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A SHORT NOTE ON FUZZY ALMOST RESOLVABLE AND FUZZY ALMOST IRRESOLVABLE SPACES

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Abstract. In this paper we study several characterizations of fuzzy almost resolvable and fuzzy almost irresolvable spaces and the conditions under which a fuzzy almost resolvable space becomes a fuzzy σ-first category space and a fuzzy first category space, are investigated. The interrelation between fuzzy almost irresolvable and fuzzy σ-second category space is also studied.

1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by L. A. Zadeh [**11**] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. This inspired mathematicians to fuzzify mathematical structures. The concepts of fuzzy topology was defined by C. L. Chang [**3**] in the year 1968. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics. E. Hewitt [**5**] introduced the concepts of resolvability and irresolvability in topological spaces. A. G. El'Kin [**4**] introduced open hereditarily irresolvable spaces in classical topology. The concept of almost resolvable spaces was introduced by Richard Bolstein [**6**] as a generalization of resolvable spaces of E. Hewitt. The concept of almost resolvable spaces in fuzzy setting was introduced and studied by the authors in [**10**]. In this paper several characterizations of fuzzy almost resolvable and fuzzy almost irresolvable spaces are studied and the relations

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between fuzzy almost resolvable, fuzzy almost irresolvable spaces and fuzzy σ-first category spaces and fuzzy Baire spaces are also investigated.

2. Preliminaries

DEFINITION 2.1. *Let λ and μ be any two fuzzy sets in a fuzzy topological space (X,T). Then we define*

(i) $\lambda \vee \mu : X \to [0,1]$ *as follows :* $(\lambda \vee \mu)(x) = Max\{\lambda(x), \mu(x)\}$ *where* $x \in X$,
(ii) $\lambda \wedge \mu : X \to [0,1]$ *as follows :* $(\lambda \wedge \mu)(x) = Min\{\lambda(x), \mu(x)\}$ *where* $x \in X$,
(iii) $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$ *where* $x \in X$.

More generally, for a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X,T), the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined respectively as $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

DEFINITION 2.2. *Let (X,T) be a fuzzy topological space. For a fuzzy set λ of X, the interior and the closure of λ are defined respectively as $int(\lambda) = \vee\{\mu/\mu \leqslant \lambda, \mu \in T\}$ and $cl(\lambda) = \wedge\{\mu/\lambda \leqslant \mu, 1 - \mu \in T\}$.*

LEMMA 2.1. [**1**] *Let λ be any fuzzy set in a fuzzy topological space (X,T). Then $1 - cl(\lambda) = int(1 - \lambda)$ and $1 - int(\lambda) = cl(1 - \lambda)$.*

DEFINITION 2.3. [**7**] *A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is., $cl(\lambda) = 1$.*

DEFINITION 2.4. [**7**] *A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is., $intcl(\lambda) = 0$.*

DEFINITION 2.5. [**2**] *Let (X,T) be a fuzzy topological space and λ be a fuzzy set in X. Then λ is called a fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, for each $\lambda_i \in T$.*

DEFINITION 2.6. [**2**] *Let (X,T) be a fuzzy topological space and λ be a fuzzy set in X. Then λ is called a fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, for each $1 - \lambda_i \in T$.*

DEFINITION 2.7. [**7**] *A fuzzy topological space (X,T) is called a fuzzy first category space if $\vee_{i=1}^{\infty}(\lambda_i) = 1$ where the fuzzy sets (λ_i)'s are fuzzy nowhere dense sets in (X,T). A topological space, which is not of fuzzy first category, is said to be of fuzzy second category.*

DEFINITION 2.8. [**9**] *Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy σ-nowhere dense set if λ is a fuzzy F_σ-set in (X,T) such that $int(\lambda) = 0$.*

DEFINITION 2.9. [**9**] *A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy σ-first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$ where (λ_i)'s are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set which is not of fuzzy σ-first category set in (X,T) is said to be of fuzzy σ-second category.*

LEMMA 2.2. [**2**] *For a family* $\mathcal{A} = \{\lambda_\alpha\}$ *of fuzzy sets of a fuzzy topological space* X, $\vee cl(\lambda_\alpha) \leqslant cl(\vee\lambda_\alpha)$. *In case* $\mathcal{A}$ *is a finite set,* $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. *Also* $\vee int(\lambda_\alpha) \leqslant int(\vee\lambda_\alpha)$, *where* $\alpha \in A$.

DEFINITION 2.10. [**8**] *Let* (X,T) *be a fuzzy topological space. Then* (X,T) *is called a fuzzy Baire space if* $int\big(\vee_{i=1}^{\infty}(\lambda_i)\big) = 0$, *where* (λ_i)*'s are fuzzy nowhere dense sets in* (X,T).

DEFINITION 2.11. [**2**] *A fuzzy topological space* (X,T) *is called a fuzzy submaximal space if for each fuzzy set* λ *in* (X,T) *such that* $cl(\lambda) = 1$, *then* $\lambda \in T$ *in* (X,T).

3. Fuzzy almost resolvable and fuzzy almost irresolvable spaces

DEFINITION 3.1. [**10**] *A fuzzy topological space* (X,T) *is called a fuzzy almost resolvable space if* $\vee_{i=1}^{\infty}(\lambda_i) = 1$, *where the fuzzy sets* (λ_i)*'s in* (X,T) *are such that* $int(\lambda_i) = 0$. *Otherwise* (X,T) *is called a fuzzy almost irresolvable space.*

THEOREM 3.1. [**8**] *If* λ *is a fuzzy nowhere dense set in* (X,T), *then* $1-\lambda$ *is a fuzzy dense set in* (X,T).

PROPOSITION 3.1. *If* $\vee_{i=1}^{N}(\lambda_i) = 1$, *where the fuzzy sets* (λ_i)*'s are fuzzy nowhere dense sets in a fuzzy topological space* (X,T), *then* (X,T) *is a fuzzy almost resolvable space.*

Proof. Let $\vee_{i=1}^{N}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s are fuzzy nowhere dense sets in a fuzzy topological space (X,T). Then $int\big(\vee_{i=1}^{N}(\lambda_i)\big) = int(1) = 1$. This implies that $1 - int[\vee_{i=1}^{N}(\lambda_i)] = 0$. Then we have $cl\big(\wedge_{i=1}^{N}(1-\lambda_i)\big) = 0 \longrightarrow (A)$. Since (λ_i)'s are fuzzy nowhere sets in (X,T), by theorem 3.1, $(1-\lambda_i)$'s are fuzzy dense sets in (X,T). Let (μ_i)'s ($i = 1$ *to* ∞) be fuzzy dense sets in (X,T) in which the first N fuzzy sets be $1-\lambda_i$. Then $\wedge_{i=1}^{\infty}(\mu_i) \leqslant \wedge_{i=1}^{N}(1-\lambda_i) \leqslant cl\big(\wedge_{i=1}^{N}(1-\lambda_i)\big)$. Then we have from (A), $\wedge_{i=1}^{\infty}(\mu_i) \leqslant 0$. That is., $\wedge_{i=1}^{\infty}(\mu_i) = 0$. This implies that $1 - \wedge_{i=1}^{\infty}(\mu_i) = 1$. Then $\vee_{i=1}^{\infty}(1-\mu_i) = 1$. Now $cl(\mu_i) = 1$ implies that $int(1-\mu_i) = 0$. Let $\delta_i = 1-\mu_i$. Then we have $\vee_{i=1}^{\infty}(\delta_i) = 1$, where $int(\delta_i) = 0$. Therefore, (X,T) is a fuzzy almost resolvable space. □

PROPOSITION 3.2. *If each fuzzy set* λ_i *is a fuzzy* F_σ*-set in a fuzzy almost resolvable space* (X,T), *then* $\wedge_{i=1}^{\infty}(1-\lambda_i) = 0$, *where* $(1-\lambda_i)$*'s are fuzzy dense sets in* (X,T).

Proof. Let (X,T) be a fuzzy almost resolvable space. Then, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s in (X,T) are such that $int(\lambda_i) = 0$. This implies that $1 - [\vee_{i=1}^{\infty}(\lambda_i)] = 0$ and $1 - int(\lambda_i) = 1$. Then, $\wedge_{i=1}^{\infty}(1-\lambda_i) = 0$ and $cl(1-\lambda_i) = 1$. Since the fuzzy sets (λ_i)'s in (X,T) are fuzzy F_σ-sets, $(1-\lambda_i)$'s are fuzzy G_δ-sets in (X,T). Hence we have $\wedge_{i=1}^{\infty}(1-\lambda_i) = 0$, where $(1-\lambda_i)$'s are fuzzy dense and fuzzy G_δ-sets in (X,T). □

THEOREM 3.2. [**8**] *Let* (X,T) *be a fuzzy topological space. Then the following are equivalent :*

(1) (X,T) *is a fuzzy Baire space.*

(2) $int(\lambda) = 0$ *for every fuzzy first category set* λ *in* (X, T).
(3) $cl(\mu) = 1$ *for every fuzzy residual set* μ *in* (X, T).

THEOREM 3.3. [**10**] *If the fuzzy topological space* (X, T) *is a fuzzy Baire space, then* (X, T) *is a fuzzy almost irresolvable space.*

The following proposition establishes under what conditions a fuzzy Baire space becomes a fuzzy almost resolvable space.

PROPOSITION 3.3. *If* $\wedge_{i=1}^{\infty}(\lambda_i) = 0$, *where the fuzzy sets* (λ_i)*'s are fuzzy residual sets in a fuzzy Baire space* (X, T), *then* (X, T) *is a fuzzy almost resolvable space.*

Proof. Let (X, T) be a fuzzy Baire space in which $\wedge_{i=1}^{\infty}(\lambda_i) = 0$, where the fuzzy sets (λ_i)'s are fuzzy residual sets in (X, T). Since (X, T) is a fuzzy Baire space and (λ_i)'s are fuzzy residual sets in (X, T), by theorem 3.2, $cl(\lambda_i) = 1$. Then, $1 - cl(\lambda_i) = 0$ and hence $int(1 - \lambda_i) = 0$. Now, $\wedge_{i=1}^{\infty}(\lambda_i) = 0$, implies that $\vee_{i=1}^{\infty}(1 - \lambda_i) = 1$. Let $1 - \lambda_i = \mu_i$. Hence $\vee_{i=1}^{\infty}(\mu_i) = 1$, where $int(\mu_i) = 0$. Therefore (X, T) is a fuzzy almost resolvable space. □

PROPOSITION 3.4. *If* $\vee_{i=1}^{\infty}(\lambda_i) = 1$, *where the fuzzy sets* (λ_i)*'s are non-zero fuzzy open sets in a fuzzy topological space* (X, T), *then* (X, T) *is a fuzzy almost irresolvable space.*

Proof. Suppose that $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s are non-zero fuzzy open sets in (X, T). Since (λ_i)'s are non-zero fuzzy open sets $int(\lambda_i) = \lambda_i \neq 0$. Hence we have $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where $int(\lambda_i) \neq 0$ for all $i = 1$ *to* ∞. Therefore (X, T) is a fuzzy almost irresolvable space. □

THEOREM 3.4. [**8**] *If* λ *is a fuzzy dense and fuzzy open set in a fuzzy topological space* (X, T), *then* $1 - \lambda$ *is a fuzzy nowhere dense set in* (X, T).

PROPOSITION 3.5. *If each fuzzy* G_δ*-set is fuzzy open and fuzzy dense set in a fuzzy topological space* (X, T), *then* (X, T) *is a fuzzy almost irresolvable space.*

Proof. Let λ be a fuzzy G_δ-set in (X, T) such that λ is a fuzzy dense and fuzzy open in (X, T). Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where the fuzzy sets (λ_i)'s in (X, T) are fuzzy open in (X, T). Now $1 - \lambda = 1 - [\wedge_{i=1}^{\infty}(\lambda_i)]$. Then, $cl(1 - \lambda) = cl\big(1 - [\wedge_{i=1}^{\infty}(\lambda_i)]\big) = cl\big(\vee_{i=1}^{\infty}(1 - \lambda_i)\big)$ and hence $cl(1 - \lambda) = cl\big(\vee_{i=1}^{\infty}(1 - \lambda_i)\big) \geqslant \vee_{i=1}^{\infty} cl(1 - \lambda_i)$. Since (λ_i)'s are fuzzy open sets in (X, T), $(1 - \lambda_i)$'s are fuzzy closed sets in (X, T) and hence $cl(1 - \lambda_i) = 1 - \lambda_i$. Then $cl(1 - \lambda) \geqslant \vee_{i=1}^{\infty}(1 - \lambda_i)$. Now $cl(1 - \lambda) \geqslant \vee_{i=1}^{\infty}(1 - \lambda_i)$, implies that $intcl(1 - \lambda) \geqslant int[\vee_{i=1}^{\infty}(1 - \lambda_i)] \longrightarrow (1)$. Since $\vee_{i=1}^{\infty} int(1 - \lambda_i) \leqslant int[\vee_{i=1}^{\infty}(1 - \lambda_i)]$, we have $intcl(1 - \lambda) \geqslant \vee_{i=1}^{\infty} int(1 - \lambda_i) \longrightarrow (2)$.

Since λ is a fuzzy dense and fuzzy open set in (X, T), by theorem 3.4, the fuzzy set $1 - \lambda$ is a fuzzy nowhere dense set in (X, T). Then, we have $intcl(1 - \lambda) = 0$. Hence, from (2), $0 \geqslant \vee_{i=1}^{\infty}[int(1 - \lambda_i)]$. That is., $\vee_{i=1}^{\infty}[int(1 - \lambda_i)] = 0$. Then, $int(1 - \lambda_i) = 0$, for each $i = 1$ *to* ∞. Hence $intcl(1 - \lambda_i) = 0$, (since $cl(1 - \lambda_i) = 1 - \lambda_i$). This implies that $(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X, T). From (1) , we have $0 \geqslant int[\vee_{i=1}^{\infty}(1 - \lambda_i)]$. That is., $int[\vee_{i=1}^{\infty}(1 - \lambda_i)] = 0$. Hence (X, T) is a fuzzy Baire space. Therefore, by theorem 3.3, (X, T) is a fuzzy almost irresolvable space. □

PROPOSITION 3.6. *If $cl\big(\wedge_{i=1}^{\infty}(\lambda_i)\big) = 1$, where the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in a fuzzy submaximal space (X, T), then (X, T) is a fuzzy almost irresolvable space.*

Proof. Suppose that $cl\big(\wedge_{i=1}^{\infty}(\lambda_i)\big) = 1$, where the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in a fuzzy submaximal space (X, T). Now $cl\big(\wedge_{i=1}^{\infty}(\lambda_i)\big) = 1$ implies that $int\big(\vee_{i=1}^{\infty}(1-\lambda_i)\big) = 0$. Since (λ_i)'s are fuzzy dense in a fuzzy submaximal space, (λ_i)'s are fuzzy open sets in (X, T) and hence $(1-\lambda_i)$'s are fuzzy closed sets in (X, T). Then $cl(1-\lambda_i) = 1-\lambda_i$. Since (λ_i)'s are fuzzy dense in (X, T), $cl(\lambda_i) = 1$. Then $1 - cl(\lambda_i) = 0$ and hence $int(1-\lambda_i) = 0$. Now $intcl(1-\lambda_i) = int(1-\lambda_i)$ implies that $intcl(1-\lambda_i) = 0$. Then $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Hence we have, $int\big(\vee_{i=1}^{\infty}(1-\lambda_i)\big) = 0$ where $(1-\lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Then (X, T) is a fuzzy Baire space and then by theorem 3.3, (X, T) is a fuzzy almost irresolvable space. □

PROPOSITION 3.7. *If each fuzzy set is a fuzzy F_σ-set in a fuzzy almost resolvable space (X, T), then (X, T) is a fuzzy σ-first category space.*

Proof. Let (X, T) be a fuzzy almost resolvable space such that each fuzzy set in (X, T) is a fuzzy F_σ-set. Since, (X, T) is a fuzzy almost resolvable space, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s in (X, T) are such that $int(\lambda_i) = 0$. By hypothesis, (λ_i)'s are fuzzy F_σ-sets in (X, T). Hence the fuzzy sets (λ_i)'s in (X, T) are fuzzy F_σ-sets such that $int(\lambda_i) = 0$ $(i = 1 \ to \ \infty)$. This implies that (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Thus, we have, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Therefore (X, T) is a fuzzy σ-first category space. □

PROPOSITION 3.8. *If $clint(\lambda) = 1$, for each fuzzy dense set λ in a fuzzy almost resolvable space (X, T), then (X, T) is a fuzzy first category space.*

Proof. Let (X, T) be a fuzzy almost resolvable space such that $clint(\lambda) = 1$, for each fuzzy dense set λ in (X, T). Since (X, T) is fuzzy almost resolvable, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s in (X, T) are such that $int(\lambda_i) = 0$. Now $1 - int(\lambda_i) = 1$, implies that $cl(1-\lambda_i) = 1$. Then, by hypothesis, $clint(1-\lambda_i) = 1$, for the fuzzy dense set $1-\lambda_i$ in (X, T). This implies that $intcl((\lambda_i) = 0$. Hence $((\lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Therefore $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s are fuzzy nowhere dense sets in (X, T), implies that (X, T) is a fuzzy first category space. □

PROPOSITION 3.9. *If $clint(\lambda) = 1$ for each fuzzy dense set (X, T) is a fuzzy almost resolvable space (X, T), then (X, T) is not a fuzzy Baire space.*

Proof. Let (X, T) be a fuzzy almost resolvable space such that $clint(\lambda) = 1$, for each fuzzy dense set λ in (X, T). Then by proposition 3.8, $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where the fuzzy sets (λ_i)'s are fuzzy nowhere dense sets in (X, T), implies that (X, T) is a fuzzy first category space. This implies that $int\big(\vee_{i=1}^{\infty}(\lambda_i)\big) = int(1) = 1 \neq 0$. Hence (X, T) is not a fuzzy Baire space. □

THEOREM 3.5. [**9**] *If the fuzzy topological space (X, T) is a fuzzy almost irresolvable space, then (X, T) is a fuzzy σ-second category space.*

PROPOSITION 3.10. *If the fuzzy topological space (X, T) is a fuzzy σ-second category space, then (X, T) is a fuzzy almost irresolvable space.*

Proof. Let (X, T) be a fuzzy σ-second category space. Then, $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where the fuzzy sets (λ_i)'s ($i = 1$ *to* ∞) are fuzzy σ-nowhere dense sets in (X, T). Since (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T) , (λ_i)'s are fuzzy F_σ-sets in (X, T) such that $int(\lambda_i) = 0$. Hence we have $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where $int(\lambda_i) = 0$. Therefore (X, T) is a fuzzy almost irresolvable space. □

REMARK 3.1. In view of theorem 3.5 and proposition 3.10, we have the following: "A fuzzy topological space (X, T) is a fuzzy almost irresolvable space if and only if (X, T) is a fuzzy σ-second category space".

References

[1] Azad, K.K., *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl., **82** (1981), 14-32.

[2] Balasubramanian, G., *Maximal fuzzy topologies*, Kybernetika, **(5)** (1995), 459-464.

[3] Chang, C.L., *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182-190.

[4] El'Kin, A.G., Ultrafilters and undecomposable spaces, Vestnik Mosk. Univ. Mat., **24**(5) (1969), 51-96.

[5] Hewitt, E., *A problem of set-theoretic topology*, Duke Math. J., **10** (1943), 309-333.

[6] Richard Bolstein, *Sets of points of discontinuity*, Proc. Amer. Math. Soc., Vol. **38**. No. 1, (March 1973), 193-197.

[7] Thangaraj, G., Balasubramanian, G., *On somewhat fuzzy continuous functions*, J. Fuzzy Math., Vol. **11**, No. 2,(2003), 725-736.

[8] Thangaraj, G., Anjalmose, S., *On fuzzy Baire spaces*, J. Fuzzy Math., Vol. **21**, No. 3, (2013), 667-676.

[9] Thangaraj, G., Poongothai, E., *On fuzzy σ-Baire spaces*, Inter. J. Fuzzy Math. Sys., Vol. **3**, No. 4 (2013), 275-283.

[10] Thangaraj, G., Vijayan, D., *On fuzzy almost resolvable and fuzzy almost irresolvable spaces*, Inter. J. Statistika and Mathmatika, Vol. **9**, No. 2, (2014), 61-65.

[11] Zadeh, L.A., *Fuzzy sets*, Information and Control, Vol. **8** (1965), 338-353.

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