International Journal of Quality research
UDK – 681.2.088
Scientific Rewiew Paper (1.02)

Case Studies Concerning the Bad Influence of the Limit Measurement Error on Control Objectivity

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Abstract: *When the total measurement limit error goes beyond 20% of the prescribed tolerance for the parameter to be checked here is the risk accepting some parts which in reality are waste and there is the risk of rejecting some other parts which in fact are good. These risks, take place with a certain probability, according to the ration between the measurement limit error and the tolerance prescribed for the parameter to be checked. This paper make an deep analyze related to the dependence of the control uncertainty degree on the asymmetry* α_d *of the 6σ- errors' distribution compared to tolerance.*

Keywords: *design, uncertainty, 6σ, mechanical measurement, control devices, dynamics of machine*

1. GENERAL VIEWPOINTS CONCERNING CONTROL UNCERTAINTY

Measurement uncertainty reflects the impossibility of exactly knowing the true value of dimensions through measurement or control operations.

Uncertainty is defined through the value dimension which includes, with a certain probability, the true value of the dimension to be checked. Therefore, the measurement uncertainty is caused by measurement errors $\pm\Delta L$ and it can be determined on the basis of statistical calculations (Sturzu, Popescu 2002) .

The true value (C_i) of a dimension can be known on the basis of many measurements. It will be approximately equal to the $\overline{X}$ -weighted mean of the values ($C_{i\ ef}$) acquired through the repeated measurement of the respective dimension In. this case, the measurement uncertainty $6\sigma_U$ will be:

$$6\sigma_U \cong 2\Delta L \tag{1}$$

In what concerns the interpretation of the measurement result (accepting or rejecting the result of the measurement), two bad situations can occur. During the control process, when limit error of casual measurement $\pm\Delta L$ is present, two disadvantageous situations can occur:

- **the risk of accepting as accurate, some parts which in reality are outside the prescribed tolerance t_p,** which will disadvantage the beneficiary. We can call this case: "*the beneficiary's risk*";
- **the risk of rejecting other parts which may be accurate in reality,** which will disadvantage the producer . We can call this case: *" the producer's risk"*.

This means that, when error $\pm\Delta L$ is present, the control process will have some degree of uncertainty, bringing disadvantageous functional and economical implications both for the producer and the beneficiary.

Consequently, in order to correctly evaluate the manufacturing precision parameters, it is obligatory that the uncertainty degree of the control is known and compared to the admissible uncertainty degree for the respective case. In our opinion, this justifies developing a methodology and a generalized determination relation of $\pm \Delta L$ with direct application, both for conceiving, and analyzing the control methods.

2. CASE STUDY REGARDING THE INFLUENCE OF THE LIMIT MEASUREMENT ERROR ON CONTROL OBJECTIVITY

When the total measurement limit error $\pm \Delta L$ goes beyond 10 … 20% of the prescribed tolerance t_p for the parameter to be checked, there is the risk accepting some parts which, in reality, are waste, and there is the risk of rejecting some other parts which, in fact, are good.

These risks take place with a certain probability, according to the ratio between the measurement limit error and the tolerance prescribed for the parameter to be checked. The value of the mentioned probabilities also depends on the distribution type of the manufacturing or measurement errors, and the asymmetry of the manufacturing errors' distribution compared to the prescribed tolerance t_p.

If we consider as example the dispersion range 6σ of the manufacturing errors equals the prescribed tolerance t_p for the parameter to be checked, and the control means is considered with a measurement limit error $\pm \Delta L$, the situation looks like in figure1.

2.1. The influence of $\pm \Delta L$ at the inferior limit L_i of the tolerance

This case is well represented in the scheme from the figure1. On this scheme, we put together and we have done the correlation between the characteristic elements for the checked parts, like the tolerance t_p, having L_i and L_s the inferior and superior tolerance's limits, the values' dispersion field 6σ with respect of $\overline{X}$, and the characteristic elements for the control process, like measurement limit error $\pm \Delta L$. The true value of a dimension is noted as C_i (C'_i, C''_i) and the values acquired through the measurement of the respective dimension, is noted as $C_{i\,ef}$; the i subscript having values :1,2, 3, 4...

- *Case study:* $C_{1ef} = L_i$

When checked, the parts having the dimension C_{1ef} equal with the inferior limit L_i of tolerance t_p will be accepted, even though some of them, when error $-\Delta L$ is present, may have dimension $C'_1 < L_i$, being in fact parts that are in reality waste.

At the same time, due to error $+\Delta L$, parts may have dimension $C''_1 > L_i$, but they may be better parts than the measurement showed;

- *Case study:* $C_{2ef} < L_i$

In the case of inferior limit, when checked, the parts having $C_{2ef} < L_i$ will be rejected, although, when errors $\pm \Delta L$ are present, they may be accurate ($C''_2 = L_i$) or they may be worse ($C'_2 << L_i$), under the influence of the measurement error $-\Delta L$.

2.2. The influence of $\pm \Delta L$ at the superior limit L_s of the tolerance

- *Case study:* $C_{3ef} = L_s$

The parts having when checked the dimension $C_{3ef} = L_S$ are accepted, although some of them, when error $\pm \Delta L$ is present, may have a dimension $C''_3 > L_S$, being a waste in reality. Some of these parts may have dimension $C'_3 < L_S$, being better than it was determined.

- *Case study:* $C_{4ef} > L_s$

Parts having the dimension $C_{4ef} > L_S$ are rejected, although, when incidental error $-\Delta L$ appears, they may have in reality $C'_4 = L_S$, being good parts, and when error $+\Delta L$ appears, they may have in reality $C''_4 >> L_S$, being worse manufactured than the check up showed.

Consequently, based on what we mentioned above, the parts having when

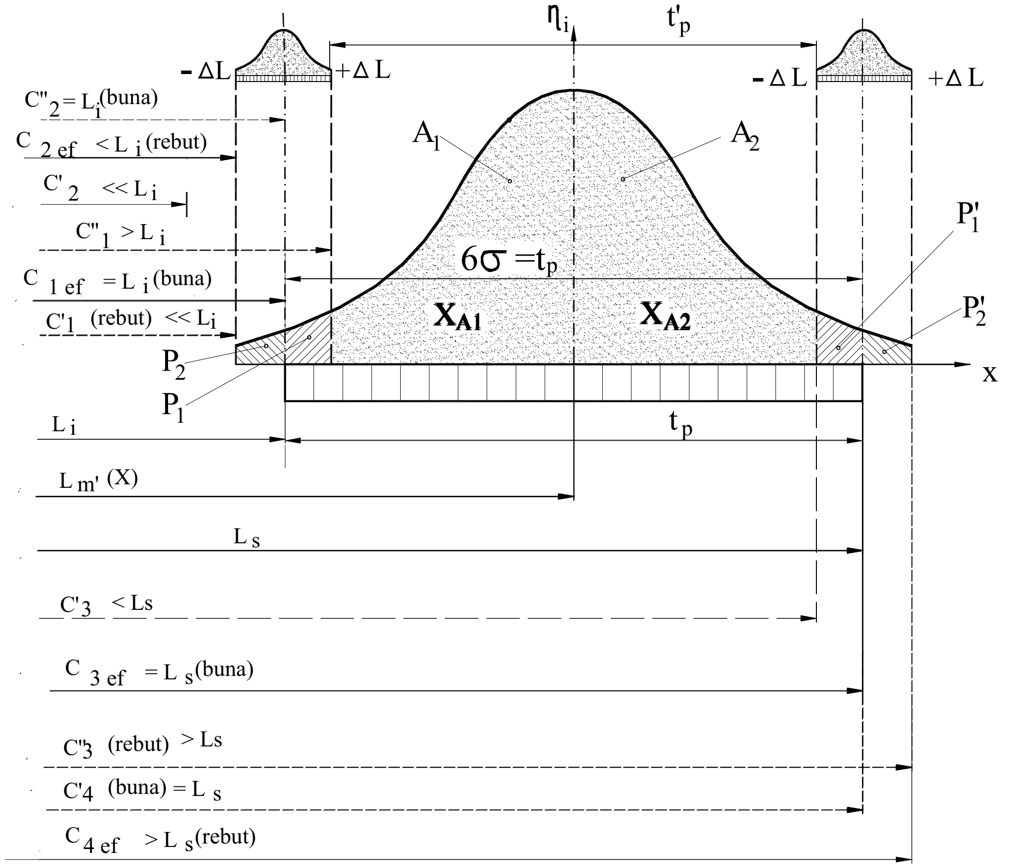

Figure 1. Dependence of the control uncertainty on limit measurement error

checked values within segments P_1 and P'_1 will be accepted, although, in reality, when incidental measurement error $\pm\Delta L$ is present, some of them may be waste, and the parts having when checked values within segments P_2 and P'_2 will be rejected, although, in reality, when incidental measurement error $\pm\Delta L$ is present, some of them may be good parts (with values within segments P_1 and P'_1).

2.3. Calculus for the probabilities of control uncertainties due to limit measurement error

Segments P_1, P'_1, P_2 *and* P'_2 also represent at the same time the probabilities of control uncertainties due to limit measurement error. These probabilities are determined through the following relation:

$$P_1 + P_2 = 0.5C_d - A_1 \qquad (2)$$

where: C_d – represents the distribution curve value equal to 1 or 100% and A_1 – represents the area below the distribution curve, unaffected by the measurement error $\pm\Delta L$, area which can be determined through the integration of the function:

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i - \overline{X})^2}{2\sigma^2}} \qquad (3)$$

where: x_i – represents the incidental measurement to be studied; $\overline{X}$ – represents the weighted mean of the dimension x_i; σ – represents the average square deviation of the values compared to $\overline{X}$; e – is the natural logarithmic base (e = 2.718).

The calculation relations of the main

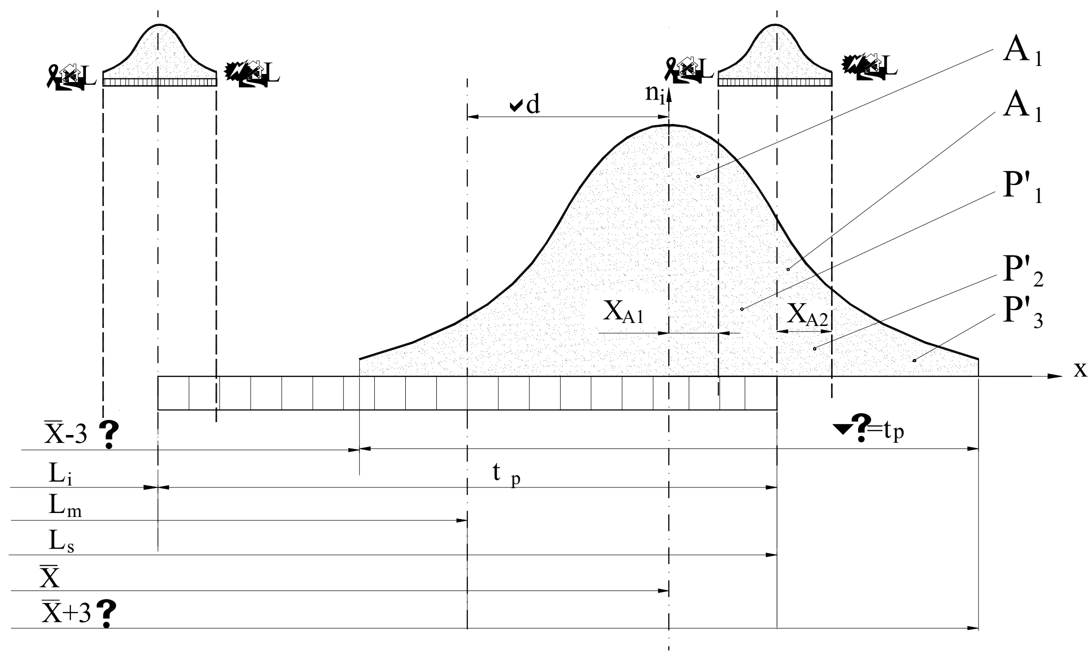

Figure2. Dependence of the control uncertainty degree on the asymmetry α_d of the manufacturing errors' distribution compared to tolerance t_p.

parameters for the manufacturing errors' distribution, $\overline{X}$ and σ, are:

$$\overline{X} = \sum x_i \frac{n_i}{N} \tag{4}$$

$$\sigma = \sqrt{\sum (x_i - \overline{X})^2 \frac{n_i}{N}} \tag{5}$$

where: n_i – the absolute frequency on the value intervals for the incidental measurement to be studied (in other words, n_i represents the number of cases which favours the occurrence of an incident on the considered interval); N – total number of cases; n_i/N – represents the probability of the expected event's occurrence within the considered interval.

On the basis of what we showed, area A_1 is determined by the following relation:

$$A_1 = \int_{L_i+\Delta L}^{\overline{X}} y dx = \frac{1}{\sqrt{2\pi}} \int_{L_1+\Delta L}^{\overline{X}} e^{-\frac{x_{A_1}^2}{2\sigma^2}} \frac{dx_{A_1}}{\sigma} \tag{6}$$

Relation (6) shows that the value of the integral depends on the $\frac{x_{A_1}}{\sigma}$ ratio, which can be considered the argument of a $\Phi(Z_{A_1})$ function. Consequently, we can change the variable:

$$Z_{A_1} = \frac{x_{A_1}}{\sigma} = \frac{\overline{X} - (L_i + \Delta L)}{\sigma} \tag{7}$$

Accordingly, relation (6) becomes a $\Phi(Z_{A_1})$ function, meaning $A_1 = f(Z_{A_1})$. Relation (6) becomes:

$$A_1 = \Phi(Z_{A_1}) = \frac{1}{\sqrt{2\pi}} \int_0^{Z_{A_1}} e^{-\frac{Z_{A_1}^2}{2}} dZ_{A_1} \tag{8}$$

The values for $\Phi(Z_{A_1})$ function are presented in table (Dumitras, Popescu, Bendic,1997), for known values of Z_{A_1} argument, according to relation (7).

On the basis of the above mentioned, relation (2) becomes:

$$P_1 + P_2 = 0.5 - \Phi(Z_{A_1}) \tag{9}$$

Similarly, we can easily determine probabilities $P'_1 + P'_2$ (see figure1):

$$P'_1 + P'_2 = 0.5 - A_2 = 0.5 - \Phi(Z_{A_2}) \tag{10}$$

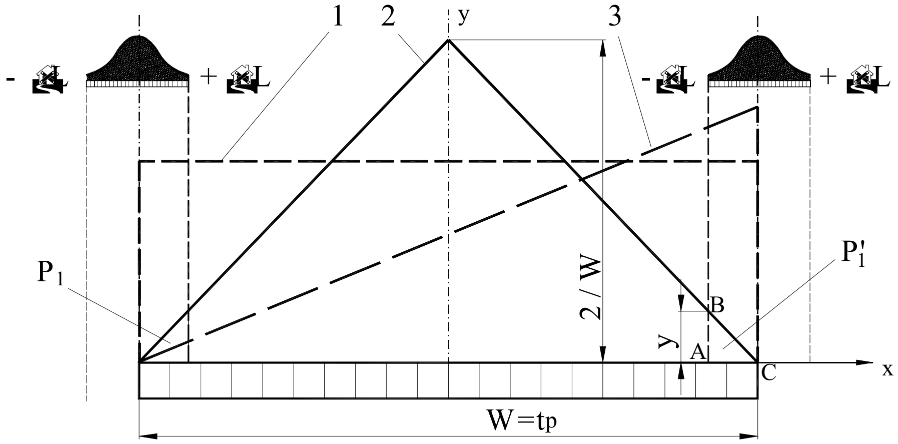

Figure 3. Dependence of the control uncertainty degree on the distribution type

Argument of $\Phi(Z_{A_2})$ function, as for $\Phi(Z_{A_1})$ function, will be:

$$Z_{A_2} = \frac{x_{A_2}}{\sigma} = \frac{L_S - \Delta L - \overline{X}}{\sigma} \qquad (11)$$

Z_{A_1} concrete value is easily determined from table $\Phi(Z_{A_2})$ in the specialised literature ((Dumitras, Popescu, Bendic,1997). This value is inserted in relation (10) and, by multiplying the result by 100, we will obtain $P'_1+P'_2$ expressed in percents.

Taking into account the placement pattern of the dispersions characteristic for the measurement errors $(\pm\Delta L)$ and manufacturing errors (6σ), we consider relations (9) and (10) better expressed as:

$$P_1 + P_2 = \frac{0.5 - \Phi(Z_{A_1})}{2} \qquad (12)$$

respectively:

$$P'_1 + P'_2 = \frac{0.5 - \Phi(Z_{A_2})}{2} \qquad (13)$$

The control process is considered correct from precision point of view only if:

$$(P_1 + P_2 + P'_1 + P'_2) \le (10...20)\% t_p \qquad (14)$$

According to figure 2, it results that the probabilities of control uncertainty will also depend on the asymmetry α_d of the manufacturing errors' distribution compared to tolerance t_p, and according to figure 3 they will also depend on the distribution type, numbered in the chart: 1.- rectangular distribution, with equal probability, situation in which probabilities P_1, P_2, P'_1 and P'_2 will have higher values than the normal distribution; 2.- triangular distribution (isosceles triangle) with uniform ascending probability - the Simpson distribution, when P_1, P_2, P'_1 and P'_2 are approximately equal, as in the case of normal distribution; 3.- right-angled triangle distribution (uniform ascending distribution), when probabilities P_1, P_2, P'_1 and P'_2 are higher than the normal distribution, but lower than the rectangular distribution.

Besides the normal distribution and the abnormal distributions mentioned above, both in the specialised literature, and in practice, we can find other distributions such as trapeze, Maxwell or apparently abnormal distributions.

In the case of abnormal distributions, the control uncertainty probabilities are determined by taking into account the error size $\pm\Delta L$ and the shape of the portion under the distribution curve situated on the influence area of the measurement error. For example, in case of distribution nr. 2, in the shape of an isosceles triangle (Simpson distribution) in figure 3, probabilities P'_1 şi P'_2 are according to the "*y*" ordinate and to the $2\Delta L$, from the resemblance of triangles ABC:

$$\frac{y}{\frac{2}{W}} = \frac{\Delta L}{\frac{t_p}{2}}; \quad y = \frac{\Delta L \cdot \frac{2}{W}}{\frac{t_p}{2}} = \frac{4\Delta L}{W \cdot t_p} \tag{15}$$

where, for $W = t_p$ relation (15) becomes:

$$y = \frac{4\Delta L}{t_p^2} \tag{16}$$

Probability P'_1, (figure 3) will be equal to the ABC triangle area:

$$P'_1 = \frac{y \cdot \Delta L}{2} = \frac{2\Delta L^2}{W \cdot t_p} \tag{17}$$

On symmetry reasons, $P_1 = P'_1$.

In the situation of the non-symmetry of the manufacturing distributions compared to tolerance t_p, probability P_1 or P'_1 will correspondently increase, according to the situation. We can similarly determine the control uncertainty for other abnormal distributions of the manufacturing error within or outside t_p.

3. CONCLUSIONS

In the case that the relation (14) is not respected, the control precision is not compatible to tolerance t_p and, consequently, the control has no objectivity - instead of informing, it will misinform.

In this situation, the following arguments can be phrased:

- *Case 1: the rejected parts disadvantage the producer and advantage the beneficiary*.

In this situation, it can be recommended for the rejected parts which have values close to the prescribed tolerances to be checked again, using more precise methods and means;

- *Case 2: the parts accepted when the measurement error ±ΔL is present, will disadvantage the beneficiary.*

In reality, some of this accepted parts, may be waste. In this situation, it is recommended that these parts are checked again using more precise methods and means so that the beneficiary may avoid losses for the company from Taguchi's concept.

Also these analised cases may lead to an inappropriate fit forming between machine parts. This is also the cause of unbalancing for rotating devices, as turbine, compressors, pumps elements, where the centrifugal force effects have bad influence on the global dynamics and performance (Senatore, 2006).

Non-compliance with this recommendation may lead to losses both for the client or losses for the company, and also to damaging the producer's image on the market or even the total loss of the market, which may even cause bankruptcy.

Following the demands imposed by the Quality Concepts, presented, and in order to eliminate the mentioned risks, we recommend the following:

- Accepting to be checked only the parts situated within the tolerance $t'_p < t_p$ presented in figure1, having the value:

$$t'_p = t_p - 2\,\Delta L \tag{18}$$

Solving the above mentioned problems will have as a consequence raising manufacturing costs, according to the chart of manufacturing cost variation compared to tolerance (Shoemaker 1988).(Popescu 2002);

- The rejected parts can be checked again with a more precise (trustworthy) control means, which increases the costs for the control works;

- The beneficiary should check his products with his own control methods and means which must have the accurate control precision for the respective target. The renowned companies, which collaborate with other companies, strictly follow this rule because they do not want to take any risks in what concerns ensuring the quality of their products (they do not buy and do not assemble anything on mere trust);

- Using from the very beginning the control methods and means capable to satisfy the demands of relation (14) and accepting 2 – 3% of parts which do not respect tolerance t_p. This approach can be taken only following some quality – price analysis achieved through Value Engineering methods and only with the beneficiary's approval (Sturzu, Popescu 2002).

Another condition which must be fulfilled when analysing these cases is that the control productivity must be higher that the manufacturing productivity and, at the same time, the control costs must not exceed the manufacturing costs.

The general methodology and the general relations for calculating error ΔLt make possible the rational interpretation of all the phenomena which appear during the control process, thus laying the theoretical foundations for conception, choosing and correct destination of control equipments.

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Received: 25.08.2006 Accepted: 07.11.2006 Open for discussion: 1 Year