

Analysis and Selection of Appropriate Aggregation Function for Calculating of Leachate Pollution Index of Landfill Lysimeter

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Abstract: A technique to quantify the leachate pollution potential of solid waste landfills on a comparative scale using an index known as the leachate pollution index (LPI) developed. The LPI is a quantitative tool by which the leachate pollution data of the landfill sites can be reported uniformly. It is an increasing scale index and has been formulated based on Delphi technique. The formulation process involved selecting variables, deriving weights for the selected pollutant variables, formulating their sub-indices curves and finally representing the pollutant variables to arrive at LPI. The aggregation function is one of the most important steps in calculating any environmental index. If aggregation function is ambiguous, the result will raise an unnecessary alarm, indicating a comparatively less polluted environmental situation as mere contaminated. Similarly, if the aggregation function is eclipsed a false sense of security may be created, indicating a highly polluted environmental situation as less polluted. In this paper, the concept of LPI is briefly described. In order to select the most appropriate aggregation function, various possible aggregation functions are described and used to calculate LPI values for pilot scale landfill lysimeter at KUET campus, Khulna, Bangladesh. Based on obtained results, it is concluded that the weighted linear sum aggregation function is the best possible aggregation function for calculating LPI. Sensitivity analysis of the six short-listed aggregation functions is performed to substantiate this conclusion.

Key words: Landfill lysimeter • Leachate • Aggregation • Leachate pollution index • Sensitivity analysis

INTRODUCTION

The term 'landfill' can be treated as synonymous to 'sanitary landfill' of MSW, only if the latter is designed on the principle of waste containment and is characterized by the presence of a liner and leachate collection system to prevent ground water contamination [1]. Sanitary landfill is one of the secure and safe facilities for the disposal of MSW. However, it needs high standard of environment protection in operation of landfill [2].

Lysimeter is a simulate form of sanitary landfill in the sense of control device. The word lysimeter is a combination of two Greek words "*Lusis*" means "*Solution*" and "*Metron*" means "*Measure*" and the original aim is to measure soil leaching [3]. Chemically contaminated leachate is one of the byproducts in landfill

degradation reactions. One of the severe problems associated with open dump of MSW, percolating leachate into the surrounding environment, subsequent contamination of land and water bodies [4].

Leachate pollution from closed and active landfills is an important issue, as it affects human health and the environment to a great extent. The leachate produced from a landfill may enter the underlying groundwater or the adjoining surface water bodies; can seriously degrade the water quality [5-7]. Once groundwater is contaminated, it is not desired to be used for drinking and domestic usage. It has already become necessary to shut down thousands of drinking water wells across the United States due to contamination from landfills. The problem is more acute in the developing nations, where the landfills do not have any base liners or leachate collection and treatment systems [3].

A strong need is presently being felt to take appropriate remedial measures to avoid contamination of the underlying soils and groundwater water aquifers from the leachate generated from the landfills. State regulatory authorities, in almost all the countries in the world, have framed regulations to safeguard against the contamination of groundwater sources from the leachate generated from the landfills [8]. A system needs to be developed to prioritize actions and to establish which landfills needs immediate attention for remediation works. An index for easy comparison of leachate contamination potential of landfills would be a useful tool in this regard. But necessary remedial and preventive measures can't be undertaken at all the existing closed and active landfill sites in one go because of financial constraints. In an effort to quantify the leachate pollution potential of the landfill sites, an index known as Leachate Pollution Index (LPI) was developed using Delphi technique [9].

The formulation of an environmental index involves four basic steps: (1) selection of variables; (2) derivation of weights; (3) formulation of their sub-indices equation and (4) aggregation of the sub-indices. The aggregation process is one of the most important steps. It is here where most of the simplification (reduction of information) takes place and most of the distortion is likely to be introduced [8].

In this paper, the concept of LPI was described in brief and various possible aggregation methods are reviewed and applied in an effort to select the most appropriate one for calculating LPI. The LPI based on leachate characteristics of a pilot scale landfill lysimeter at KUET campus, Khulna, Bangladesh were calculated using the various aggregation functions. Sensitivity analysis of six aggregation functions is also performed by the author to select the most appropriate aggregation function.

Concept of Leachate Pollution Index: In an effort to develop a system to compare the leachate contamination potential of various landfill sites in a given geographical area, 80 panelists, which included academicians in environmental engineering, environmental regulatory authority scientists, consulting engineers and members of International Solid Waste Association (ISWA) from around the world, were surveyed [9]. The survey was conducted using multiple questionnaires to develop a LPI.

- The index is a mathematical method of calculating a single value from multiple chemical and biological test results of the landfill leachate.

Table 1: Significance and weights of pollutant variables included in LPI [9]

Sl. No.	Pollutant	Significance	Pollutant weight
1	Total Chromium	4.057	0.064
2	Lead	4.019	0.063
3	COD	3.963	0.062
4	Mercury	3.923	0.062
5	BOD ₅	3.902	0.061
6	Arsenic	3.885	0.061
7	Cyanide	3.694	0.058
8	Phenol Compound	3.627	0.057
9	Zinc	3.585	0.056
10	pH	3.509	0.055
11	Total Kjeldahl Nitrogen	3.367	0.053
12	Nickel	3.321	0.052
13	Total Coliform Bacteria	3.289	0.052
14	Ammonia Nitrogen	3.250	0.051
15	Total Dissolved Solids	3.196	0.050
16	Copper	3.170	0.050
17	Chlorides	3.078	0.048
18	Total Iron	2.830	0.045
Total		63.165	1.000

- The single value LPI have a grade that expresses the overall leachate contamination potential of a landfill, based on several leachate pollution parameters at a given time.
- It is an increasing scale index, wherein a higher index value indices a poorer environmental condition.

Total of 18 leachate parameters were selected for inclusion in LPI and their weights factors based on significance levels given by the panelists on a scale of 1 to 5 and are summarized in Table 1. A selected group of panelists were asked to draw curves for the pollutant variables included in LPI with respect to leachate pollution ranging from 5 (the best) to 100 (the worst). Levels of leachate pollution from 0 to 100 were indicated on ordinate of each graph, while various levels of concentration of the particular variable, up to the maximum limits reported in literature, were indicated on the abscissa. The curves drawn by the panelists were averaged to obtain "average sub-index" curves for each parameter. The averaged sub-index curves are illustrated and reported in the literature [9].

Aggregation Function: Aggregation methods are crucial in the field of environmental indices, as they affect the quality of result in many ways. Aggregation has been defined as adding variables the same properties resulted in an overall value of individual component. Aggregation function usually consists of any of the following three forms:

- Additive form (summation function), in which individual variables are added together;
- Multiplicative form (multiplication function) in which a product is formed of some or all of the variables
- Maximum or minimum operator form, in which just the maximum or the minimum sub-index value of the variable directly accepted.
- An overestimation (ambiguity) problem, where the aggregate index I exceeds the critical level without any of the sub-indices exceeding the critical levels.
- An underestimation (eclipsing) problem, where the aggregate index I does not exceed the critical level despite one or more of the sub-indices exceeding the critical levels.

The type of aggregation function is selected based on the function and the use of the index. Because most of the air pollution indices reported in the literature are of the increasing scale form, they mostly use the additive form aggregation function [10-12] or the maximum operator form aggregation function [13]. Some of the water quality indices are of the decreasing scale from [14-18] and the others are of the increasing scale form [19]. The water quality indices, independent of their functional forms, use all three forms of aggregation functions. The additive form of aggregation function was used by Brown *et al.* [15] for developing a water quality index; by Horton [14] for Horton's Water Quality Index; by Prati and Pesarin [19] for Prati's Implicit Index of Pollution; and by Gilianovic [20] for the Water Quality Index for Dalmatia. The multiplicative form of aggregation function was used by Landwehr for the National Sanitation Foundation's Water Quality Index [13, 21] and by Walski and Parker [17] for Walski and Parker's Index. The minimum operator form of aggregation function was used by Smith [18] for developing a decreasing scale water quality index for New Zealand. Harkins [22] proposed an aggregation function based on Kendal's nonparametric multivariate ranking procedure for the National Sanitation Foundation water quality index developed by Brown *et al.* [15], which was later criticized by Landwehr and Deininger [23].

Procedure for Selecting Appropriate Aggregation Function: The following aspects are to be considered for selection of the appropriate aggregation method.

Functional Form of Index: An index can be an increasing scale index or a decreasing scale index. In case of an increasing scale index, usually called an environmental pollution index. The higher values of index indicate a worse state than lower values. In decreasing scale indices, higher values are associated with a better state than lower values. The decreasing scales are referred to as environmental quality indices.

Strength and Weakness of Aggregation Function: The two potential problems associated with aggregation functions are [13]:

These two problems crop up only with dichotomous sub-indices. The most appropriate aggregation function will minimize one or more both the overestimation and underestimation problems.

Parsimony Principle: When competing aggregation functions produce similar results w.r.t. overestimation and underestimation, the most appropriate aggregation function will be that which is mathematically simple [24].

Transparency of Aggregation Function: Finally, an aggregation approach is successful if all assumptions and sources of data are identified, the methodology is transparent and publicly and an index can be readily disaggregated into the separate components with no information lost [25]. In addition to the aforementioned procedure, the aggregation function selected for any environmental index shall also meet the following criteria. It should

- Be sensitive to the changes in an individual variable throughout its range;
- Not be biased towards good or poor environmental quality ;
- Consider weighting factors, as all variables included in the index are not equal contributors to environment pollution; and
- Be relatively easy to use.

Selecting Appropriate Aggregation Function for Leachate Pollution Index: To select the most appropriate aggregation for LPI, the various possible aggregation functions are applied by the author and also represents in Table 2. For the present study, the leachate characteristics of a pilot scale landfill lysimeter at KUET campus, Khulna, Bangladesh, have been considered represents in Table 3, column 3.

The leachate generated from the landfill lysimeter is neither collected nor treated at present. However, guidelines and standards do exist in India for the discharge of leachate from municipal landfills. The LPI values for the treated leachate are also calculated using all these aggregation functions.

Table 2: Aggregation functions used by different researchers for leachate pollution indices

Eq. No.	Aggregation function	Function expression	Users	Specific remarks	Reported Table/column
1	Unweighted Additive Form	$LPI_{ua} = \frac{1}{n} \sum_{i=1}^n p_i$	[15, 19, 26]	Ambiguous function; shows eclipsing region; simple but little flexibility; unsuitable for dichotomous sub-indices.	Table 3, Column 5
2	Weighted Linear Additive Form	$LPI_{wa} = \sum_{i=1}^n w_i p_i$	[14-16, 19, 27-31]	Ambiguity free functions; shows small eclipsing with large number of variables; not suitable for dichotomous sub-indices; widely used aggregation function.	Table 3, Column 6
3	Root Sum Power Additive form	$LPI_r = \left(\sum_{i=1}^n p_i^r \right)^{1/r}$	[31,32]	Shows reduced eclipsing but exhibit ambiguity problem; with increase in r , ambiguity decreases. If $r \rightarrow \infty$, it becomes ambiguity and eclipsity free function; use of aggregation function for $r > 2$ is not practiced for aggregation of water pollution indices.	
3(a)	Root Sum Power Additive form ($r=2$)	$LPI_2 = \left(\sum_{i=1}^n p_i^2 \right)^{1/2}$			Table 3, Column 7
3(b)	Root Sum Power Additive form ($r=4$)	$LPI_4 = \left(\sum_{i=1}^n p_i^4 \right)^{1/4}$			Table 3, Column 8
3(c)	Root Sum Power Additive form($r=10$)	$LPI_{10} = \left(\sum_{i=1}^n p_i^{10} \right)^{1/10}$			Table 3, Column 9
4	Weighted Root Sum Power Additive Form	$LPI_{wr} = \left(\sum_{i=1}^n w_i p_i^r \right)^{1/r}$	[31]	Exhibits slightly reduced ambiguity, unwidely used aggregation function.	
4(a)	Weighted Root Sum Power Additive Form ($r=4$)	$LPI_{w4} = \left(\sum_{i=1}^n w_i p_i^4 \right)^{1/4}$			Table 3, Column 10
4(b)	Weighted Root Sum Power Additive Form ($r=10$)	$LPI_{w10} = \left(\sum_{i=1}^n w_i p_i^{10} \right)^{1/10}$			Table 3, Column 11
5	Root Mean Square Additive Form	$LPI_{rm} = \left(\sum_{i=1}^n \frac{1}{n} p_i^2 \right)^{1/2}$	[11,31]	Exhibits small ambiguity problems.	Table 3, Column 12

Tbale 1: Continued

Eq. No.	Aggregation function	Function expression	Users	Specific remarks	Reported Table/column
6	Weighted Root Sum Square Aggregation Function	$LPI_{wr} = \frac{\left(\sum_{i=1}^n w_i p_i^2\right)^{0.5}}{\sum_{i=1}^n w_i}$	[31,33]	Exhibits small eclipsing problems.	Table 3, Column 13
7	Maximum Operator Function	$LPI_{wr} = \max. \{p_1, p_2\}$	[18, 31, 32]	No eclipsing problem but exhibit ambiguity for large number of variables; suitable for aggregation of air pollution ; limited application for water quality indices.	Table 3, Column 14
8	Ambiguity And Eclipsity Free Aggregation Function	$LPI_{aef} = \sum_{i=1}^n \left(p_i^{1/r}\right)^r$	[31, 32]	Eclipsity and ambiguity free function, limited application for air pollution indices; minimal ambiguity for $r=0.4$.	
8(a)	LPI _{2.5} Ambiguity and Eclipsity Free Aggregation Function	$PLI_{2.5} = \sum_{i=1}^n (p_i^{2.5})^{0.4}$			Table 3, Column 15
9	Weighted Ambiguity and Eclipsity Free Aggregation Function	$LPI_{w2.5} = \left(\sum_{i=1}^n w_i p_i^{2.5}\right)$	[31]	Eclipsity & Embiguity free function; limited application for leachate pollution indices.	Table 3, Column 16
10	Multiplicative Aggregation Function	$LPI_{wm} = \prod_{i=1}^n p_i w_i$	[17,31,32, 34,35]	Nonlinear; ambiguity free but exhibits eclipsing at low weights and increasing scale indices; insensitive when applied to large number of variables.	Table 3, Column 17
11	Unweighted Multiplicative Aggregation Function	$LPI_{um} = \left(\prod_{i=1}^n p_i\right)^{1/n}$	[23, 31, 36]	Exhibits small eclipsity problem, applied for comparison purposes only.	Table 3, Column 18
12	Geometric Aggregation Function	$LPI_{gm} = \left(\prod_{i=1}^n p_i^{g_i}\right)^{1/\gamma}$	[17]	Nonlinear; ambiguity free but exhibits eclipsing at low weights and increasing scale indices; insensitive when applied to large number of variables.	

Table 3: LPI values for leachate characteristics of landfill lysimeter, using different Aggregation Functions

Pollutant	w _i	Conc.	p _i	LPI _{min}	LPI _{max}	LPI ₂	LPI ₄	LPI ₁₀	LPI ₂₅	LPI ₅₀	LPI ₇₅	LPI _{max}	LPI _{2.5}	LPI _{w2.5}	LPI _{wm}	LPI _{lm}		
Cr	0.064	1.75	6.5	6.5	0.416	42.25	1.79E+03	1.35E+08	1.14E+02	8.62E+06	42.25	2.7	6.5	1.08E+02	6.89E+00	1.13	6.5	
Pb	0.063	0.68	9	9	0.567	81	6.56E+03	3.49E+09	4.13E+02	2.20E+08	81	5.1	9	2.43E+02	1.53E+01	1.15	9	
COD	0.062	8425	70	70	4.34	4900	2.40E+07	2.82E+18	1.49E+06	1.75E+17	4900	303.8	70	4.10E+04	2.54E+03	1.30	70	
Hg	0.062	0.4	59	59	3.658	3481	1.21E+07	5.11E+17	7.51E+05	3.17E+16	3481	215.8	59	2.67E+04	1.66E+03	1.29	59	
BOD ₅	0.061	1398	35	35	2.135	1225	1.50E+06	2.76E+15	9.15E+04	1.68E+14	1225	74.73	35	7.25E+03	4.42E+02	1.24	35	
As	0.061	0.01	5.5	5.5	0.335	30.25	9.15E+02	2.53E+07	5.58E+01	1.55E+06	30.25	1.85	5.5	7.09E+01	4.33E+00	1.11	5.5	
CN	0.058	1.3	10	10	0.58	100	1.00E+04	1.00E+10	5.80E+02	5.80E+08	100	5.8	10	3.16E+02	1.83E+01	1.14	10	
Phenol	0.057	4	8.5	8.5	0.484	72.25	5.22E+03	1.97E+09	2.98E+02	1.12E+08	72.25	4.1	8.5	2.11E+02	1.20E+01	1.13	8.5	
Zn	0.056	1.3	5	5	0.28	25	6.25E+02	9.77E+06	3.50E+01	5.47E+05	25	1.4	5	5.59E+01	3.13E+00	1.09	5	
pH	0.055	7.3	6	6	0.33	36	1.30E+03	6.05E+07	7.13E+01	3.33E+06	36	1.98	6	8.82E+01	4.85E+00	1.10	6	
TKN	0.053	3000	98	98	5.194	9604	9.22E+07	8.17E+19	4.89E+06	4.33E+18	9604	509.0	98	9.51E+04	5.04E+03	1.28	98	
Ni	0.052	0.23	8	8	0.416	64	4.10E+03	1.07E+09	2.13E+02	5.58E+07	64	3.33	8	1.81E+02	9.41E+00	1.11	8	
TCB	0.052	8000	92	92	4.784	8464	7.16E+07	4.34E+19	3.73E+06	2.26E+18	8464	440.13	92	8.12E+04	4.22E+03	1.27	92	
NH ₄ -N	0.051	1300	100	100	5.1	10000	1.00E+08	1.00E+20	5.10E+06	5.10E+18	10000	510	100	1.00E+05	5.10E+03	1.26	100	
TDS	0.05	12540	28	28	1.4	784	6.15E+05	2.96E+14	3.07E+04	1.48E+13	784	39.2	28	4.15E+03	2.07E+02	1.18	28	
Cu	0.05	0.98	7	7	0.35	49	2.40E+03	2.82E+08	1.20E+02	1.41E+07	49	2.45	7	1.30E+02	6.48E+00	1.10	7	
Cl	0.049	3597	30	30	1.47	900	8.10E+05	5.90E+14	3.97E+04	2.89E+13	900	44.1	30	4.93E+03	2.42E+02	1.18	30	
Fe	0.045	82	9.5	9.5	0.427	90.25	8.15E+03	5.99E+09	3.67E+02	2.69E+08	90.25	4.1	9.5	2.78E+02	1.25E+01	1.11	9.5	
Total	1.00				587	32.267	39948	3.03E+08	2.28E+20	1.61E+07	1.19E+19	39948	2169.6	100	3.62E+05	1.95E+04	18.04	4.79E+22
LPI value					32.61	32.27	199.87	131.93	108.61	63.36	80.82	47.11	46.53	100	167.30	52.05	18.04	18.20

Note: All the values are in mg/L except for pH and TCB (cfu/100ml).

Table 4: Leachate pollution index (LPI) values for treated leachate using different aggregation functions

Pollutant	w _i	Conc. ^b	p _i	LPI _{min}	LPI _{max}	LPI ₂	LPI ₄	LPI ₁₀	LPI ₂₅	LPI ₅₀	LPI ₇₅	LPI _{max}	LPI _{2.5}	LPI _{w2.5}	LPI _{wm}	LPI _{lm}		
Cr	0.06	2	9	9	0.58	81	6.56E+03	3.49E+09	4.20E+02	2.23E+08	81	5.18	9	243.00	15.55	1.15	9	
Pb	0.06	0.1	5	5	0.32	25	6.25E+02	9.77E+06	3.94E+01	6.15E+05	25	1.58	5	55.90	3.52	1.11	5	
COD	0.06	250	10	10	0.62	100	1.00E+04	1.00E+10	6.20E+02	6.20E+08	100	6.20	10	316.23	19.61	1.15	10	
Hg	0.06	0.01	6	6	0.37	36	1.30E+03	6.05E+07	8.04E+01	3.75E+06	36	2.23	6	88.18	5.47	1.12	6	
BOD ₅	0.06	30	6	6	0.37	36	1.30E+03	6.05E+07	7.91E+01	3.69E+06	36	2.20	6	88.18	5.38	1.12	6	
As	0.06	0.2	5	5	0.31	25	6.25E+02	9.77E+06	3.81E+01	5.96E+05	25	1.53	5	55.90	3.41	1.10	5	
CN	0.06	0.2	6	6	0.35	36	1.30E+03	6.05E+07	7.52E+01	3.51E+06	36	2.09	6	88.18	5.11	1.11	6	
Phenol	0.06	1	5	5	0.29	25	6.25E+02	9.77E+06	3.56E+01	5.57E+05	25	1.43	5	55.90	3.19	1.10	5	
Zinc	0.06	5	6	6	0.34	36	1.30E+03	6.05E+07	7.26E+01	3.39E+06	36	2.02	6	88.18	4.94	1.11	6	
pH	0.06	5.5-9	5	5	0.28	25	6.25E+02	9.77E+06	3.44E+01	5.37E+05	25	1.38	5	55.90	3.07	1.09	5	
TKN	0.05	100	6	6	0.32	36	1.30E+03	6.05E+07	6.87E+01	3.20E+06	36	1.91	6	88.18	4.67	1.10	6	
Ni	0.05	3	10	10	0.52	100	1.00E+04	1.00E+10	5.20E+02	5.20E+08	100	5.20	10	316.23	16.44	1.13	10	
TCB	0.05	3 ^a	10	10	0.52	100	1.00E+04	1.00E+10	5.20E+02	5.20E+08	100	5.20	10	316.23	16.44	1.13	10	
NH ₄ -N	0.05	50	7	7	0.36	49	2.40E+03	2.82E+08	1.22E+02	1.44E+07	49	2.50	7	129.64	6.61	1.10	7	
TDS	0.05	2100	7	7	0.35	49	2.40E+03	2.82E+08	1.20E+02	1.41E+07	49	2.45	7	129.64	6.48	1.10	7	
Cu	0.05	3	18	18	0.90	324	1.05E+05	3.57E+12	5.25E+03	1.79E+11	324	16.20	18	1374.62	68.73	1.16	18	
Cl	0.05	1000	8	8	0.39	64	4.10E+03	1.07E+09	2.01E+02	5.26E+07	64	3.14	8	181.02	8.87	1.11	8	
Fe	0.05	100 ^a	7	7	0.32	49	2.40E+03	2.82E+08	1.08E+02	1.27E+07	49	2.21	7	129.64	5.83	1.09	7	
Total	1.00				136	7.47	1196	1.62E+05	3.61E+12	8.40E+03	1.81E+11	1196	64.61	18	3800.76	203.34	7.05	2.16E+15
LPI value					7.56	7.47	34.58	2.01E+01	1.80E+01	9.57E+00	1.34E+01	8.15	8.03	18	27.04	8.38	7.05	7.11

Note: All the values are in mg/L except for pH and TCB (cfu/100ml).

In fact, the data set considered is not for the treated leachate of any municipal landfill site, but it is for the maximum permissible discharge limits for the various pollutant variables according to Indian regulations, assuming these to be the characteristics of the treated leachate. The LPI for treated leachate using all the aggregation functions are computed and summarized in Table 4.

RESULTS AND DISCUSSIONS

The LPI values for the landfill lysimeter calculated using all the aggregation methods explained earlier. Figure 1 shows the LPI values for raw and treated leachate of active landfill sites in of New Delhi, India. The LPI values of the treated leachate are also plotted in Figure 2.

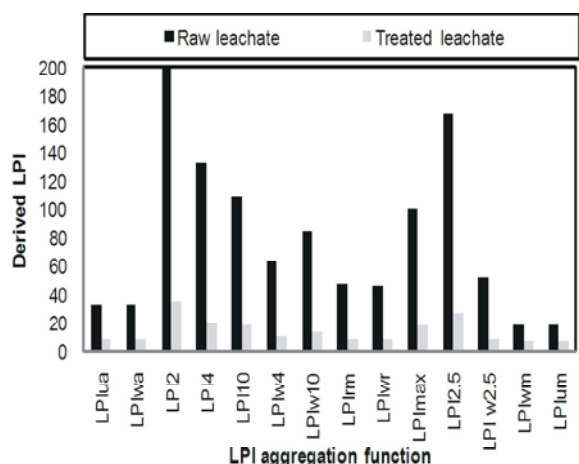


Fig. 1: LPI values of raw and treated leachate of active landfill site in New Delhi, India, using different aggregation methods where series 1 indicates Raw leachate and series 2 indicates Treated Leachate.

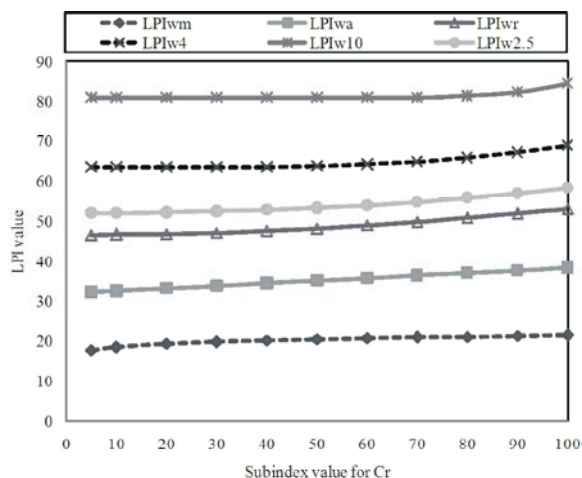


Fig. 2: Sensitivity of weighted additive, weighted square root, weighted ambiguity eclipsity free, weighted fourth root, weighted tenth root and weighted multiplicative aggregation function with respect to changes in sub-index of chromium.

The LPI values calculated using various aggregation methods indicate that root sum power, 4th root power, 10th root power and the calculated LPI values exceed the maximum reported individual pollutant sub-index value. All these values also exceed the theoretical range of LPI, i.e., 5-100. However, the 10th root power aggregation function results in the least ambiguous results, followed by the 4th root, the ambiguity and eclipsity free (2.5th root) and the root sum square aggregation functions.

The maximum operator aggregation function does not show ambiguity of results, but it cannot be used for calculating the LPI values, as its results cannot be used to compare the fine gradations of leachate pollution. The results of the two multiplicative aggregation functions, the weighted multiplicative aggregation function and the unweighted multiplicative aggregation function, indicate high eclipsing of the data. The two LPI values for these aggregation functions are 22.63 and 22.35, respectively. The values are very low as compared to the other additive form aggregation functions.

The unweighted additive form and root mean square additive form aggregation functions suffer from the drawback that the weight of the variables are not considered and all the variables are assumed to be of the same weight. Though the weighted linear sum aggregation functions also suffer from the eclipsity problem, the eclipsity produced is small, as the number of the variables included in the aggregation function is large. The eclipsity problem is associated with this aggregation function when the dichotomous state of the index is to be reported, which is not the case here. The weighted sum aggregation, weighted root mean square, weighted 2.5th root, weighted 4th root, weighted 10th root and weighted multiplicative aggregation functions, however, fulfill other criteria, such as weights of all the pollutant variables being considered and are easy to ascertain. The sensitivity of these six aggregation function to the changes of the individual pollutant is further conducted to select the best possible aggregation function.

Sensitivity Analysis: The sensitivity analysis of the six aggregation functions, which takes into consideration the weights attached to the pollutants, with respect to the change in strength or concentration of two pollutants is performed independently. The six aggregation functions selected for the sensitivity analysis are:

- Weighted sum additive aggregation function, LPI_{wa}
- Wighted root sum square aggregation function, LPI_{wr}
- Weighted 2.5throot (ambiguity and eclipsity free) aggregation function, LPI_{w2.5}
- Weighted 4th root sum aggregation function, LPI_{w4}
- Weighted 10th root sum aggregation function, LPI_{w10}
- Weighted multiplicative aggregation function, LPI_{wm}

The two pollutants selected are chromium, which is the most significant variable and thus has the highest “weight” value and total iron which is least significant pollutant with the lowest “weight” value.

Table 5: Sensitivity Analysis Results of Selected Aggregation Functions for Change in Sub-index Values of Chromium and Total Iron from 5 to 100

No.	Aggregation function	Changes in LPI for Cr (%)	Changes in LPI for Fe (%)
1	Weighted sum additive aggregation function, LPI_{wa}	18.89	13.31
2	Weighted multiplicative aggregation function, LPI_{wm}	21.18	14.48
3	Wighted root sum square aggregation function, LPI_{wr}	13.76	0.09
4	Weighted 2.5 th root (ambiguity and eclipsity free) aggregation function, $LPI_{w2.5}$	11.99	8.63
5	Weighted 4 th root sum aggregation function, LPI_{w4}	8.73	6.35
6	Weighted 10 th root sum aggregation function, LPI_{w10}	4.42	3.25

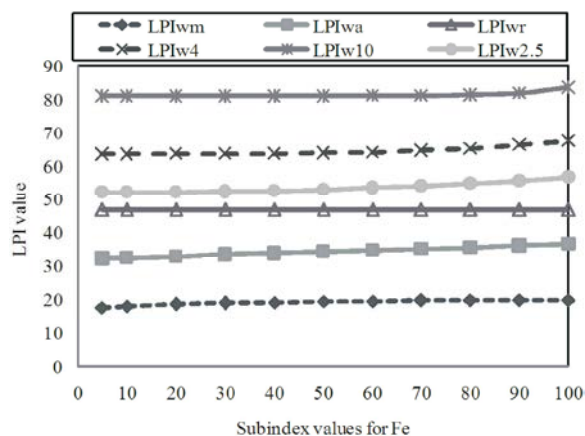


Fig. 3: Sensitivity of weighted additive, weighted square root, weighted ambiguity eclipsity free, weighted fourth root, weighted tenth root and weighted multiplicative aggregation function with respect to changes in sub-index of total Iron.

It is hypothetical to assume that all the leachate pollutant variables included in LPI for a particular landfill will take on the highest possible sub-index value of 100. Therefore, it is felt more practical to study the sensitivity analysis on actual landfill leachate data. For performing the sensitivity analysis, the sub-index value of chromium is varied from 5 to 100 in the same data set for the landfill lysimeter and the LPI using these six aggregation functions are calculated. The variations in the LPI of this six aggregation functions with respect to the change in the sub-index of chromium are shown in Figure 2.

The variations of the LPI values for all six aggregation functions with respect to the change in the sub-index value of total iron from 5 to 100 for the same data set are shown in Figure 3. The calculated percentage variation of the LPI values over the minimum value for the sub-index variation of chromium and total iron are shown in Table 5.

From Table 5, it can be concluded that the weighted multiplicative aggregation function is the most sensitive one in comparison to other aggregation functions, showing change in LPI values of 21.18 and 14.48% for

chromium and total iron, respectively. The next most sensitive aggregation function is the weighted linear sum aggregation function, which shows 18.89 and 13.31% variation in LPI values for the two pollutants, followed by the weighted root square, weighted 4th root and weighted 10th root aggregation function. The weightedth10 root aggregation function is least sensitive to the sub-index changes of chromium and total iron.

Further, the behavior of LPI_{w10} , LPI_{w4} , $LPI_{w2.5}$ and LPI_{wr} values with respect to the changes in sub-index values of chromium and total iron, shown in Figures 2 and 3, clearly indicates that these aggregation functions are least sensitive to changes in the sub-index values, particularly for the lower values of the sub-index. The LPI_{w10} value remains almost constant for a change in the sub-index value of chromium from 5 to 70, while the LPI_{w4} value remains almost constant for a change in the sub-index value of chromium from 5 to 60. Similarly, the $LPI_{w2.5}$ and LPI_{wr} are also insensitive for the changes in the sub-index value of chromium from 5 to 40. The behavior of these aggregation functions with respect to the changes in the sub-index values of total iron is similar. Therefore, these aggregation functions may not be useful when the fine gradation in leachate pollution of different landfill sites spread over a given area is to be compared.

Though the sensitivity analysis shows that the variation of LPI_{wm} values for the change in sub-index values of chromium and total iron is highest, it suffers from the drawback that the function is nonlinear. Figures 2 and 3 indicate that the LPI_{wm} values hardly change when the sub-index values of chromium and total iron vary from 50 to 100, but the changes in LPI_{wm} values is more rapid when the sub-index value changes from 5 to 50. Thus the LPI_{wm} curve does not represent the change in sub-index values as effectively as LPI_{wa} . Moreover, the weighted multiplication aggregation function shows far higher eclipsity as compared to the weighted linear sum aggregation function and thus may not be the most appropriate aggregation function for calculating LPI.

The variation in LPI_{wa} values is comparatively sensitive and linear to changes in the sub-index value of chromium and total iron throughout their range. Thus it can be concluded that the weighted linear sum aggregation function is the most appropriate aggregation function for calculating the leachate pollution index.

CONCLUSIONS

From analysis of obtained data it was concluded that:

- The unweighted linear sum, unweighted multiplicative, root sum square, root mean square, ambiguity and eclipsity free, fourth root and tenth root sum aggregation functions are not suitable for aggregating sub-indices, as these aggregation functions do not take into consideration the importance / significance of all the variables and assume that all the pollutant variables have some importance.
- The square root, ambiguity and eclipsity free (2.5^{th} root), 4^{th} and 10^{th} root additive form aggregation functions also produce ambiguous results.
- The maximum operator aggregation function is ambiguity and eclipsity free, but it cannot be used as an aggregation function for LPI, as it is least sensitive to fine gradations of changes in leachate pollution.
- The two multiplicative aggregation functions (the unweighted aggregation function and the weighted aggregation function) produce highly eclipsed results.
- The weighted square root, weighted ambiguity and eclipsity free, weighted fourth root and weighted tenth root aggregation functions are insensitive and nonlinear to variations of individual pollutants.
- Though the weighted multiplicative aggregation function is most sensitive to the changes in pollutant concentration, is nonlinear and shows biased results for higher sub-index values.
- Although the weighted linear sum does cover an underestimation region, it is less than that of the weighted multiplicative aggregation function.
- The weighted sum aggregation function is more parsimonious than the weighted multiplicative aggregation function.
- Hence, it can be concluded that the linear weighted sum aggregation method is the most suitable aggregation function for estimation of the most suitable aggregation function for estimation of the Leachate Pollution Index.

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