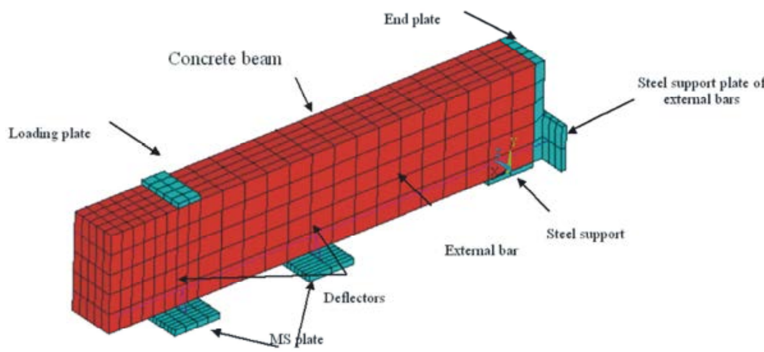

Fig. 7: Constituent elements of strengthened reinforced concrete beam in ANSYS

that steel loading plate. External reinforcing bars are strengthening reinforcing bars with threads at the ends which were attached to both sides of the beam, at the level of internal flexural reinforcement, by means of deflectors.

To prevent shear failure of the beams and to ensure flexural failure, all beams were reinforced by shear ties. All the details are shown in Figure (7).

The type of element, material properties and real constants of a strengthened reinforced concrete beam are show in Table (2).

The element *Solid65* which is used to model concrete, is a three dimensional element which is capable of cracking in tension and crushing in compression.

According to the reported literature [10] this element requires linear isotropic and multi-linear isotropic material properties for properly model concrete. The multi-linear isotropic material uses the Von Mises failure criterion along with the William and Warnke [11] model to define the failure of the concrete. Implementation of the William and Warnke [11] material model in ANSYS requires that different constants be defined. These 9 constants are:

- Shear transfer coefficients for open crack;
- Shear transfer coefficients for closed crack;
- Uniaxial tensile cracking stress;
- Uniaxial crushing stress;
- Biaxial crushing stress;
- Ambient hydrostatic stress state for use with constant 7 and 8;
- Biaxial crushing stress under the ambient hydrostatic stress state;
- Uniaxial crushing stress under the ambient hydrostatic stress state;
- Stiffness multiplier for cracked tensile condition.

Typical shear transfer coefficients range from 00 to 10 which representing a smooth crack and rough crack, respectively. Razaghi [8] has recommended for open crack its range is from 0.15 to 0.3 and for closed crack is from 0.7 to 1.00. The uniaxial cracking stress was based upon the modulus of rupture. Also the uniaxial crushing stress in this model was based on the uniaxial unconfined compressive strength f_c and is denoted as f_t. It was entered as -1 to turn of the crushing capability of the

Table 2: Type of element, material properties and real constants of a strengthened reinforced concrete

Model parts	Element type	Material properties	Real constant
Steel support of ex-bar	Solid45	Linear isotropic	-
End plate	Soli`d45	Linear isotropic	-
Steal support	Solid45	Linear isotropic	-
MS plate	Solid45	Linear isotropic	-
Steal loading plate	Solid45	Linear isotropic	-
Concrete beam	Solid 65	Linear isotropic, Concrete, Multilinear isotropic	Properties of bar exist in solid65 element
Compression bar	Link8	Linear isotropic , Bilinear isotropic	Cross-sectional area , initial strain
Tension bar	Link8	Linear isotropic , Bilinear isotropic	Cross-sectional area , initial strain
External bar	Link8	Linear isotropic , Bilinear isotropic	Cross-sectional area , initial strain
Shear tie	Link8	Linear isotropic , Bilinear isotropic	Cross-sectional area , initial strain
Deflector	Link8	Linear isotropic , Bilinear isotropic	Cross-sectional area , initial strain

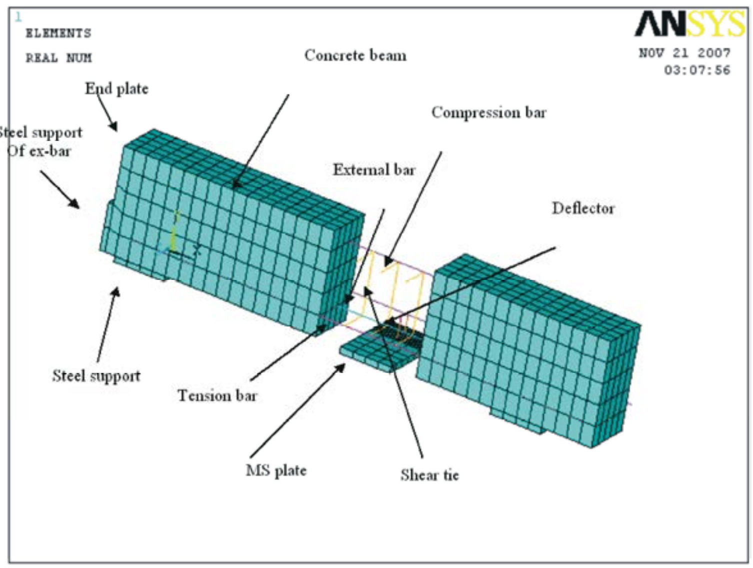

Fig. 8: ANSYS beam model

concrete element as suggested by past researchers [12]. Convergence problems have been repeated when the crushing capability was turned on. Other coefficients entered 0 as suggested by Wolanski [13].

In this element the reinforcement can be modeled by one dimensional rod with compressive and tensile behavior. This bar can be introduced in the middle of the element in all three directions [9]. As recommended by Razaghi *et al*. [8]. *Solid65* with zero percent reinforcement is used to model reinforced concrete beam. Tension and compression reinforcement and shear ties in the beam are separately modeled by using element *Link8*. To ensure the necessary conditions for transfer of load *Link8* element should be located between two or more elements modeled by *Solid65*, otherwise, the forces would be only transferred at the nodes of *Solid65* and the element *Link8* is not effective in representing a reinforcing bar, as if it does not exist [8]. On the other hand since all the tested beams are symmetrical in two directions, as it is recommended in the literature [3], only a quarter of the beam is modeled due to symmetry. This would save both process time and the memory capacity.

A modeled beam is shown in Figure (8).

Support Conditions and Loading: To ensure that modeled beam behave as test beam, boundary conditions at points of symmetry, supports and load points should be satisfied.

Boundary conditions for symmetry will be set first. The constructed model is symmetrical in two directions. To model the symmetry, points in the plane of symmetry should be constraint in the perpendicular direction [13]. The boundary conditions for both planes of symmetry are shown in Figure (9a).

Considering the connection between MS plate and concrete in the lab, it is seen that there was no connection between deflectors and concrete and they were just touched with each other so for modeling in ANSYS, firstly

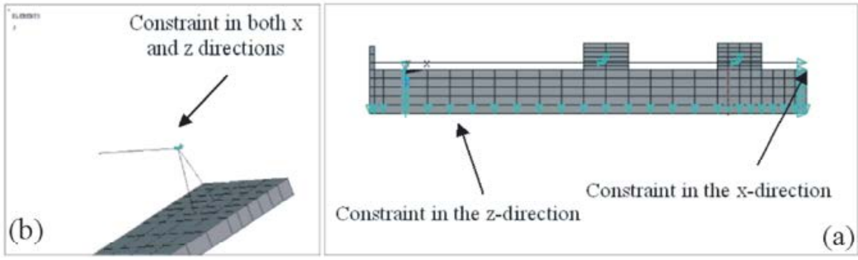

Fig. 9: Boundary conditions for planes of symmetry and the point of attachment of external strengthening reinforcing bars and deflector

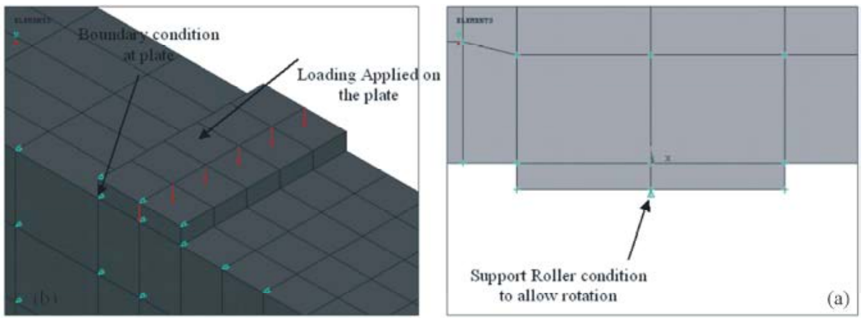

Fig. 10: a) Boundary conditions of the support, b) loading on the steel plates under concentrated load

Table 3: Nnumber of steps of loading in each beam

Number of steps of loading	One step loading	Two step loading	//Three step loading
Name of beam	Reference beams A,B and C	Strengthened beams A_1,,A_2,B_1,,B_2,C_1	Strengthened beams A_3,,B_3,C_2 and C_3

we created two volumes that have a common surface, then by using glue command, the connection was created. So no friction element was used.

To model the connection between external reinforcing bar and deflector, we consider a common node, since this node should only be displaced in vertical direction, therefore, all of the connecting nodes should be constraint in X and Z direction as shown in Figure (9b).

The support was modeled in such a way that a roller was created. A single line of nodes on the plate were given constraints in the UY and UZ directions and applied as constant values of 0. By doing this, the beam will be allowed to rotate at the support. The support condition is shown in Figure (10a).

The force, P, applied at the steel plate is applied across the entire centerline of the plate. Figure (10b) illustrates the plate and applied loading.

The load is applied in one, two, or three steps corresponding to the test beam type. The required number of steps of loading for each test beam is shown in Table (3). Figure (11) shows the way the loads are applied.

Analysis: To analyze the constructed models, static analysis is used and since the materials behavior is nonlinear, the nonlinear elastic analysis is carried out. Analysis is also carried out on the basis of small displacements. Finite element analysis is organized in a way that three different behaviors of material, primary crack of the beam, yielding of the internal flexural reinforcement and ultimate capacity of the beam, can be determined. Newton-Raphson method is used to consider nonlinear response [9].

It is important to note that test results and results from analysis using ANSYS software are not completely the same and the possibility of error always exists. It is possible that the calculated service or ultimate load of beams differ considerably with corresponding real (tested) values. This is because of many reasons including difference in design material properties and sizes of the beam cross sections to those of real beam. This means the applied load of 77% of the calculated load might not really be 77% of that load. The same is true for 65 and 93%. As a result the effect of strengthening cannot be measured accurately. It was therefore decided to use the percentage of the pre-load and not the percentage of

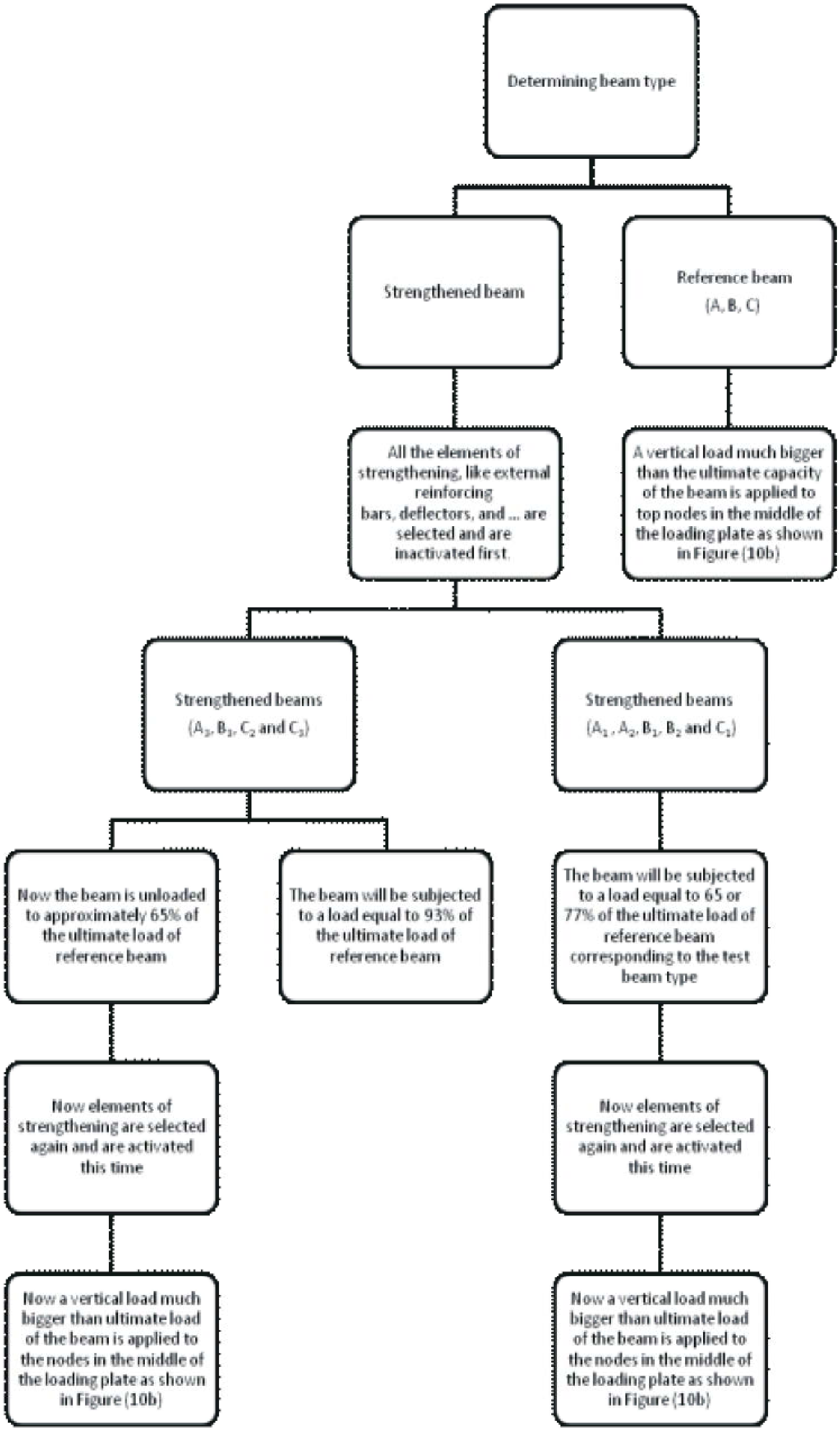

Fig. 11: Algorithm of how applying loads

Table 4: Ultimate capacity of strengthened beams prior to strengthening

Group name	Test beam name	Model name	Ultimate load *Ton*
A	A_1	*NFourteen*	10.338
	A_2	*Nfifteen*	10.91
	A_3	*Nsixteen*	9.62
B	B_1	*Neighteen*	10.066
	B_2	*Nnineteen*	8.22
	B_3	*Ntwenty*	9.216
C	C_1	*Ntwentytwo*	6.91
	C_2	*Ntwentythree*	7.35
	C_3	*Ntwentyfour*	6.517

Table 5: Results from ANSYS and tests

Test beam name	The percentage of pre-loading prior to strengthening %	The percentage of pre-loading when strengthening %	Type of beam	Ultimate load of beam *Ton* *Ansys*	*Exp*	The percentage of increase of strength *Ansys*	*Exp*	The percentage of error with respect to test
A	-	-	Reference	11.134	13	-	-	14.354
A_1	65.4	65.4	Strengthened	14.72	17	32.208	30.769	13.39
A_2	77	77	Strengthened	12.35	15.9	10.922	22.308	22.33
A_3	95	62	Strengthened	12.34	17.6	10.832	35.385	29.9
B	-	-	Reference	9.18	10.3	-	-	10.874
B_1	68	68	Strengthened	12.862	15	40.109	45.631	14.253
B_2	77.7	77.7	Strengthened	12.08	14.5	31.59	40.777	16.69
B_3	97	68	Strengthened	12.75	14	38.889	35.922	8.93
C	-	-	Reference	6.656	8.2	-	-	23.2
C_1	67	67	Strengthened	11.349	14.2	70.508	73.17	20.08
C_2	93.3	67	Strengthened	9.934	12.6	49.249	53.659	21.16
C_3	93	67	Strengthened	9.248	13.7	38.942	67.073	32.5

Table 6: Amount of load at first crack and at yielding of internal flexural tension reinforcement

Test beam name	Amount of load at first crack Ton *Ansys*	*Exp*	Amount of load at yielding of internal flexural tension reinforcement Ton *Ansys*	*Exp*
A	2.28	2	10.44	12
A_1	2	2	9.31	15~15.5
A_2	2.22	2	9.58	13
A_3	2.25	2	10.69	16.5
B	2.25	2	8	10
B_1	2.1	2	10.72	12.3
B_2	2.22	2	10.75	11
B_3	2.29	2	10.54	-
C	2.1	2	9.97	6~7
C_1	1.69	2	5.5	12
C_2	1.7	2	5.04	7~7.5
C_3	1.66	2	5.1	7.5

design load, as a major factor for study. On the other hand since ANSYS software has the capability of calculating the ultimate capacity of all beams prior to strengthening, in modeling of the strengthened beams, ultimate capacity of each beam is calculated first and then each test beam is subjected to the percentages of its own ultimate load. Table (4) shows the ultimate capacity calculated for strengthened beams prior to strengthening.

Test and Analytical Results For Beams: To verify the validity of modeling and analysis, analytical results obtained using ANSYS were compared to those obtained by test. A summery of the results is shown in Table (5).

The Amount of Load at First Crack and at Yielding of Internal Flexural Tension Reinforcement: Load-deflection diagrams can be used to determine linear region (from the beginning of the test to the attainment of the first crack of the beam), secondary linear region (from first crack to yielding of internal flexural tension reinforcement) and failure. The values of load at first crack and load at yielding of internal flexural tension reinforcement can also be determined using load-deflection diagrams. Table (6) shows the values of load at first crack and load at yielding of internal flexural tension reinforcement. The accuracy of the study can be obtained by comparison of load-deflection diagrams resulted from test and finite element

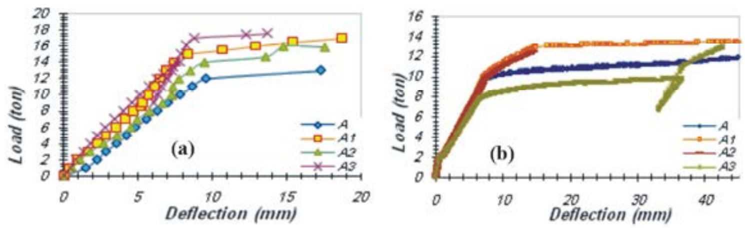

Fig. 12: Load-deflection diagram for group A beams, a) test beams, b) modeled beams

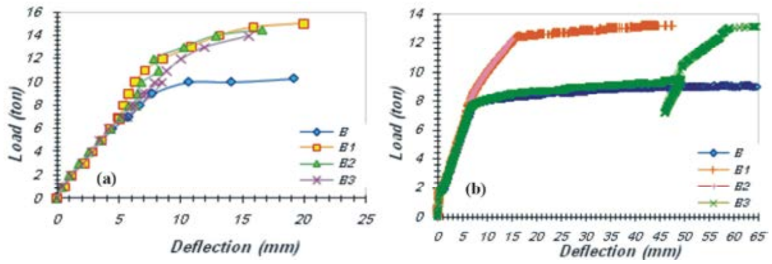

Fig. 13: Load-deflection diagram for group B beams, a) test beams, b) modeled beams

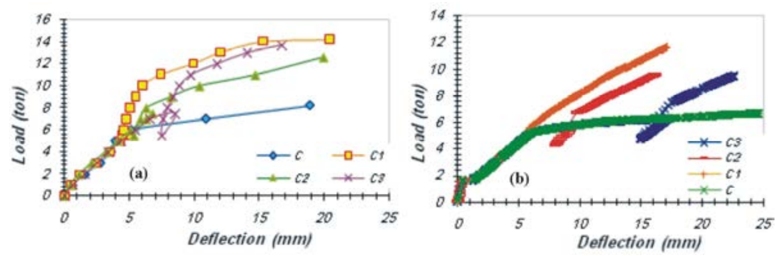

Fig. 14: Load-deflection diagram for group C, a) test beams, b) modeled beams

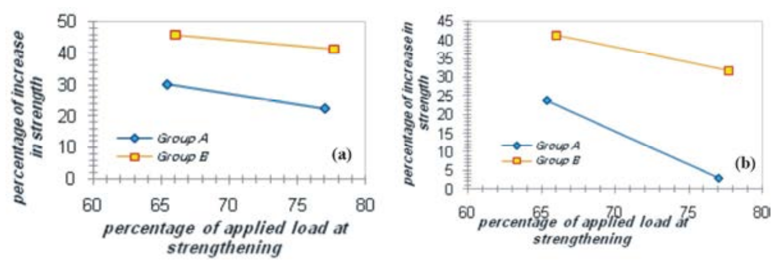

Fig. 15: Diagram of the percentage of strength increase of the beam versus the percentage of applied load at strengthening, a) test beams, b) modeled beams

analysis using ANSYS. Load- deflection diagram for group A, B and C were shown in Figures (12), (13) and (14), respectively. It can also be concluded that the rigidity of above beams were increased considerably.

Study of Strength of Specimen: Figure (15) shows the diagram of the percentage of increase in strength versus the percentage of the applied load at strengthening. The latter is a ratio of the percentage of dead load at strengthening of the beam and the ultimate load of reference beam. The diagrams show the decrease in the percentage of strength increase with respect to the increase of the applied load at the time of strengthening of the beam.

Figure (16) shows the changes in the percentage increase in strength versus the percentage of pre-loading for different percentages of internal flexural tension reinforcement in the beams. The beams were loaded to 65% of ultimate reference beam load at strengthening.

Figure (17) shows the effect of internal flexural tension reinforcement area on the percentage increase in strengthening of pre-loaded beam up to yielding of the internal flexural tension bars, while other beam parameters and the amount of external strengthening reinforcing bars are kept constant. In this diagram ρ_{ub} = 0.227 ρ_{bal} and $\frac{\rho b}{\rho bal} = K(b)$ show the ratio of internal flexural tension reinforcement and reinforcement at balance condition.

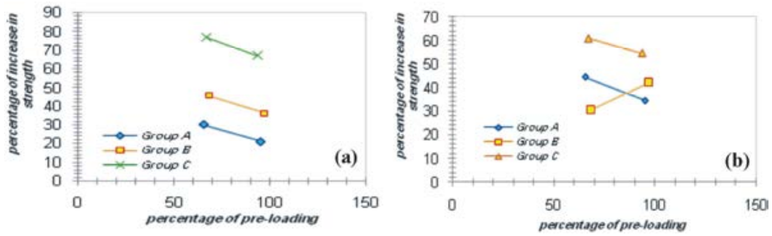

Fig. 16: Diagram of the percentage increase in strength versus the percentage of pre-loading at strengthening, a) test beams, b) modeled beams

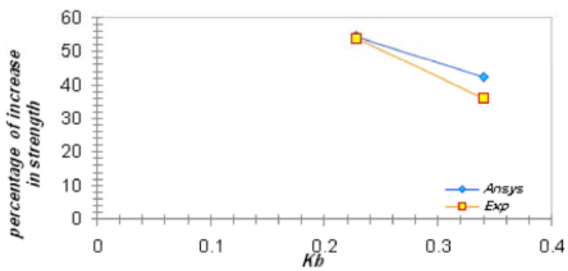

Fig. 17: Diagram of the percentage increase of strengthening versus the percentage of internal flexural tension reinforcement at $\frac{\rho_{ub}}{\rho_{bal}} = 0.227$

Study of Load-Deflection Diagrams of Specimens: Load-deflection diagrams for group A, B and C were shown in Figures (12), (13) and (14), respectively.

In load-deflection diagrams of reference beams, the beams show linear behavior up to the first crack of the beam and a nonlinear behavior after that. The deflection of the beam at the middle of the span will increase considerably with yielding of longitudinal reinforcing bars.

In strengthened beams, the load-deflection diagrams up to yielding of internal flexural tension reinforcement are similar to those of reference beams. But after yielding because of the existence of external strengthening bars, the rigidity of beams increase and depending on the amount of external strengthening reinforcing bars these external bars may yield first (if the amount of external reinforcement is low) or the concrete in compression region crushes (for large amount of external reinforcement).

Study of Specimens Rigidity: Load-deflection diagrams for group A, B and C were shown in Figures (12), (13) and (14), respectively.

Comparing load-deflection diagram of reference beam of each group with those of the rest of the beams in the same group, it can be concluded that existence of the external strengthening reinforcing bars causes a considerable decrease in deflection of the beam, i. e.; it causes an increase in beam rigidity and a decrease in beam ductility.

Study of Beams Pre-loaded over 90% of Their Ultimate Capacity: According to the results obtained in beams pre-loaded over 90% of their ultimate capacity (beams A_3, B_3, C_3 and C_2), it can be concluded that strengthening of beams by external strengthening reinforcing bars can be very effective and increases the strength of beams considerably. This holds true even in beams pre-loaded to approximately their ultimate capacity which has caused an extensive damages.

For beams pre-loaded to yielding of internal flexural tension reinforcement prior to strengthening and strengthened while sustaining 65% of ultimate load, the percentage increase in strength is decreased appreciably. This decrease is directly proportional to the amount of internal flexural tension reinforcement.

For beams pre-loaded to more than 90% of their ultimate load and then strengthened at the load approximately equal to 65% of their ultimate load, for equal amount of external strengthening reinforcing bars, the increase in strength is in reverse proportion with the amount of the internal flexural tension reinforcement.

CONCLUSION

The results of the study are summarized below:

- Strengthening of reinforced concrete beams by unbounded external reinforcing bars increases flexural strength of beams. The increase in strength is in reverse proportion with the percentage of internal flexural tension reinforcement.
- The percentage increase in strength of strengthened beams decreases with the increase in applied load at strengthening. This decrease in strength is in direction proportion with the amount of internal flexural tension reinforcement.

- The percentage increase in strength of pre-loaded beams is in reverse proportion with the amount of internal flexural tension reinforcement.
- The increase in strength increases with the increase in the percentage of external strengthening reinforcing bars.
- Strengthening of reinforced concrete beams by external strengthening reinforcing bars can be used as an effective method even in cases where beams are under load. The advantages of the method are; speedy application, simplicity of employment, almost no increase in weight of the structure and economic advantages.
- Since there exists close agreement between load-deflection diagrams of modeled and test beams, the models constructed can be used for future research and the model is valid for modeling of reinforced concrete beams strengthened by external strengthening bars.
- The results show that in most beams, there is a close agreement between the load-deflection diagrams of modeled and test beams, especially before the yielding of internal flexural tension reinforcement.

Notations:

a_v	Shear span	mm
A_{sb}	Cross sectional area of bonded reinforcement mm^2	
A_{sub}	Cross sectional area of unbounded reinforcement	mm^2
b	Width of beam	mm
d	Effective depth	mm
d'	Compression effective depth	mm
d_{ub}	Unbounded effective depth at ultimate load	mm
E_c	Elastic modulus of concrete	N/mm^2
E_s	Elastic modulus of steel	N/mm^2
f_c	Cylindrical compressive strength of concrete	N/mm^2
f_{cu}	Compressive strengths of concrete	N/mm^2
f_{sb}	Stress in bonded reinforcement	N/mm^2
f_{st}	Post tensioned stress	N/mm^2
f_{sub}	Stress in unbounded reinforcement	N/mm^2
f_{yb}	Yield strength of bonded tension reinforcement	N/mm^2
f_{yc}	Yield strength of compression reinforcement	N/mm^2
I	Moment of inertia of cracked section	mm^4
K	Defined factor	
K_1	Defined factor	
K_2	Defined factor	
K_3	Defined factor	
L	Length of beam	mm
L_1	Beam span	mm
M	Flexural moment in specified section	$N.mm$
M_u	Ultimate moment	$N.mm$
M'_u	Defined factor	
P_u	Ultimate load	N
Q_b	Defined factor	
Q_c	Defined factor	

Q_t	Defined factor	
Q_{ub}	Defined factor	
x	The distance of the extreme compressive fiber of concrete from neutral axis of the beam	*mm*
z	The internal lever arm	*mm*
β	Defined factor	
Δ	Deflection at service load	*mm*
ε_c	Strain in concrete at the level of reinforcement	
ε_{sb}	Strain in bounded reinforcement	
ε_{sub}	Strain in unbounded reinforcement	
ρ_b	Bonded tension reinforcement ratio	
ρ_c	Compression reinforcement ratio	
ρ_{ub}	unbounded reinforcement ratio	

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